Information Acquisition and the Equilibrium Incentive Problem

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Abstract

I study the optimal incentive provision in a principal-agent relationship with costly information acquisition by the agent. I emphasize that adverse selection or moral hazard is an endogenous choice of the principal via inducing or deterring information acquisition. The principal designs the contract not only to cope with an existing incentive problem, but also to implement its presence. Implementation of adverse selection relies on a steeper information rent to the agent than the standard menu, so that he is motivated to distinguish the efficient state of nature from the inefficient. Moral hazard is implemented by replacing the benchmark debt contract with a debt-with-equity-share contract, so that the agent does not attempt to acquire information for neither avoiding debt nor rent extraction. With imperfect information acquisition or private knowledge of information acquiring cost, the contract offered to the uninformed agent is qualitatively robust, and that to the informed exhibits countervailing incentives.

Keywords: Incentive contract, Information acquisition, Adverse selection, Moral hazard, Countervailing incentives.

JEL Classification: D82, D83, D86

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1 Introduction

The incentive problems, adverse selection and moral hazard, have been the essence of standard agency theories, in which either or both are exogenously present. The exogeneity of incentive problems is attributed to the assumption that productive information structure is exogenous.\textsuperscript{1} Information, however, often realizes as a result of an endogenous and costly activity of acquisition, in the following spirit of Arrow.

\textit{A key characteristic of information costs is that...they typically represent an irreversible investment...I am thinking of the need for having made an adequate investment of time and effort to be able to distinguish one signal from another.} (Arrow, 1974: 39)

With the information structure being endogenous, so is the underlying incentive problem in the principal-agent relationship. Consider a principal contracting with an agent protected by limited liability, and both players are risk neutral. The principal’s revenue is generated by the agent’s hidden productive effort, whose productivity depends on the stochastic state of nature. The agent can acquire information on the realized state of nature at a sunk cost. The principal may thus implement adverse selection with a contract that induces the agent to acquire information, so that information is asymmetric and the agent manipulates the output with productive effort to communicate the state of nature to the principal. The principal may also implement moral hazard with a contract that deters the agent from acquiring information, in which scenario the principal and the agent are symmetrically informed, and the publicly observed output is an imperfect measurement of the agent’s private effort. The conventional theories of incentive discussed the optimal contract to cope with an existing incentive problem; in addition to which, I study how it is designed to implement the presence of the incentive problem.

The emergence of the incentive problem and its interaction with the principal’s information management has only received attention at one end: endogenous information acquisition which generates adverse selection.\textsuperscript{2} Moral hazard as a consequence of

\begin{itemize}
\item \textsuperscript{1}Consider a principal contracting with an agent to execute a project that yields output to the principal, which depends on the agent’s private productive effort and stochastic productive state of nature. Adverse selection arises when the agent has private information on the productive state of nature, while the principal observes only its stochastic distribution. The agent then manipulates output through private effort to communicate such information to the principal. Moral hazard, on the other hand, arises when the productive effort is imperfectly measured by the output since the productive state of nature is imperfectly and symmetrically observed.
\item \textsuperscript{2}Please refer to Lewis and Sappington (1997), Crémér, Khalil, and Rochet (1998a), and Terstiege (2012) in the literature review for more details.
\end{itemize}
deterrence of information acquisition has been, to the best of my knowledge, absent from contract theory. I fill this gap by investigating how information management interacts with the equilibrium incentive problem, and how the optimal contract is modified from the benchmark second best in response to such interaction.

Two information acquiring technologies are examined. Perfect information acquisition is defined in the sense that the correct signal on the productive state of nature is available to the agent as long as he acquires it. Imperfect information acquisition refers to an information acquiring effort that improves the probability for the agent to observe the correct signal on the realized state of nature, and no signal otherwise. With perfect information acquisition, the equilibrium incentive problem is deterministic; either adverse selection or moral hazard is implemented. With imperfect information acquisition, the equilibrium incentive problem is stochastic, whose density is implemented by the principal through inducing information acquisition to a specific precision.

To implement adverse selection through inducing perfect information acquisition, the principal offers a menu contract that motivates the agent to distinguish the efficient states of nature from the inefficient, as well as to reveal the truth. In this contract, a higher (lower) output than the second best is specified for a sufficiently efficient (inefficient) state of nature, to implement a steeper rent than its second best counterpart.

To implement moral hazard through deterring information acquisition, the principal must deter the agent’s opportunistic motives to acquire information off the equilibrium path. Given the second best debt contract, the agent attempts to acquire information in order to distinguish a sufficiently inefficient state of nature to avoid the debt by rejecting the contract, and to discover a relatively efficient state of nature to extract maximal rent. The optimal contract is thus characterized by a downward distortion of debt from its second best and a lower equity share of output residual to the agent, to restrict his ability to extract rent by acquiring information. The former implies a larger output residual, which motivates productive effort in equilibrium, whereas the latter discourages it. This results in an upward distortion of productive effort from the second best with sufficiently large cost of information acquisition, and a downward distortion otherwise. Deterrence of information acquisition is complementary to higher-powered incentive for sufficiently large cost of information acquisition.

3Contracting with a risk neutral agent protected by limited liability, the second best contract with the presence of moral hazard is a debt contract, which prescribes a debt paid to the principal, leaving the agent the claimant of output residual. Please refer to Innes (1990) for the pioneer treatment, and Poblete and Spulber (2012) with a weaker assumption.
which is different from Crémer, Khalil, and Rochet (1998a), who did not introduce moral hazard in production when information is deterred.

The key tradeoff behind the decision to induce or to deter information acquisition, to implement adverse selection or moral hazard at the production stage, involves rent and efficiency. The agent’s acquisition of information benefits the principal as it allows for more efficient production, yet an information rent is given to induce hidden information acquiring effort and truthful revelation. For sufficiently small cost of information acquisition, it is optimal to induce information acquisition and implement adverse selection as the improvement in efficiency exceeds the net information rent; for sufficiently costly information acquisition, it is optimal to deter information acquisition and implement moral hazard as the improvement in efficiency falls short of the net information rent.

Consider in a firm-employee relationship, inducing information acquisition to implement adverse selection is optimal if the agent is an “expert” in the field, who is able to acquire productive information at a lower cost. Deterring information acquisition to implement moral hazard is optimal if the agent is a “mediocre,” who acquires productive information at a higher cost. Applying to investment banking, the investment bank (principal) finds it optimal to induce the funds-seeking firm (agent) to conduct costly market investigation and reveal its finding through a menu of funding options if the market is well-established and sufficiently transparent, and finds it optimal to deter the private firm from conducting costly market investigation with a single debt-with-equity-share contract if the firm participates in a newly-established market in which past data is limited. The next question is then whether the contracts above is robust to imperfect information acquisition (e.g. there is a positive probability that the agent firm finds nothing from the market investigation), or to private knowledge of information acquiring cost (e.g. the agent employee knows if he is an expert or a mediocre).

The optimal debt-with-equity-share contract to deter information acquisition is qualitatively robust to imperfect information acquisition, as well as to private knowledge of the information acquiring cost. The main difference to perfect information acquisition with common knowledge of information acquiring cost is that the principal does not know perfectly upon offering the contract whether the agent is informed of the state of nature or not, which itself is an information advantage of the agent. The optimal contracts under these two remedies are thus designed such that the informed agent truthfully reveals being informed, and vice versa for the uninformed. The additional incentive compatibility constraints distort the contract designed to the
uninformed agent towards the same direction as does the constraint to deter perfect information acquisition with common knowledge of information acquiring cost. This is intuitive as the agent’s opportunistic motive to acquire information still remains.

The optimal menu contract designed to the informed agent, however, exhibits pooled output menu over some intermediate states of nature, which is absent in the optimal menu contract to induce perfect information acquisition with common knowledge of information acquiring cost. This is due to the technical resemblance of the truth telling constraints for the informed agent to the type-dependent participation constraints that generate countervailing incentives\(^4\), although the reservation utility is assumed to be identical across states of nature.

The paper is organized as the following. The model is outlined in Section 2. Given perfect information acquisition, I derive in Section 3 the optimal menu contract when information acquisition is induced, and Section 4 is devoted to the optimal contract to deter information acquisition. Optimal information management and equilibrium incentive problem in the contractual relationship is discussed in Section 5. I examine the robustness of the optimal contract with imperfect information acquisition in Section 6, and that with private knowledge of the cost of information acquisition in Section 7. The paper is concluded in Section 8.

1.1 Related Literature

Information acquisition in the environment with adverse selection has gained much attention in contract theory, and is roughly categorized into two forms of information acquisition: strategic and productive information gathering. The former refers to circumstances where information is realized at no cost at the production stage, but can be acquired at a cost ex ante to facilitate the agent’s decision on accepting the contract, which affects the form of the agent’s individual rationality but not truthful revelation of information. Crémer and Khalil (1992), Crémer, Khalil, and Rochet (1998b), and Szalay (2009) study this sort of information acquisition. As information would realize at the stage of production, information acquisition is only for strategic purpose and the incentive problem at the production stage is exogenously adverse selection.

I build my propositions on the latter form of information acquisition, which corresponds to situations where information is realized only if it is acquired at a cost.\(^4\)

Thus, information acquisition affects both participation and incentive compatibility of the agent. Lewis and Sappington (1997), Crémer, Khalil, and Rochet (1998a), Kessler (1998), Krähmer and Strausz (2011), Zermeño (2011), Terstiege (2012), and Hoppe and Schmitz (2013) fall into this category. They, however, either do not consider deterrence of information acquisition, or assume a deterministic output as a perfect measurement of the agent’s productive effort when information acquisition is deterred. Adverse selection endogenously arises as a consequence of inducing information acquisition, whereas moral hazard is assumed away. The interaction between deterring information acquisition and moral hazard is absent from the principal’s optimization program in these papers.

A considerable literature is also devoted to inducing information acquisition on productive noise in an environment with moral hazard and risk adverse agent, to explain the empirical puzzle that a higher-powered incentive is given under a riskier environment. Information on productive noise is assumed to be a mean-preserving imperfect signal that is unable to be communicated through a contract; truthful revelation is absent. Regardless of the level of information acquisition, the fundamental incentive problem is moral hazard. In this literature, a higher-powered incentive in a riskier environment is attributed to inducing information acquisition, implicitly implying a lower-powered incentive if information acquisition is deterred. With risk neutrality, I show in the current paper that higher-powered incentive to deter information acquisition may be optimal, provided that the cost of information acquisition is not too small. As an appendix, I also claim that deterring a risk-averse agent from information acquisition does not necessarily rely on a lower-powered incentive; it depends on the density of the state of nature.

The idea that information availability on state of nature distinguishes adverse selection from moral hazard is also emphasized by Sobel (1993) and Chu and Sappington (2009a). The former compares the principal’s payoff given various timing that information becomes available: pre-contract, post-contract prior to production, or after production. The latter develops a dynamic model in which information becomes available at an interim stage, before which, the incentive problem is due to hidden action, and asymmetric information thereafter. In both papers, however, information is not acquired by the agent, and the timing of information availability is exogenous, so does the underlying incentive problem. What the present paper contributes to the literature is the endogenous choice of the incentive problem through inducing/deterring

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information acquisition, which provides a refutable modification to the standard contracts.

Endogeneity of incentive problems due to productive information acquisition is also noticed by an independent work of Iossa and Martimort (2013). They study imperfect information acquisition in a similar fashion to mine in Section 6. Their propositions rely on the assumptions that i) the agent is pessimistic about his own information acquisition; ii) the transfer scheme is linear in output regardless of whether the agent is informed; iii) information is acquired after the contract is signed; iv) pooling is defined over the slope of transfer instead of output schedule. Thus, the main conclusions are drawn differently from mine. First, costly information acquisition itself does not distort the optimal contract and equilibrium productive effort, it is the joint effect with the agent’s pessimistic attitude that makes a difference. In contrast, I show that even with a neutral attitude of the agent, information management distorts the optimal contract from the second best provided that the cost of information acquisition is not too extreme. Second, I characterize the optimal contract without assuming linear transfer. Finally, they find pooling to be optimal in sufficiently inefficient states of natures instead of in the intermediate states of nature, due to the assumed linear transfer and a different definition of pooling.

2 Model

A principal hires an agent to execute a project that yields publicly observable and contractible output $q(e, \theta)$, depending on the agent’s privately observed productive effort ($e$) and the state of nature ($\theta$). Let $q_e(e, \theta) > 0^6$, $q_\theta(e, \theta) > 0$, $q_{ee}(e, \theta) < 0$, $q_{\theta\theta}(e, \theta) < 0$, and $q_{e\theta}(e, \theta) > 0$ for $(e, \theta) > (0, 0)$, i.e. the output function is concave in both effort and state of nature, and higher $\theta$ indicates a relatively efficient state of nature with higher total and marginal output. $\theta$ follows prior distribution $F(\theta)$ defined over $[0, \bar{\theta}]$. The agent’s cost of effort is given by the convex cost function $c(e)$, where $c(0) = 0$, $c_e(e) > 0$ and $c_{ee}(e) \geq 0$ for $e > 0$.

The principal and the agent are both risk neutral, with the principal’s payoff defined as the output net of contingent transfer specified in the contract, $u^P = q(e, \theta) - t(q(e, \theta))$, and the agent’s payoff defined as the contingent transfer net of cost of effort, $u^A = t(q(e, \theta)) - c(e)$. The agent is protected by limited liability.\textsuperscript{7}

Upon being offered a contract, the agent can invest effort $a$ in information acquisition.

\textsuperscript{6}Subscripts denote partial derivatives.

\textsuperscript{7}Contracting with a risk averse agent without limited liability is discussed in Appendix B.
Figure 1: Time-line

quisition, which allows him to observe the correct signal of the state of nature with probability $a \in [0, 1]$, or no signal otherwise, at a (sunk) non-monetary cost $d(\kappa, a)$, before accepting the contract, $\kappa$ being the cost parameter of information acquisition. The acquired information is private to the agent, as well as his information acquiring action, but the cost of information acquisition is common knowledge. The non-monetary sunk cost of information acquisition captures the characteristic of information acquisition as “an irreversible investment of the agent’s effort to distinguish one signal from another,” which is unconstrained by his limited liability.

I proceed with the case of perfect information acquisition, i.e. $a \in \{0, 1\}$. The agent knows the realized state of nature perfectly upon exerting information acquiring effort. This corresponds to the equilibrium information acquisition with $d(\kappa, a) = \kappa a$, the agent being risk neutral in information acquisition. I then discuss in Section 6 the implementation of imperfect information acquisition, when the optimal information acquiring effort is interior, given $d_a(\kappa, a) \geq 0$, with equality at $a = 0$, $d_{aa}(\kappa, a) > 0$, $d_a(\kappa, 1) \to \infty$.

The cost parameter of information acquisition, $\kappa$, can be interpreted accordingly in different applications of the model. For instance, in a firm-employee relationship, it represents the agent’s expertise in his field, with a lower cost corresponding to a higher level of expertise as the agent is able to distinguish between productive signals at a lower cost. In investment banking, it captures the cost of market investigation (or more broadly, due diligence), which depends on market transparency or availability of data and past experiences, where a lower cost may be due to a well-established market with high level of information transparency. In insurance market, such cost of information acquisition may reflect the cost of conducting genetic test or other health examination, which, comparing with identifying accident, is less costly to reach the same accuracy.

If the principal induces perfect information acquisition with the contract, the agent has private information on the state of nature prior to accepting the contract and there is no productive uncertainty. He communicates the acquired information to the
principal by producing a certain level of output specified in the menu contract. The incentive problem is due to ex-ante asymmetric information, the adverse selection. If the principal deters information acquisition with the contract, the principal and the agent have symmetric information, and the publicly observed output is an imperfect measurement of the agent’s hidden action. The incentive problem is then moral hazard. The time-line of the game is given by Figure 1.

To capture my main arguments, the following assumptions are made so that the moral hazard problem when information acquisition is deterred is relevant, and such assumptions do not affect the adverse selection problem when information acquisition is induced.

**Assumption 1.** \( q(e,0) \) is a constant, normalized to zero, e.g. \( q(e,\theta) = (m(\theta) - m(0))n(e) \), where \( n(0) = 0 \).

**Assumption 2.** \( \rho_1(e,\theta) \equiv \frac{f(\theta)}{1-F(\theta)} q_e(e,\theta) \) is increasing in \( \theta \) and \( \rho_2(e,\theta) \equiv \frac{f(\theta)}{F(\theta)} q_e(e,\theta) \) is decreasing in \( \theta \) for \( (e,\theta) > (0,0) \).

Assumption 1 is imposed to assume out moving support in lower realization of output when \( \theta \) is a stochastic variable at the production stage. Assumption 2 assumes a monotone hazard rate weighted by marginal rate of substitution in production. It guarantees a second best separating equilibrium in an adverse selection environment, as well as the optimality of a feasible debt contract with risk neutrality and limited liability if moral hazard is the underlying incentive problem.

### 3 Inducing Information Acquisition

Suppose perfect information acquisition, which allows the agent to distinguish the realized state of nature upon acquisition. Applying the Revelation Principle, a feasible contract to induce information acquisition and truthful revelation consists of a menu of options, \( \{t(\theta), q(\theta)\} \), in which the agent in state \( \theta \) accepts the contract (is individually rational), is incentive compatible not to produce \( q(\theta') \) for any \( \theta' \neq \theta \), and acquires information on the state of nature at a cost \( \kappa \) before acceptance. Let \( h(q(\theta'),\theta) \equiv e, \dot{c}(q(\theta'),\theta) \equiv c(h(q(\theta'),\theta)) \), where \( q(\theta') = q(e,\theta) \). Precisely,

\[
t(\theta) - \dot{c}(q(\theta),\theta) \geq 0, \quad \forall \theta \in [0,\bar{\theta}] \quad (IR_\theta),
\]

\[
\theta \in \arg \max_{\theta'} t(\theta') - \dot{c}(q(\theta'),\theta), \quad \forall \theta \in [0,\bar{\theta}] \quad (IC_\theta),
\]

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This is phrased as “critical ratio” in Poblete and Spulber (2012).
and
\[
\int_{0}^{\hat{\theta}} (t(\theta) - \hat{c}(q(\theta), \theta))dF(\theta) - \kappa \geq \max_{e} \int_{0}^{\hat{\theta}} (t(q(e), \theta))dF(\theta) - c(e) \quad (II).
\]

As the agent’s utility \( u^A(t(\theta'), q(\theta'), \theta) \) satisfies single crossing property\(^9\), \((IC_{\theta})\) can be replaced by local incentive compatibility and monotonicity constraints:
\[
t_\theta(\theta) - \hat{c}_q(q(\theta), \theta)q_\theta(\theta) = 0 \quad (LIC_{\theta})
\]
and
\[q_\theta(\theta) \geq 0 \quad (M).\]

The principal’s optimization program to induce information acquisition is thus
\[
\mathcal{P}_{II} : \max_{t(\theta), q(\theta)} \int_{0}^{\hat{\theta}} q(\theta) - t(\theta)dF(\theta)
\]
subject to
\[(IR_{\theta}), (LIC_{\theta}), (M), (II).\]

The optimal contract to induce information \( C_{II} = \{t_{II}(\theta), q_{II}(\theta)\} \) in comparison to the second best menu \( C_{SM} = \{t_{SM}(\theta), q_{SM}(\theta)\} \) is characterized in Proposition 1.

**Proposition 1.** For \( \kappa > \kappa^a \), if inducing information acquisition is optimal, \( C_{II} \) exhibits a higher-powered (lower-powered) incentive than \( C_{SM} \) in sufficiently efficient (inefficient) states, i.e. \( q_{II}(\theta) > q_{SM}(\theta) \) for \( \theta > \hat{\theta} \) and \( q_{II}(\theta) \leq q_{SM}(\theta) \) for \( \theta \leq \hat{\theta} \), where \( \hat{\theta} \) is the state of nature that the agent would have expected to reveal ex-ante if he did not acquire information.

**Proof.** Appendix A.1, or Crémé, Khalil, and Rochet (1998a) and Lewis and Sappington (1997) for the case where the stochastic state of nature is a cost parameter. \( \square \)

When information acquisition is induced, I am able to replicate the proposition of Crémé, Khalil, and Rochet (1998a) and Lewis and Sappington (1997). The principal implements additional (ex-ante) information rent when \((II)\) is violated at the second

\[g \frac{d(-\frac{q_{\theta}}{q_{e\theta}})}{d\theta} = \frac{d(c_{\theta}b_{\theta})}{d\theta} = \frac{1}{q_{e\theta}}(c_{\theta}q_{\theta} - c_{\theta}q_{e\theta}) < 0, \text{ i.e. marginal cost of output, relative to marginal utility of transfer, decreases in } \theta.\]
best, as if the agent is rewarded rent to utilize his expertise. The implemented information rent is steeper than its second best counterpart, to motivate the agent to distinguish the relatively efficient states of nature from the relatively inefficient.

4 Deterring Information Acquisition

A feasible contract to deter information acquisition and to implement productive effort $e$ prescribes a transfer contingent on output, $t(q)$, such that it satisfies the agent’s limited liability, that both players earn payoffs non-decreasing in output\textsuperscript{10}, that the agent is incentivized to exert productive effort $e$, and that he does not acquire information on the state of nature before acceptance.

If deterring information acquisition is optimal, the principal offers the contract which solves the following program subject to limited liability, non-decreasing payoff, incentive compatibility, and deterring information acquisition constraints.

$$\mathcal{P}_{DI} : \max_{t(q(e,\theta)), e} \int_0^{\hat{\theta}} q(e, \theta) - t(q(e, \theta)) dF(\theta)$$

subject to

$$t(q) \geq 0 \quad (LL),$$

$$0 \leq t_q(q) \leq 1 \quad (NDP),$$

$$e \in \arg \max_y \int_0^{\hat{\theta}} t(q(y, \theta)) - c(y) dF(\theta) \quad (IC),$$

$$\int_0^{\hat{\theta}} t(q(e, \theta)) - c(e) dF(\theta) \geq \int_0^{\hat{\theta}} 1_{\theta \geq \hat{\theta}} t(q(e(\theta), \theta)) - c(e(\theta)) dF(\theta) - \kappa \quad (DI),$$

where $e(\theta) \in \arg \max_y t(q(y, \theta)) - c(y)$ and $\hat{\theta}$ is such that $t(q(e(\hat{\theta}), \hat{\theta})) - c(e(\hat{\theta})) = 0$, i.e. for $\theta < \hat{\theta}$, the agent who acquired information off the equilibrium path finds it optimal to reject the contract.\textsuperscript{11}

Simply by the right-hand-side of (DI) one can have a glimpse of the agent’s opportunistic motives to acquire information off the equilibrium path: to distinguish a sufficiently inefficient state of nature to avoid exerting effort at a loss, and to discover

\textsuperscript{10}Defining feasible contract as satisfying limited liability and non-decreasing payoffs follows Innes (1990) and Poblete and Spulber (2012).

\textsuperscript{11}This is by the envelope theorem of the informed agent’s optimization problem off the equilibrium path.
a relatively efficient state of nature to extract maximal rent.

This section is devoted to the discussion on how deterring information acquisition interacts with the moral hazard problem in the contractual relationship, by characterizing the distortion of the optimal contract from the second best. In Section 4.1, I focus on a tractable example assuming a specific form of contingent transfer consisting of a debt and a share of output residual, and turn to the general case in Section 4.2. I assume in both sections risk neutrality, leaving the case with a risk averse agent to Appendix B.

4.1 Example

In the standard moral hazard problem with risk neutrality, limited liability, and non-decreasing payoff, the optimal second best contract is a debt contract, i.e. \( t(q) = \max\{q(e, \theta) - q, 0\} \), \( q \geq 0 \). In this subsection, I focus on a simplified example in which the contract to deter information acquisition, \( C^{DI} \), has a contingent transfer in the form \( t^{DI}(q) = T^{DI} + \max\{s^{DI}(e^{DI}, \theta) - q^{DI}, 0\} \), and discuss how deterring information acquisition modifies this contract from the second best debt contract \( C^{SD} \), in which \( t^{SD}(q) = \max\{q(e^{SD}, \theta) - q^{SD}, 0\} \), leaving a general contractual form to the next subsection.

Lemma 1. \( T^{DI} = 0 \).

**Proof.** If \((DI)\) is violated at the second best, \( T > 0 \) does not bind \((DI)\), as off the equilibrium path, the agent who acquires information can always accept the contract and exert any \( e \geq 0 \) to earn \( T \), i.e. regardless of whether acquiring information or not, the agent’s expected utility increases by \( T \). \( T < 0 \) violates \((LL)\) for \( q(e, \theta) < q \). \( \square \)

For the convenience of interpretation, I would phrase the simplified contract as a duo of debt \((q)\) and equity share of output residual \((s)\).

The production and cost functions being well-behaved, and the transfer being linear except at \( q \), the first order approach is valid. \((IC)\) can be replaced by local incentive compatibility without loss of generality,

\[
\int_2^\theta s q_e(e, \theta) dF(\theta) - c_e(e) = 0 \quad \text{\((LIC')\)}
\]

\(12\)Please refer to Innes (1990) and Poblete and Spulber (2012). The latter has shown the optimality of debt contract when \( \rho_1(e, \theta) = \frac{f(\theta)}{1-F(\theta)} \) is increasing in \( \theta \), under a similar model to this paper, without the possibility to acquire information.
where $\tilde{\theta}$ is such that $q(e, \tilde{\theta}) \equiv q$. ($NDP$) is expressed as

$$0 \leq s \leq 1 \quad (NDP').$$

**Lemma 2.** $\bar{q} \equiv q(e(\tilde{\theta}), \tilde{\theta})) > q$ and $\tilde{\theta} > \bar{\theta}$.

**Proof.** If $\bar{q} \leq q$, $t(\bar{q}) - c(e(\tilde{\theta})) = -c(e(\tilde{\theta})) < 0$, contradicting to the definition of $\tilde{\theta}$. Thus, $\bar{q} > q$. Off the equilibrium path, an informed agent exerts effort $e(\theta)$ where $sq(e(\theta), \theta) = c(e(\theta))$. An uninformed agent exerts effort $e^*$ such that $\int_{\bar{\theta}}^{\theta} sq(e^*, \theta)dF(\theta) = c_e(e^*)$. Let $\theta^0$ be such that $\int_{\bar{\theta}}^{\theta} sq(e^*, \theta)dF(\theta) = sq(e^*, \theta^0)$, i.e. $e^* = e(\theta^0)$. If $\theta^0 \leq \bar{\theta}$, $q(e(\theta^0), \theta^0) \leq q(e(\theta^0), \tilde{\theta}) = q$. Thus, by definition of $\tilde{\theta}$, $\theta^0 < \tilde{\theta}$ for all $\theta^0 \leq \bar{\theta}$, implying that $\tilde{\theta} < \bar{\theta}$. If $\theta^0 > \bar{\theta}$, $e(\theta^0) > e(\tilde{\theta})$; hence, $q(e(\theta^0), \tilde{\theta}) = q > q(e(\tilde{\theta}), \tilde{\theta})$, implying that $\tilde{\theta} < \bar{\theta}$. \qedsymbol

Lemma 2 gave a preliminary hint on one of the agent’s motives to acquire information off the equilibrium path: to distinguish a sufficiently inefficient state of nature to avoid the debt. It, along with Lemma 1, is applied to rewrite constraint ($DI$) into

$$\int_{\bar{\theta}}^{\theta} s(q(e, \theta) - q)dF(\theta) - c(e) \geq \int_{\bar{\theta}}^{\theta} s(q(e(\theta), \theta) - q) - c(e(\theta))dF(\theta) - \kappa \quad (DI').$$

The principal’s optimization program to deter a risk neutral agent from acquiring information with the simplified contract is thus reduced to

$$\mathcal{P}_{DI'}: \max_{s,q,e} \int_{0}^{\theta} q(e, \theta)dF(\theta) - \int_{\bar{\theta}}^{\theta} s(q(e, \theta) - q)dF(\theta)$$

subject to

$$(NDP'), (LIC'), (DI').$$

Characterization of the optimal simplified contract with binding constraint ($DI$) is given in Proposition 2 below.

**Proposition 2.** There exists $\kappa^s$ and $\kappa^d$, $\kappa^s \leq \kappa^d$, such that for $\kappa \in [\kappa^s, \kappa^d]$, the optimal contract to deter information acquisition is a debt contract which has a lower debt than the second best, $\frac{q^{DI}}{\bar{q}^{SD}} < \frac{s^{DI}}{s^{SD}} = 1$; for $\kappa < \kappa^s$, the optimal contract to deter information acquisition is a debt-with-equity-share contract, in which $\frac{q^{DI}}{\bar{q}^{SD}} < \frac{q^{DI}}{\bar{q}^{SD}}$ and $s^{DI} < s^{SD} = 1$, if $\int_{\bar{\theta}}^{\theta} q(e^{SD}, \theta) - s^{SD}dF(\theta) < \int_{\bar{\theta}}^{\theta} q(e^{SD}, \theta) - s^{SD}dF(\theta)^{13}$, where

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13 The agent’s expected full residual when uninformed is smaller than that when informed, given the second best debt contract, i.e. information is valuable to the agent. For instance, a Cobb-Douglas production function with uniformly distributed state of nature would satisfy this.
θ_{SD} is such that \( q(e_{SD}, θ_{SD}) \equiv q_{SD} \). (Illustrated in Figure 2)

Proof. Given Lemma 2, lowering the debt \( q < q^{SD} \) increases expected output residual more significantly on the equilibrium path than it does off the equilibrium path. Suppose that \( s = 1 \), for sufficiently small \( κ \) such that \( q \) is arbitrarily close to zero to deter information acquisition, the principal earns arbitrarily close to nothing. Lowering \( s \) gives the principal a positive share of a smaller expected output. The complete proof is in Appendix A.2.

When the second best contract violates constraint \((DI)\), there are two opportunistic motives of the agent to acquire information: to discover a sufficiently inefficient state of nature to avoid the debt, and to distinguish a relatively efficient state of nature to extract maximal rent by exerting \( e(θ) \). The former motive is relaxed by lowering the debt from the second best, and the latter by reducing the equity share of output residual to the agent to restrict his rent extracting ability.

For sufficiently large cost of information acquisition, \( κ^a < κ < κ^q \), lowering the debt has a second-order effect to the principal’s payoff, whereas lowering the equity share of output residual has a first-order effect. The optimal contract to deter information acquisition is a debt contract yet with a lower debt than the second best. This is as if the principal trades off a larger output residual to the agent as his rent, in order to incentivize productive effort and discourage him from acquiring information for opportunistic purpose. As a claimant for a larger output residual, the agent unambiguously exerts higher effort than that under the second best.

A larger output residual to the agent, however, amplifies the rent extraction motive to acquire information, which is of significant concern to the principal when the cost
of information acquisition is sufficiently small, $\kappa < \kappa^s < \kappa^q$, and the expected output residual to the uninformed agent is smaller than that to the deviated informed under the second best debt contract, $\int_{\theta}^{\bar{\theta}} q(e^{SD}, \theta) - q^{SD} dF(\theta) < \int_{\theta}^{\bar{\theta}} q(e(\theta), \theta) - q^{SD} dF(\theta)$. The principal, instead of granting the entire output residual to the agent, finds it optimal to reduce the agent’s equity share of output residual. The optimal contract to deter information acquisition thus has a debt-with-equity-share contract to replace the second best debt contract.

For a lower debt incentivizes productive effort yet a reduced equity share of output residual discourages it, there exists $\kappa^e < \kappa^s$ such that for $\kappa < \kappa^e$, deterring information acquisition trades off productive effort, whereas for $\kappa > \kappa^e$, deterring information acquisition is accompanied by a higher-powered incentive than the second best, summarized as the following corollary.

**Corollary 1.** There exists $\kappa^e < \kappa^s$ such that for $\kappa \geq \kappa^e$, $e^{DI} \geq e^{SD}$, and $e^{DI} < e^{SD}$ otherwise.

The possible complementarity between deterring information acquisition and a higher-powered incentive is absent from what Crémer, Khalil and Rochet (1998a) suggests. This is because of the key departure from Crémer et al.: whether a moral hazard problem is present when information acquisition is deterred. In Crémer et al., productive uncertainty is absent even when information acquisition is deterred; contractible output is a perfect measurement for the agent’s effort, as if the effort itself can also be contracted upon. In the current model, effort level is implemented with a transfer contingent on realization of contractible output. That is, in Crémer et al., the principal has two instruments to motivate productive effort and to deter information acquisition, whereas in this paper, the principal has only one instrument, the transfer.

In terms of agent’s motive to acquire information, in Crémer et al., the second best transfer to an uninformed agent is a fixed payment, so the agent’s motive to acquire information on the cost parameter is to use the information to reduce the cost of effort. Thus, to deter information acquisition, the principal increases the fixed payment, along with a lower contracted output (hence, a lower effort) if it is not too costly to acquire information. Here, the second best contract is a debt contract, given which the agent’s motive to acquire information is to use the information to avoid the debt or to extract maximal rent. Therefore, the principal lowers the debt to deter information acquisition, accompanied by a reduced equity share of output residual for insufficiently large cost of information acquisition. The former incentivize effort
whereas the latter discourages it.

4.2 General Contract

The readers at this point may question the optimality of the proposed debt-with-equity-share contract with the presence of binding constraint to deter information acquisition. I respond by showing that the result of a lower debt than its second best counterpart is indeed optimal, and a reduced share of output residual is qualitatively robust, yet in a different form of transfer, in which \( s \in \{0, 1\} \) for different output intervals beyond the debt, as the principal’s objective function is linear in the slope of transfer. The principal’s optimization program is as the following.

\[
\mathcal{P}_{DI} : \max_{t(q(e, \theta)), e} \int_0^\theta q(e, \theta) - t(q(e, \theta))dF(\theta)
\]

subject to

\( (LL), (NDP), (IC), (DI). \)

**Proposition 3.** The optimal contract to deter a risk neutral agent protected by limited liability from acquiring information is a debt-with-equity-share contract, with \( t^D(q) = 0 \) for \( q \leq q^{DI} < q^{SD} \) and \( t^D(q) \leq q - q^{DI} \) for \( q > q^{DI} \).

**Proof.** Appendix A.3. Precise form of \( t^D(q) \) is derived in the same appendix with the proof and is illustrated in Figure 3.

The intuition discussed in the previous example prevails. Recall that in the second best environment, the agent’s opportunistic motive to acquire information is to distinguish the inefficient states of nature to avoid the debt, and to discover the efficient states of nature to extract maximal rent. A lower debt, \( q^{DI} < q^{SD} \), is implemented to account for the former motive, and the transfer for sufficiently high realization of output is lowered to demotivate the latter, which violates monotonicity for some intermediate outputs. Thus, a low-powered incentive \( t^D(q) = 0 \) is optimal for some intermediate outputs if information acquisition is sufficiently costly, as illustrated in Figure 3(a), 3(b).\(^{14}\) Under sufficiently costly information acquisition, the optimal contract to deter information acquisition provides a higher-powered incentive than does

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\(^{14}\)Alternatively, if we model the problem in the context of procurement or regulation as in Lewis and Sappington (1997), the optimal form of procurement contract to deter information acquisition in Figure 3(a) and 3(b) would coincide qualitatively to what Chu and Sappington (2009b) characterizes in a model with adverse selection. Their driving force for the optimality is the shape of the density function of the state of nature, whereas in this paper, it is due to deterrence of information acquisition.
the second best debt contract if \((1 - F(\theta^{SD}))q_0(e^{SD}, \theta^{SD})\) is sufficiently large, as illustrated in Figure 3(b) and summarized in Corollary 2. If the cost of information acquisition is sufficiently low, the rent extraction motive of the agent is so significant that the optimal contract to deter information acquisition is capped at a threshold level, i.e. \(t^D_{q}(q) = 0\) for \(q > q^c\), as illustrated in Figure 3(c).

**Corollary 2.** The principal implements \(e^{DI} > e^{SD}\) when deterring information acquisition if information acquisition is sufficiently costly and \((1 - F(\theta^{SD}))q_0(e^{SD}, \theta^{SD})\) is sufficiently large.

One can also regard the contract in Proposition 3 as a second-degree contract discrimination, in a similar fashion to the second-degree price discrimination. For sufficiently low output level, the agent is paid according to the transfer schedule \(t^a(q) = \max\{q - q^{DI}, 0\}\), a simple debt contract with the level of debt smaller than the second best. Intuitively, the principal rewards the agent in the form of a lower debt for sufficiently small output realization, which could have been avoided if he acquired information. For sufficiently high level of output, the agent receives \(t^b(q) = T + \)
max\{q - q^b, 0\}, T = q^a - q^{DI}, \text{ i.e. a debt-with-fixed-payment contract, if the cost of information acquisition is sufficiently high, or he receives a fixed payment } T \text{ if the cost of information acquisition is sufficiently low. The agent’s motive to acquire information off the equilibrium path contributes to the explanation of this second-degree contract discrimination. If the state of nature is sufficiently inefficient, the agent’s benefit of acquiring information mainly comes from avoiding the debt. If, instead, the state of nature is relatively efficient, the agent’s benefit of acquiring information is mostly attributed to rent extraction. Thus, from the principal’s perspective, it is optimal to deter information acquisition with different transfer schemes associated to different sets of output.}

5 The Equilibrium Incentive Problem – the Rent-Efficiency Tradeoff

The following lemma indicates that the principal prefers some information management, either to induce information acquisition or to deter it, to null information management.

**Lemma 3.** Null information management is suboptimal.

**Proof.** Consider $\kappa > \kappa^a$ such that $(II)$ is strictly violated under $C^{SM}$. Without inducing information acquisition, $C^{SM}$ implements the same outcome as $C^0 = \{t^0(q(e^0, \theta))\}$, where $e^0 \in \arg\max e \int_0^\theta t^{SM}(q(e, \theta))dF(\theta) - c(e)$ and $t^0(q) = t^{SM}(q)$ for all $q$. $C^0$ satisfies $(LL)$, $(IC)$, and $(DI)$ by construction, which must not be preferred to $C^{DI}$ for the principal. Consider $\kappa < \kappa^a$ such that $(DI)$ is strictly violated under $C^{SD}$. Without deterring information acquisition, $C^{SD}$ implements the same outcome as $C^1 = \{t^1(q^1(\theta)), q^1(\theta)\}$, where $q^1(\theta) = q(e(\theta), \theta)$ for all $\theta \geq \tilde{\theta}$, zero otherwise, and $t^1(q^1(\theta)) = t^{SD}(q(e(\theta), \theta))$ for $\theta \geq \tilde{\theta}$, zero otherwise. $C^1$ by construction satisfies $(IR_\theta)$, $(IC_\theta)$, and $(II)$, which the principal does not prefer to $C^{II}$.

Lemma 3 allows us to restrict our attention to the comparison between inducing information acquisition and deterring it when studying the endogenous implementation of incentive problem. Define the principal’s net value of information, $V(\kappa)$, as the difference between her ex ante expected utility when information acquisition is induced and that when it is deterred,

$$V(\kappa) \equiv E(u^P(C^{II}, \kappa)) - E(u^P(C^{DI}, \kappa)).$$
It is equivalent to the expected improvement in efficiency minus the expected net information rent given to the agent to incentivize information acquisition,

\[ V(\kappa) = \int_0^{\theta} q^{II}(\theta) - c(h(q^{II}(\theta), \theta))dF(\theta) - \int_0^{\theta} q(e^{DI}, \theta) - c(e^{DI})dF(\theta) \]

\[ - \left[ \int_0^{\theta} u^A(t^{II}(q^{II}(\theta)), q^{II}(\theta), \kappa)dF(\theta) - \int_0^{\theta} u^A(t^{DI}(q(e^{DI}, \theta)), \kappa)dF(\theta) \right] . \]

For an agent with the cost of information acquisition \( \kappa \), if information is crucial in the sense that the principal benefits more from an improvement in efficiency relative to the net information rent to motivate the agent to acquire and use the information, the principal finds it optimal to induce information acquisition and implement adverse selection at the production stage. Otherwise, it is optimal for her to deter information acquisition to avoid a high net information rent, at the expense of efficiency, and implement moral hazard at the production stage. The principal’s information management and endogenous implementation of incentive problem exhibits a rent-efficiency trade-off. In a standard adverse selection problem, efficient production from the inefficient types of agent is traded off to save on information rent given to the efficient types of agent, and in the scope of information management, efficient use of information is traded off to save on rent given to the agent obtaining such information.

Straightforward from the optimization problem of the principal, \( V_*(\kappa) < 0 \). For \( \kappa \to 0 \), the principal earns second best payoff if she induces the agent to acquire information, and she can only deter information acquisition by an extremely low-powered transfer scheme, i.e. effort is distorted far away from the efficient level. For \( \kappa \to \infty \), the principal earns second best payoff if she deters the agent from acquiring information, and if she intends to induce information acquisition, the information rent goes to infinite. Proposition 4 is thus obtained.

**Proposition 4.** There exists \( 0 < \kappa^I < \infty \) such that for \( \kappa < \kappa^I \), improvement in efficiency exceeds the net information rent, and it is optimal to induce information acquisition and implement adverse selection at the production stage; for \( \kappa > \kappa^I \), improvement in efficiency falls short of the net information rent, and it is optimal to deter information acquisition and implement moral hazard at the production stage.
Application: Expert and Mediocre in Production. Interpreting the cost of information acquisition as the agent’s expertise in this field, an “expert” is able to acquire information at a sufficiently low cost, while a “mediocre” is able to acquire information at a sufficiently high cost. The principal finds it optimal to induce an expert to acquire productive information and implement adverse selection at the production stage, as by acquiring this information, improvement in efficiency is more significant than the net information rent. If the agent is a mediocre, it is then optimal to deter him from acquiring information and implement moral hazard at the production stage, to avoid a significantly large information rent at the expense of efficiency.

In terms of the contractual form, if \( \kappa^I \in (\kappa_a, \kappa_q) \), it is optimal i) to induce information acquisition with the second best menu contract, \( C^{SM} \) for \( \kappa \leq \kappa_a \) (an agent with extremely high expertise); ii) to induce information acquisition with a steeper menu contract, \( C^{II} \), in which \( q^{II}(\theta) > q^{SM}(\theta) \) for \( \theta \geq \tilde{\theta} \) and \( q^{II}(\theta) < q^{SM}(\theta) \) for \( \theta < \tilde{\theta} \), for \( \kappa_a < \kappa \leq \kappa^I \) (an agent with high expertise); iii) to deter information acquisition with a debt-with-equity-share contract, \( C^{DI} \), in which \( q^{DI} < q^{SD} \) and \( t^{DI}(q) \leq q - q^{DI} \) for \( q \geq q^{DI} \), for \( \kappa^I < \kappa < \kappa_q \) (an agent with mild expertise); to deter information acquisition with a second best debt contract, \( C^{SD} \), for \( \kappa_q \leq \kappa \) (an agent with poor expertise). However, level of \( \kappa^I \) depends on the exact functional form and the distribution of the state of nature, and is not guaranteed to be within the above-mentioned interval. If \( k^I \leq k_a \), interval ii) does not exist, and if \( k^I \geq k_q \), interval iii) does not exist. For example, given production function \( q(e, \theta) = \sqrt{\theta e} \), cost of effort \( c(e) = \frac{e^2}{2} \), and \( \theta \sim Unif(0, \tilde{\theta}) \), a modified menu contract is never optimal if \( \tilde{\theta} \) is sufficiently low, i.e. if information on the state of nature does not improve efficiency significantly relative to the net information rent, and a debt-with-equity-share contract is never optimal if \( \tilde{\theta} \) is sufficiently high, where information is crucial in production.

Application: Investment Banking. One can also apply the model I present here to address the agency problems in investment banking, where an investment bank (the principal) makes decision on funding a project executed by a private firm (the agent), the profitability of which depends on the firm’s non-observable investment (human and physical capital) and stochastic market condition. The firm can, before accepting the contract, conduct market investigation (information acquisition)\(^{15}\) at a sunk cost.

The cost of market investigation may be related to the characteristics of the market

\(^{15}\) I restrict information acquisition to only market investigation for explanatory convenience. Information acquisition by the private firm may also include interior investigation such as management and production audit.
where the firm participates, such as market transparency, or whether the market is a newly formed or a well-established one. If the investment bank is contracting with a firm in a well-established market with high level of transparency, the firm is able to collect data and past experience at a sufficiently low cost. It is optimal for the investment bank to offer a menu of funding options that induce the firm to conduct market investigation prior to acceptance. On the other hand, contracting with a firm in a newly formed market or in one with low level of transparency, data and past experience is limited or sufficiently costly for the firm to acquire. It is optimal for the investment bank to propose a state-independent debt-with-equity-share contract, so that the firm is deterred from conducting market investigation.

I am aware of the complexity of the real investment banking industry than in this model, e.g. there involves more competition among investment banks and firms instead of a simple principal-agent relationship, the investment bank itself may acquire information as well, and there may also be a regulator involved, yet this model serves as a benchmark for more sophisticated studies in which incentive problem is optimally chosen with information management.

6 Imperfect Information Acquisition

In the previous sections, information acquisition is perfect as long as acquiring effort is exerted, or equivalently, it is taken as a special case with linear cost of information acquisition. To emphasize the difference between perfect and imperfect information acquisition, in this section I examine the case with an interior solution of information management, assuming that \( d(\kappa, a) \) has \( d_a(\kappa, a) \geq 0 \), with equality at \( a = 0 \), \( d_{aa}(\kappa, a) > 0 \), \( d_a(\kappa, 1) \to \infty \) for all \( \kappa \). That is, in equilibrium, the incentive problem at the production stage is stochastic, whose density is implemented by the contracts offered. Denote the contracts \( C^I = \{q^I(\theta), t^I(\theta)\} \) to an informed agent and \( C^U = \{t^U(q)\} \) to an uninformed agent.

Given \( a \), if the agent observes productivity signal and is induced to reveal it truthfully, he has information advantage at the production stage and earns information rent \( u^I(\theta) = t^I(\theta) - \hat{c}(q^I(\theta), \theta) \). If he does not observe any signal, output is an imperfect measurement of his productive effort and he earns \( u^U(\theta) \equiv t^U(q(e^U, \theta)) - c(e^U) \), where \( e^U \) is the implemented effort by \( C^U \). The optimal investment in information acquisition is thus \( a \in \arg\max a' \int_0^\bar{\theta} u^I(\theta)dF(\theta) + (1 - a') \int_0^\bar{\theta} u^U(\theta)dF(\theta) - d(\kappa, a') \), or by
the first order condition,

\[ \int_0^{\bar{\theta}} u^I(\theta) dF(\theta) - \int_0^{\bar{\theta}} u^U(\theta) dF(\theta) = d_a(\kappa, a) \quad (A). \]

Information on productivity is not the only information advantage the agent enjoys, however. Whether the agent observes a correct signal or nothing is also his private information. The feasible contracts \( \{C^I, C^U\} \) are designed such that an informed agent prefers \( C^I \) and the uninformed finds \( C^U \) more attractive. Respectively,

\[ u^I(\theta) \geq \max \limits_{e} t^U(q(e, \theta)) - c(e) \quad \forall \theta \in [0, \bar{\theta}] \quad (TT_I) \]

and

\[ \int_0^{\bar{\theta}} u^U(\theta) dF(\theta) \geq \max \limits_{e} \int_0^{\bar{\theta}} t^I(q^I(e, \theta)) dF(\theta) - c(e) \quad (TT_U). \]

Adjusting notation accordingly for \((LIC_\theta), (M), (IC), (LL), \) and \((NDP), \{C^I, C^U\} \) solves the following program to implement imperfect information acquisition,

\[ \mathcal{P}_M : \max \limits_{\{q^I(\theta), t^I(\theta), e^I, q^U(\theta)\}, a} \int_0^{\bar{\theta}} q^I(\theta) - t^I(q^I(\theta)) dF(\theta) \]

\[ + (1 - a) \int_0^{\bar{\theta}} q(e^U, \theta) - t^U(q(e^U, \theta)) dF(\theta) \]

subject to

\((LIC_\theta), (M), (IC), (LL), (NDP), (A), (TT_I), (TT_U).\)

**Lemma 4.** If \((TT_I)\) is binding for some states of nature, it is binding at \( \theta^T, \tilde{\theta} < \theta^T \leq \bar{\theta} \), where \( \tilde{\theta} \) is such that \( e(\theta) \in \max \limits_{e} t^U(q(e, \theta)) - c(e) = 0 \) for \( \theta < \tilde{\theta} \).

**Proof.** Appendix A.4.

**Proposition 5.** Optimal contract \( \{C^I, C^U\} \) with imperfect information acquisition has the following properties

1. \( q^I(\theta) \geq q^{SM}(\theta) \) for \( \theta > \hat{\theta} \), \( q^I(\theta) \geq q^{SM}(\theta) \) for \( \theta \leq \hat{\theta} \), and \( q^I(\theta) \) exhibits a gap at \( \hat{\theta} \), where \( \hat{\theta} \) is the state that an uninformed agent expected to reveal ex ante if he pretends to be informed.

2. If \( \theta^T < \bar{\theta} \), there exists interval \((\theta^a, \theta^b)\) containing \( \theta^T \) such that \( q^I_\theta(\theta) = 0 \) for \( \theta \in (\theta^a, \theta^b) \).
3. $t^U(q)$ takes the form of debt-with-equity-share, with a lower debt than its second best counterpart, $t^U(q) = 0$ for $q \leq q^U < q^{SD}$ and $t^U(q) \leq q - q^U$ for $q > q^U$.


As the agent has private information in whether a correct signal or a null signal is observed, the optimal contract in comparison to the second best\textsuperscript{16} incorporates this dimension of truthful revelation. To induce truthful revelation of receiving no signal, $q^I(\theta)$ is lowered for $\theta < \hat{\theta}$ to restrict the ex ante expected rent given to an uninformed agent claiming to be informed of state $\hat{\theta}$, and the debt in $t^U(q)$, $q^U$, is lowered to give a higher rent to an uninformed agent who truthfully report receiving no signal. In addition, an informed agent in $\theta < \hat{\theta}$ has no attempt to pretend to be uninformed and give up his rent. Thus, to induce truthful revelation of receiving a correct signal, an equity share of output residual in $t^U(q)$ is offered in equilibrium to limit an informed agent’s ability to extract rent by claiming to be uninformed. $q^I(\theta)$ for $\theta < \theta^T$ is raised to give an informed agent a higher rent so that it is more costly for him to pretend uninformed, which violates monotonicity near $\theta^T$. Pooled output schedule is then optimal for some intermediate states of nature containing $\theta^T$. Whether $q^I(\theta)$ for $\theta < \min\{\hat{\theta}, \theta^T\}$ is above or below the second best then depends on the relative magnitude of the effects from inducing truthful revelation of an informed agent and that of an uninformed agent.

I thus conclude the qualitative robustness of the debt-with-equity-share contract in $C^U$, with a lower debt than the second best debt contract. The intuition, however, is different from that under perfect information acquisition. With perfect information acquisition, a debt-with-equity-share contract is offered to deter different use of information off the equilibrium path. With imperfect information acquisition, a lower debt is offered to induce the uninformed agent to truthfully report his informativeness, and an equity share of output residual is offered to deter the informed agent from claiming to be uninformed. The pooled output schedule for intermediate states of nature in $C^I$ is attributed to the joint effect of truthful revelation of being informed of states $\theta \in (\theta^a, \theta^b)$ and the monotonicity constraint. The former technically resembles the type-dependent participation constraints that generate countervailing incentives. In fact, the contract designed for an uninformed agent is itself a type-dependent alterna-

\textsuperscript{16}The second best here is referred to the one with symmetric information on whether information is realized imperfectly. I find it more persuasive to compare the optimal contract to this second best instead of the one with perfect signal, as the latter includes the effect of information management and that of a possible null signal.
tive for an informed agent.\footnote{Please refer to Lewis and Sappington (1989) for a pioneer work and to Jullien (2000) for a general discussion of countervailing incentives. Lemma 4 and Proposition 5 here can be regarded as a justification for the presence of countervailing incentive even with type-independent reservation payoff. However, it does not perfectly coincide with Lewis and Sappington (1989) and Jullien (2000), as the “type-dependent reservation payoff” for an informed agent here depends on the principal’s endogenous choice of contract to an uninformed agent.}

In the case of perfect information acquisition, information management exhibits a rent-efficiency tradeoff. With imperfect information acquisition, an additional consideration is included: the risk for having no signal, which is either absent in the environment with perfect information acquisition, or negligible when the agent is risk neutral in acquiring information. The principal implement $a^*$ in equilibrium such that

$$\int_0^\bar{\theta} q^I(\theta) - c(h(q^I(\theta), \theta)) - u^I(\theta)dF(\theta) - \int_0^\bar{\theta} q(e^U, \theta) - c(e^U) - u^U dF(\theta) = \phi d_{aa}(a^*)$$

equivalently,

$$\int_0^\bar{\theta} q^I(\theta) - c(h(q^I(\theta), \theta)) - q(e^U, \theta) + c(e^U)dF(\theta) - \phi d_{aa}(\kappa, a^*)$$

$$\int_0^\bar{\theta} u^I(\theta) - u^U dF(\theta)$$

Information management thus involves a rent-risk-efficiency tradeoff.

Implementing a high information acquiring effort benefits the principal in the sense that, with a high probability of the agent being informed, productive effort is exerted more efficiently, and the rent given to the agent regarding whether he is informed or not is lower; it costs the principal a higher information rent to the informed agent to motivate such high information acquiring effort and reveal the truth. Implementing a low information acquiring effort benefits the principal as she pays an information rent to the informed agent with a small probability, a lower rent to the agent regarding whether he is informed or not for it is likely that the agent is uninformed, and a small transfer to motivate a low information acquiring effort, at a larger expense of efficiency. Implementing an intermediate information acquiring effort balance between efficient production, information rent to the informed agent, and information acquiring incentive, at the expense of higher rent given to the agent regarding whether information is realized.
Private Cost of Information Acquisition

I have adopted the assumption of common knowledge in the cost of information acquisition. It is not surprising that this cost, interpreted as the agent’s expertise, may also be the agent’s private information. Consider perfect information acquisition as assumed throughout the paper except in Section 6. For ease of illustration, let \( \kappa \in \{ \kappa_L, \kappa_H \} \), \( \kappa_L < \kappa_I < \kappa_H \), \( \kappa = \kappa_L \) with probability \( k \). Under common knowledge of \( \kappa \), the principal finds it optimal to implement adverse selection by inducing the agent of \( \kappa_L \) to acquire information, and to implement moral hazard by deterring the agent of \( \kappa_H \) from acquiring information.

If \( \kappa \) is private knowledge of the agent, the principal design a pair of contract \( \{ \mathbb{C}^I, \mathbb{C}^U \} \), where \( \mathbb{C}^I = \{ q^I(\theta), t^I(\theta) \} \) is designed to induce the agent of \( \kappa_L \) to acquire and reveal information truthfully, and \( \mathbb{C}^U = \{ t^U(q) \} \) is designed to keep the agent of \( \kappa_H \) uninformed and motivated to exert effort, and that the agent voluntarily reveal his cost of information acquisition. In addition to the incentive compatibility, individual rationality, inducing information acquisition, and deterring information acquisition constraints in Sections 3 and 4, the pair of contracts satisfies

\[
\begin{align*}
\mathcal{P}_p : \max_{q^I(\theta), t^I(\theta), e^U, t^U(q)} & k \int_0^{\theta} q^I(\theta) - t^I(q^I(\theta))dF(\theta) \\
& + (1 - k) \int_0^{\theta} q(e^U, \theta) - t^U(q(e^U, \theta))dF(\theta)
\end{align*}
\]

subject to

\[
(LIC_\theta), (M), (IC), (LL), (NDP), (II), (DI), (TT_I), (TT_U).
\]

Proposition 6. The debt-with-equity-share contract to deter information acquisition derived in Proposition 3 is qualitatively robust to private knowledge of \( \kappa \), and the
optimal menu contract to induce information acquisition has \( q_0'(\theta) = 0 \) for \( \theta \in (\theta_c, \theta^d) \), when \( \kappa \) is the agent’s private information.

Proof. As shown in Appendix A.4, \((TT_I)\) and \((TT_U)\) distort the contract to the uninformed agent \( C^U \) from the second best \( C^{SD} \) towards the same direction as does \((DI)\). The result predicted in Proposition 3 is re-enforced with asymmetric information on the cost of information acquisition. \((TT_U)\) distort the contract to the informed agent \( C^I \) from the second best \( C^{SM} \) towards the same direction as does \((II)\), and as pointed out in Section 6, \((TT_I)\) technically resembles a \( \theta \)-dependent reservation payoff that generates countervailing incentives. The presence of \((TT_U)\) re-enforce the result predicted in Proposition 1, and \((TT_I)\), along with monotonicity constraint, results in pooled output schedule in intermediate states of nature for the informed agent. \( \square \)

8 Conclusion

The main insights of this paper are the treatment of the two polar incentive problems as equilibrium responses via information management, and the optimal contract to implement the equilibrium incentive problem. Model-wise, this brings the two polar incentive problems under a unified framework. What’s more, this fills the gap in the literature, in which abundant analysis is focused on how existing incentive problem affects equilibrium outcome, but little is said about how such incentive problem arises, and how the optimal contract responds respectively to its emergence.

The model presented in this paper is ready to be extended towards several directions that are left off. One drawback of the present model is that, given the assumed information acquiring technology, the two incentive problems are substitutes in equilibrium, which fails to explain the possible co-existence of the two incentive problems. Information acquiring effort that generates a noisy signal, which is communicated from the agent to the principal through a menu of contingent transfers, may be a more sophisticated way to model the interaction between information management and implementation of the incentive problems, yet at the expense of model complexity, as output options in the menu contract cannot be made singletons. In addition, I only consider the agent to acquire information, implicitly assuming that it is impossible or infinitely costly for the principal to acquire information. Relaxing this assumption, one can incorporate into the model the principal’s decision on whether to delegate information acquisition to the agent, or to acquire information by herself and communicate such information to the agent. This expands the support of endoge-
nous incentive problem within the contractual relationship to include the possibility of an informed principal. A static contractual relationship was assumed throughout the paper, and the timing of information acquisition is exogenously given. It would be interesting to extend the model to a dynamic contracting relationship, in which the timing of information acquisition is endogenously implemented, and the cost of information acquisition diminishes in time as partial information may be freely observed by the agent throughout the production process.

From an empirical standpoint, I suggested the importance of identifying the cost of information acquisition as well as the essential incentive problem(s) in empirical tests on contract theory. The incentive problem within the contractual relationship is an equilibrium response, and empirical research in which it is assumed exogenously may in some occasions fail to identify the true underlying incentive problem and thus generate bias conclusions. Specifically, in a scenario where the contractible variable depends on a stochastic and a choice variable, information on the former is acquirable at a cost, e.g. production, employment relationships, and investment banking, identification of the cost of information acquisition is more likely to play an important role in the analysis as it sheds light on the equilibrium incentive problem and the form of contract. For scenarios where information on the stochastic state of nature is almost costless to acquire, e.g. a buyer’s preference in a trade contract after the object is produced, or situations where information on the stochastic state of nature is extremely costly or almost impossible to acquire, e.g. accident insurance, assuming the source of incentive problem from the outset may benefit the researcher for its simplicity.

Appendices

A Proof of Propositions

A.1 Proof of Proposition 1

\[ P_{II} : \max_{t(\theta), q(\theta)} E(u^P(q(\theta), t(\theta))) = \int_{\theta} q(\theta) - t(\theta) dF(\theta) \]

subject to

\[ t(\theta) - c(q(\theta), \theta) \geq 0 \quad (IR_\theta), \]

\[ t_\theta(\theta) - \hat{c}_q(q(\theta), \theta) q_\theta(\theta) = 0 \quad \forall \theta \in [0, \bar{\theta}] \quad (LIC_\theta), \]
The principal’s reduced program to induce information acquisition is thus

\[ q_\theta(\theta) \geq 0 \quad (M), \]

\[ \int_0^\theta t(\theta) - \hat{c}(q(\theta), \theta)dF(\theta) - \kappa \geq \max_e \int_0^\theta t(q(e, \theta)) - c(e)dF(\theta) \quad (II). \]

Subscripts stand for partial derivatives.

Let \( u^A(\theta) = \max_y t(y) - \hat{c}(q(y), \theta) = t(\theta) - \hat{c}(q(\theta), \theta) \). \( u^A_\theta(\theta) = -\hat{c}_\theta(q(\theta), \theta) > 0 \) by envelop theorem. Taking integral and by binding \((IR_0)\), \( u^A(\theta) = \int_0^\theta -\hat{c}_\theta(q(x), x)dx \).

Plug \( t(\theta) = u^A(\theta) + \hat{c}(q(\theta), \theta) \) into \( E(u^P(q(\theta), t(\theta))) \) and rearrange by integration by parts,

\[ E(u^P(q(\theta), t(\theta))) = \int_0^\theta q(\theta) - \hat{c}(q(\theta), \theta)dF(\theta) - \int_0^\theta \frac{1 - F(\theta)}{f(\theta)}(-\hat{c}_\theta(q(\theta), \theta))dF(\theta). \]

Let \( u^A(\hat{\theta}) = t(\hat{\theta}) - \hat{c}(q(\hat{\theta}), \hat{\theta}) = \max_e \int_0^\theta t(q(e, \theta)) - c(e)dF(\theta) \), the certainty equivalence of the right hand side of \((II)\), then \((II)\) can be rewritten as

\[ \int_0^\theta (1_{\theta > \hat{\theta}} - F(\theta))(-\hat{c}_\theta(q(\theta), \theta))d\theta \geq \kappa. \]

The principal’s reduced program to induce information acquisition is thus

\[ \mathcal{P}_{II} : \max_{q(\theta)} \int_0^\theta q(\theta) - \hat{c}(q(\theta), \theta) - \frac{1 - F(\theta)}{f(\theta)}(-\hat{c}_\theta(q(\theta), \theta))dF(\theta) \]

subject to

\[ \int_0^\theta (1_{\theta > \hat{\theta}} - F(\theta))(-\hat{c}_\theta(q(\theta), \theta))d\theta \geq \kappa \quad (II'). \]

Let \( \lambda \) be the Lagrange multiplier for \((II')\), \( q^{II}(\theta) \) solves

\[ \left(1 - \hat{c}_q(q(\theta), \theta)\right) - \frac{1 - F(\theta)}{f(\theta)}(-\hat{c}_\theta q(q(\theta), \theta)) + \lambda \frac{1 - F(\theta)}{f(\theta)}(-\hat{c}_\theta q(q(\theta), \theta)) = 0. \]

For \( \kappa < \kappa^a \), where \( \kappa^a = \lim_{q(\theta) \to q^{SM}(\theta)} \int_0^\theta (1_{\theta > \hat{\theta}} - F(\theta))(-\hat{c}_\theta(q(\theta), \theta))d\theta \), \((II')\) slacks and the principal is able to induce information acquisition with the second best menu contract \( C^{SM} = \{l^{SM}(\theta), q^{SM}(\theta)\} \), \( \lambda = 0 \). Note that \( \frac{1 - F(\theta)}{f(\theta)}(-\hat{c}_\theta q(q(\theta), \theta)) \)

\[ = \frac{\partial}{\partial q(\theta)} \left( c_e(h(\theta, q(\theta))) \frac{1 - F(\theta)}{f(\theta)} \frac{g_\theta(h(\theta, q(\theta)), \theta)}{g_e(h(\theta, q(\theta)), \theta)} \right) > 0 \]

which is decreasing in \( \theta \) by Assumption 2, so monotonicity constraint is strictly satisfied.

For \( \kappa > \kappa^a \), \((II')\) is violated given the second best menu contract. \( \lambda > 0 \) in
equilibrium. First claim that $\lambda \leq 1$. Suppose that the cost of information acquisition increased by $\delta$, with the optimal contract provided, the principal’s equilibrium payoff dropped by $\lambda \delta$. A weakly dominated response to this increment for the principal is to increase the transfer to the agent in any state of nature by $\delta$, which does not violate any constraint. This results in a drop of the principal’s payoff by $\delta$. $\lambda \delta \leq \delta$, and thus $\lambda \leq 1$. With $0 < \lambda \leq 1$ and $\frac{1-F(\theta)}{f(\theta)}(-c_\theta(q(\theta),\theta)) > 0$, the optimal contract to induce information acquisition $C_{II} = \{q_{II}(\theta), q_{II}(\theta)\}$ is such that $q_{II}(\theta) > q_{SM}(\theta)$ for $\theta > \hat{\theta}$, and $q_{II}(\theta) < q_{SM}(\theta)$ for $\theta \leq \hat{\theta}$.

A.2 Proof of Proposition 2

$$\mathcal{P}_D': \max_{s,q,e} \int_0^{\hat{\theta}} q(e,\theta)dF(\theta) - \int_\theta^{\hat{\theta}} s(q(e,\theta) - q)dF(\theta)$$

subject to

$$0 \leq s \leq 1 \quad (NDP'),$$

$$\int_\theta^{\hat{\theta}} sq_e(e,\theta)dF(\theta) - c_e(e) = 0 \quad (LIC'),$$

$$\int_\theta^{\hat{\theta}} s(q(e,\theta) - q)dF(\theta) - c(e) \geq \int_\theta^{\hat{\theta}} s(q(e(\theta),\theta) - q) - c(e(\theta))dF(\theta) - \kappa \quad (DI').$$

Subscripts stand for partial derivatives.

Let the Lagrange function exclusive of $(NDP')$ be

$$\mathcal{L}' = \int_0^{\hat{\theta}} q(e,\theta)dF(\theta) - \int_\theta^{\hat{\theta}} s(q(e,\theta) - q)dF(\theta) + \mu' \left( \int_\theta^{\hat{\theta}} sq_e(e,\theta)dF(\theta) - c_e(e) \right)$$

$$+ \phi' \left( \int_\theta^{\hat{\theta}} s(q(e,\theta) - q)dF(\theta) - c(e) - \int_\theta^{\hat{\theta}} s(q(e(\theta),\theta) - q) - c(e(\theta))dF(\theta) + \kappa \right),$$

where $\mu'$ and $\phi'$ are the Lagrange multipliers associated to $(LIC')$ and $(DI')$ respec-
tively. \( C^{DI} = \{ q^{DI}, s^{DI} \} \) has \( q^{DI} \) solving

\[
\frac{\partial \mathcal{L}'}{\partial q} q = \left( 1 - \mu \left( \frac{q_e(e, \theta)}{\theta} f(\theta) \right) \right) + \phi' \left( \frac{F(\theta) - F(\tilde{\theta})}{1 - F(\tilde{\theta})} \right) q = 0
\]

and \( s^{DI} \) solving

\[
\frac{\partial \mathcal{L}'}{\partial s} (1 - s) s = \left( \int_{\theta}^{\tilde{\theta}} \left( -(q(e, \theta) - q) + \mu q_e(e, \theta) \right) dF(\theta) + \phi' \left( \int_{\theta}^{\tilde{\theta}} (q(e, \theta) - q) dF(\theta) - \int_{\theta}^{\tilde{\theta}} (q(e(\theta), \theta) - q) dF(\theta) \right) \right) (1 - s) s = 0.
\]

If \( \phi' = 0 \), constraint \((D'I')\) slacks and the principal is able to deter information acquisition with the second best debt contract \( C^{SD} = \{ s^{SD} = 1, q^{SD} \} \). Define \( \kappa^q = \lim_{q \to q^{SD}} \int_{\theta}^{\tilde{\theta}} (q(e(\theta), \theta) - q) - c(e(\theta)) dF(\theta) - \int_{\theta}^{\tilde{\theta}} (q(e^{SD}, \theta) - q) dF(\theta) + c(e^{SD}) \), where \( e^{SD} \) denoting the equilibrium effort under \( C^{SD} \). For \( \kappa < \kappa^q, \phi' > 0 \). By Lemma 2, \( \tilde{\theta} > \theta \), so \( F(\tilde{\theta}) < F(\theta) < 0 \). The optimal debt contract to deter information acquisition has \( 0 \leq q^{DI} < q^{SD} \). Note that lowering \( q \) from the second best has a second order effect, while lowering \( s \) from the second best has a first order effect. There thus exist \( \kappa^s < \kappa^q \) such that for \( \kappa > \kappa^s \), the principal is able to deter information acquisition with \( s^{DI} = s^{SD} = 1 \). \( \kappa^s = \lim_{s \to s^{SD}} \int_{\theta}^{\tilde{\theta}} s(q(e(\theta), \theta) - q^s) - c(e(\theta)) dF(\theta) - \int_{\theta}^{\tilde{\theta}} s(q(e^s, \theta) - q^s) dF(\theta) + c(e^s) \), with superscript \( s \) denoting the level of equilibrium variables when \( \frac{\partial \phi'}{\partial s} \geq 0 \) is just binding. If \( \int_{\theta}^{\tilde{\theta}} (q(e^{SD}, \theta) - q^{SD}) dF(\theta) < \int_{\theta}^{\tilde{\theta}} (q(e(\theta), \theta) - q^{SD}) dF(\theta) \), for \( \kappa < \kappa^s, s^{DI} < s^{SD} \) and solves \( \frac{\partial^2 \phi'}{\partial s^2} \bigg|_{q=q^{DI}} = 0 \). The solution is optimal as \( \frac{\partial^2 \phi'}{\partial q^2} < 0 \) by Assumption 2 and \( \frac{\partial^2 \phi'}{\partial q^2} = 0 \) by Lemma 2.

\[ \square \]

### A.3 Proof of Proposition 3

Suppose that the first-order approach is valid, the principal’s optimization program is

\[
\mathcal{P}_{DI} : \max_{t(q(e, \theta)), e} \int_{0}^{\tilde{\theta}} q(e, \theta) - t(q(e, \theta)) dF(\theta)
\]

subject to

\[
\begin{align*}
t(q(e, \theta)) & \geq 0 \quad (LL), \\
0 & \leq t_q(q(e, \theta)) \leq 1 \quad (NDP),
\end{align*}
\]
\[
\int_0^\vartheta t_q(q(e, \theta)) q_e(e, \theta) dF(\theta) = c_e(e) \quad (LIC),
\]
\[
\int_0^\vartheta t(q(e, \theta)) dF(\theta) \geq \int_0^\vartheta 1_{\theta \geq \hat{\theta}} t(q(e(\theta), \theta)) - c(e(\theta)) dF(\theta) - \kappa \quad (DI).
\]

Subscripts stand for partial derivatives. Let \( \mu \), and \( \phi \) be the Lagrange multipliers associated to \((LIC)\), and \((DI)\), respectively.

With limited liability, \( t(q(e, \theta)) = \int_0^\vartheta t_q(q(e, x)) q_\theta(e, x) dx \), and by integration by parts, \( \int_0^\vartheta t(q(e, \theta)) dF(\theta) = \int_0^\vartheta (1 - F(\theta)) t_q(q(e, \theta)) q_\theta(e, \theta) d\theta \). By the envelope theorem of an informed agent off the equilibrium path and integration by parts, \( t(q(e(\theta), \theta)) - c(e(\theta)) = \int_0^\vartheta t_q(q(e(x), x)) q_\theta(e(x), x) dx \), and \( \int_0^\vartheta t(q(e(\theta), \theta)) - c(e(\theta)) dF(\theta) = \int_0^\vartheta (1 - F(\theta)) t_q(q(e(\theta), \theta)) q_\theta(e(\theta), \theta) d\theta \). The (point-wise) Lagrange function of the principal’s problem to deter an agent from acquiring information, excluding \((LL)\) and \((NDP)\), is then written as

\[
\mathcal{L} = t_q(q(e, \theta)) (-1 - F(\theta)) q_\theta(e, \theta) + \mu q_e(e, \theta) f(\theta) + \phi (1 - F(\theta)) q_\theta(e, \theta)
\]
\[
- \phi 1_{\theta' \geq \hat{\theta}} (1 - F(\theta')) q_\theta(e(\theta'), \theta')) + q(e, \theta) f(\theta) - \mu c_e(e) - \phi (c(e) + \kappa),
\]

where \( \theta' \) is such that \( q(e, \theta) \equiv q(e(\theta'), \theta') \). If \( \kappa \) is sufficiently large that \( \phi = 0 \), \( t_q(q(e, \theta)) = 1 \) if \( \mu \geq \frac{1 - F(\theta)}{f(\theta)} q_\theta(e(\theta'), \theta') \geq \frac{1}{\rho_1(e, \theta)} \), \( t_q(q(e, \theta)) = 0 \) otherwise. As \( \frac{1}{\rho_1(e, \theta)} \) by Assumption 2 is decreasing in \( \theta \), the second best contract is in the form of debt, where \( t^{SD}(q) = 0 \) for \( q \leq \tilde{q}^{SD} \), and \( t^{SD}(q) = q - \tilde{q}^{SD} \) otherwise.

Let the solution to \( \mathcal{P}_{\theta, \phi} \), ignoring monotonicity for now, be \( \hat{t}(q) \). If \( \kappa \) is sufficiently small that \( \phi > 0 \), for \( q < \hat{q} \equiv q(e(\hat{\theta}), \hat{\theta}), \) i.e. where \( \theta' < \hat{\theta}, \hat{t}_q(q) = 1 \) if \( \mu \geq \frac{1 - F(\theta)}{f(\theta)} q_\theta(e(\theta'), \theta') \geq \frac{1}{\rho_1(e, \theta)} \), \( \hat{t}_q(q) = 0 \) otherwise. For \( q \geq \hat{q} \), \( \hat{t}(q) = \max\{q - \tilde{q}^{DI}, 0\} \), where \( \tilde{q}^{DI} < q^{SD} \).

For \( q \geq \hat{q} \), i.e. where \( \theta' \geq \hat{\theta}, \hat{t}_q(q) = 1 \) if \( \mu \geq \frac{1 - F(\theta)}{f(\theta)} q_\theta(e(\theta'), \theta') \geq \frac{1}{\rho_1(e, \theta)} \), \( \hat{t}_q(q) = 0 \) otherwise. \( \frac{1 - F(\theta)}{f(\theta)} q_\theta(e(\theta'), \theta') \geq \frac{1}{\rho_1(e, \theta)} \). Let \( \theta_0 \) be such that \( q(e, \theta_0) \equiv q(e(\theta_0), \theta_0) \). For \( \theta < \theta_0 \), \( \theta < \theta' \) and for \( \theta > \theta_0, \theta > \theta' \), so \( \frac{\partial q'}{\partial \theta} < 1 \). \( \frac{1 - F(\theta)}{f(\theta)} q_\theta(e(\theta'), \theta') \) is then increasing in \( \theta \). If \( \phi = 1, -\frac{\partial}{\partial \theta} \left( \frac{1}{\rho_1(e, \theta)} \right) < 0 \), \( \phi < \hat{\phi}, \hat{\phi} < 1 \), \( \frac{1 - F(\theta)}{f(\theta)} q_\theta(e(\theta'), \theta') \) is increasing in \( \theta \). If \( 0 < \phi < \hat{\phi}, for \ q \geq \hat{q}, \hat{t}(q) = \max\{q - \tilde{q}, 0\} \), where \( \tilde{q} > q^{DI} \). As \( \hat{q} > q^{DI} \), \( \hat{t}(q) \) violates non-decreasing transfer near \( \hat{q} \). Thus, there exists an interval \([\hat{q}^*, \hat{q}^h]\) containing \( \hat{q} \) in which low-powered incentive \( (t_q(q) = 0) \) occurs. Therefore, \( t^{DI}(q) \leq q - \tilde{q}^{DI} \) for \( q > \tilde{q}^{DI} \). The optimal contract with \( 0 < \phi < \hat{\phi} \) is as the following:
\[ t^{DI}(q(e, \theta)) = \begin{cases} 
0 & \text{for } q \in [q(e, 0), q^{DI}] \\
q(e, \theta) - \hat{q}^{DI} & \text{for } q \in (q^{DI}, q^a) \\
q^a - \hat{q}^{DI} & \text{for } q \in [q^a, q^b] \\
q(e, \theta) - \hat{q} & \text{for } q \in (q^b, q(e, \theta)) 
\end{cases} \]

with \( \hat{q} \gtrsim \frac{q}{2} \) if \( \frac{1-F(q)}{1-F(q^S)} \gtrsim 1 \). If \( 1 > \phi > \hat{\phi} \), there exists \( \hat{q} \) such that \( \hat{t}_q(q) = 0 \) for \( q > \hat{q} \). The optimal contract with \( 1 > \phi > \hat{\phi} \) is as the following:

\[ \hat{t}^{DI}(q(e, \theta)) = \begin{cases} 
0 & \text{for } q \in [q(e, 0), q^{DI}] \\
q(e, \theta) - \hat{q}^{DI} & \text{for } q \in (q^{DI}, q^a) \\
\hat{q} & \text{for } q \in (q^a, q(e, \theta)) 
\end{cases} \]

As \( \hat{t}^{DI} = 0 \) wherever second-order differentiable, \( \int_0^{\hat{q}} t^{DI}(q(e, \theta)) \) is valid.

\[ \frac{1}{2} \int_0^{\hat{q}} t^{DI}(q(e, \theta))q(e, \theta)dF(\theta) < 0, \]

the first order approach is valid.

\[ \Box \]

A.4 Proof of Lemma 4 and Proposition 5

\[ u^I(\theta) = \int_0^\theta -\hat{c}_\theta(q^I(x), x)dx + u^I(0). \]

The principal’s optimization program is then

\[ \mathcal{P}_M : \max_{q^I(\theta), e^U, q^U(\theta), a} \alpha \left( \int_0^{\hat{q}} q^I(\theta) - \hat{q}(q^I(\theta), \theta) - \frac{1-F(\theta)}{f(\theta)} (-\hat{c}_\theta(q^I(\theta), \theta))dF(\theta) - u^I(0) \right) \]

subject to

\[ t^U(q^U, \theta) \geq 0 \quad (LL), \]

\[ 0 \leq t^U_q(q) \leq 1 \quad (NDP), \]

\[ q^I_\theta(\theta) \geq 0 \quad (M), \]

\[ \int_0^{\hat{q}} t^U(q^U(\theta))q_e(q^U(\theta), \theta)dF(\theta) = c_e(q^U) \quad (LIC_U), \]

\[ \int_0^{\hat{q}} u^I(\theta)dF(\theta) - \int_0^{\hat{q}} u^U(\theta)dF(\theta) = d_a(\kappa, a) \quad (A), \]
\[ u'(\theta) \geq \max_e t^U(q(e, \theta)) - c(e) \forall \theta \in [0, \bar{\theta}] \quad (TT_I), \]
\[ \int_0^{\bar{\theta}} u^U(\theta)dF(\theta) \geq \max_e \int_0^{\bar{\theta}} t^I(q^I(e, \theta))dF(\theta) - c(e) \quad (TT_U). \]

Subscripts in the functions stand for derivatives. Let \( \mu, \phi, \lambda^I(\theta), \lambda^U \) be the Lagrange multipliers for \((LIC_U), (A), (TT_I), (TT_U)\), respectively. Denote \( \hat{\theta} \) as such that \( u'(\hat{\theta}) = \max_e \int_0^{\bar{\theta}} t^I(q^I(e, \theta))dF(\theta) - c(e) \).

\( q^I(\theta) \) solves the point-wise optimization condition

\[
\left( a - a\hat{c}_q(q(\theta), \theta) - (a - \phi)\frac{1 - F(\theta)}{f(\theta)}(-\hat{c}_\theta q^I(\theta), \theta)) \right) \\
+ \frac{1}{f(\theta)} \int_\theta^{\bar{\theta}} \lambda^I(x)dx(-\hat{c}_\theta q^I(\theta, \theta)) - \frac{\lambda^U}{f(\theta)}(-\hat{c}_\theta q^I(\theta, \theta))1_{\theta < \bar{\theta}} = 0
\]

and, by similar method as Appendix A.3, \( t^U_q(q) = 1 \) if

\[
\mu > \frac{1 - a + \phi - \lambda^U}{\rho_1(e, \theta)} - \frac{\lambda^U}{\rho_1(e, \theta)} + \frac{\int_0^{\bar{\theta}} \lambda^I(\theta)q_0(e(\theta), \theta)d\theta}{q_0(e(\theta))f(\theta)},
\]

where \( \rho_1(e, \theta) \equiv f(\theta)q_0(e, \theta) \) and \( \theta' \) is such that \( q(e, \theta) = q(e(\theta'), \theta') \).

Show Lemma 4. Let \( \{\hat{C}^I, \hat{C}^U\} \) be the optimal contract excluding \((TT_I)\), in which \( t^U_q(q) = 1 \) if

\[
\mu > \frac{1 - a + \phi - \lambda^U}{\rho_1(e, \theta)}.
\]

Claim that \( 1 - a + \phi - \lambda^U \geq 0 \). Suppose that in addition to the optimal contracts, that the principal increases the transfer to the uninformed agent by \( \delta \) and adjust \( a \) downward by \( \eta \) to bound \((A)\), which does not violate any constraint excluding \((TT_I)\). Downward adjustment of \( a \) has a second order effect yet increment of transfer has a first order effect. The principal’s indirect objective function is then changed by \((-1 + a - \phi + \lambda^U)\delta \leq 0\) as she is moving from the optimal solution to the suboptimal. As \( 1 - a + \phi - \lambda^U \geq 0 \), the optimal contingent transfer excluding \((TT_I)\) to an uninformed agent is a debt contract. Thus, given implemented productive effort, there exist \( \hat{\theta} \) such that \( e(\theta) \in \max_e t^U(q(e, \theta)) - c(e) = 0 \) for \( \theta < \hat{\theta} \). Along with individual rationality of the informed agent, \((TT_I)\) is strictly satisfied for \( \theta < \hat{\theta} \). Hence, if \( u^I(\theta) \) is sufficiently convex such that \((TT_I)\) is violated for some \( \theta \in [\theta_1, \theta_2] \), it is where \( \theta_1 > \hat{\theta} \) and \( \theta_2 \leq \bar{\theta} \). To deter an informed agent in states \( \theta \in (\theta_1, \theta_2) \) from lying to be uninformed, there exists \( \theta^T \) such that \( q^I(\theta) \) for \( \theta < \theta^T \) are raised to increase the rent in these states. As
\((TT_I)\) is strictly satisfied in \(\theta \leq \hat{\theta}, \ \hat{\theta} < \theta^T \leq \overline{\theta}\).

Given Lemma 4, \(q^I(\theta)\) solves

\[
\left( a - a\hat{c}_q(q(\theta), \theta) - (a - \phi)\frac{1 - F(\theta)}{f(\theta)}(-\hat{c}_\theta q(q^I(\theta), \theta)) \right) + \frac{\lambda^I(\theta^T)}{f(\theta)} 1_{\theta \leq \theta^T}(-\hat{c}_\theta q(q^I(\theta), \theta)) = 0
\]

and \(t^U_q(q) = 1\) if

\[
\mu > \frac{1 - a + \phi}{\rho_1(e, \theta)} + \frac{\lambda^U}{\rho_1(e, \theta)} + \frac{\lambda^I(\theta^T)}{q^i(e, \theta)} 1_{\theta \leq \theta^T} \geq 0.
\]

Show i) and ii) in Proposition 5. As \(-\hat{c}_\theta(q^I(\theta), \theta) = \frac{\partial}{\partial q(\theta)} \left( c_e(h(\theta, q(\theta))) \frac{q(\theta)}{q_e(h(\theta, q(\theta)))} \right) > 0\), two binding constraints distort \(q^I(\theta)\) from \(q^{SM}(\theta)\): a) \(q^I(\theta)\) for \(\theta \leq \theta^T\) are raised to prevent an informed agent from pretending to be uninformed, implied by \(\frac{\lambda^I(\theta^T)}{f(\theta)} 1_{\theta \leq \theta^T}(-\hat{c}_\theta(q^I(\theta), \theta)) \geq 0\); b) \(q^I(\theta)\) for \(\theta \leq \hat{\theta}\) is lowered with a gap at \(\hat{\theta}\) to prevent an uninformed agent from lying to be informed, captured by \(-\frac{\lambda^U}{f(\theta)}(-\hat{c}_\theta(q^I(\theta), \theta))1_{\theta \leq \hat{\theta}} < 0\). For \(\theta > \hat{\theta}\), \(q^I(\theta) \geq q^{SM}(\theta)\) as at most the first effect is present. For \(\theta \leq \theta^T\), depending on the countervailing effect between truth telling in the informed phase and that in the uninformed phase, \(q^I(\theta) < q^{SM}(\theta)\) if the second effect is sufficiently significant to outweigh the first. If \(\theta^T < \overline{\theta}\) and \(\theta^T \neq \hat{\theta}\), monotonicity must be violated near \(\theta^T\) due to a). Optimal \(q^I(\theta)\) thus have \(q^I(\theta) = 0\) for \(\theta \in (\theta^a, \theta^b)\), where \(\theta^T \in (\theta^a, \theta^b)\).

Show iii) in Proposition 5. The same constraints also distort \(t^U(q)\) from \(t^{SD}(q) = \max\{q(e^{SD}, \theta) - q^{SD}, 0\}\): a) to prevent an uninformed agent from lying to be informed, the initial debt is lowered, \(t^U(q) = 0\) for \(q < q^U \leq q^{SD}\), as \(-\frac{\lambda^U}{\rho(e, \theta)} < 0\); b) to deter an informed agent from lying to be uninformed, for \(q \in [\tilde{q}, q^T]\), where \(\tilde{q} = q(e(\theta), \tilde{\theta})\) and \(q^T \equiv q(e(\theta^T), \theta^T)\), \(t^U(q)\) is lowered in the sense that \(t^U_q(q) = 1\) for \(q(e, \theta) > q_1 > q^U\), as \(\lambda^I(\theta^T) \frac{q(e(\theta), \theta^T)}{q_e(e(\theta)) f(\theta)} > 0\), which violates monotonicity near \(\tilde{q}\). Hence, there exist an interval \((q^a, q^b)\) containing \(\tilde{q}\), such that \(t^U_q(q) = 0\) for \(q(e, \theta) \in (q^a, q^b)\). Thus, \(t^U(q) \leq q - q^U\) for \(q > q^U\), share of output residual to the agent is reduced.

\(\square\)
B  Deterrence of Information Acquisition with a Risk Averse Agent

Suppose that the agent is risk averse in the realization of transfer, in the sense that 
\[ u^A = v(t(q(e, \theta))) - c(e), \]
where \( v_t(t) > 0 \) and \( v_{tt}(t) < 0 \). The \((IC)\) constraint can be replaced by the local incentive compatibility constraint

\[
\int_0^\theta v_t(t)q_t(e, \theta) - c_e(e)dF(\theta) = 0 \quad (LIC_a)
\]

if \( v(t) \) is sufficiently concave and \( c(e) \) is sufficiently convex, and transfer is non-decreasing, \( t_q(q) \geq 0 \).\(^{18}\) We assume the former two hold, along with the following assumption for the second best contract to be monotonically non-decreasing.

**Assumption 3.** \( v(t) \) has non-increasing absolute risk aversion, i.e. \( \frac{\partial}{\partial t} \left( -\frac{v_{tt}(t)}{v(t)} \right) \leq 0 \).

Replacing \((IC)\) by \((LIC_a)\), the principal’s optimization program to deter a risk averse agent from information acquisition is

\[
\mathcal{P}_a : \max_{t(q), e} \int_0^\theta q(e, \theta) - t(q(e, \theta))dF(\theta)
\]

subject to

\((IR), (LIC_a), (DI)\).

How the binding constraint \((DI)\) distort the optimal (non-monotonic) contract, \( t^{DI}(q) \), from the second best, \( t^{SB}(q) \) is characterized in the following proposition.

**Proposition 7.** Implementing \( e^* \),

1. for \( q(e^*, \theta) < q(e(\tilde{\theta}), \tilde{\theta}), t^{DI}(q) > t^{SB}(q) \);

2. for \( q(e^*, \theta) \geq q(e(\tilde{\theta}), \tilde{\theta}), t^{DI}(q) > t^{SB}(q) \) if \( f(\theta) > f(\theta') \), and \( t^{DI}(q) \leq t^{SB}(q) \) if \( f(\theta) \leq f(\theta') \) with equality holds at \( f(\theta) = f(\theta') \), where \( \theta' \) is such that \( q(e(\theta'), \theta') \equiv q(e^*, \theta) \);

3. there is a downward gap of \( t^{DI}(q) \) at \( q(e(\tilde{\theta}), \tilde{\theta}) \) from the left

**Proof.** Let \( \theta' > 0 \) is such that \( q(e^*, \theta) \equiv q(e(\theta'), \theta') \). For sufficiently small \( \theta > 0, e(\theta) < e^* \) and \( q(e(\theta), \theta) < q(e^*, \theta) \), for sufficiently large \( \theta, e(\theta) > e^* \) and \( q(e(\theta), \theta) >\)

\(^{18}\)This is straightforward from the second order derivative of the agent’s optimization problem.
where \( \lambda^{IR} \), \( \mu^{a} \), and \( \phi^{a} \) are the Lagrange multipliers associated to constraints \((IR)\), \((LIC_{a})\), and \((DI)\), respectively. If \( \kappa \) is sufficiently small that \( \phi^{a} > 0 \), \( 1 - 1_{\theta' \geq \tilde{\theta}} f(\theta') = 1 \) for \( \theta' < \tilde{\theta} \), i.e. for \( q(e^*, \theta) \equiv q(e(\theta'), \theta') < q(e(\tilde{\theta}), \tilde{\theta}) \), hence part 1. For \( \theta' \geq \tilde{\theta} \), i.e. for \( q(e^*, \theta) \equiv q(e(\theta'), \theta') > q(e(\tilde{\theta}), \tilde{\theta}) \), \( 0 < 1 - 1_{\theta' \geq \tilde{\theta}} f(\theta') < 1 \) for \( f(\theta) > f(\theta') \), and \( 1 - 1_{\theta' \geq \tilde{\theta}} f(\theta') < 0 \) for \( f(\theta) \leq f(\theta') \), with equality holds at \( f(\theta) = f(\theta') \), hence part 2. As \( 1 - 1_{\theta' \geq \tilde{\theta}} f(\theta') = 1 \) for \( \theta' < \tilde{\theta} \) and \( 1 - 1_{\theta' \geq \tilde{\theta}} f(\theta') < 1 \) for \( \theta' \geq \tilde{\theta} \), part 3 is straightforward.

Part 1 in Proposition 7 is intuitive: one motive for the agent to acquire information is to distinguish sufficiently inefficient state of nature to avoid exerting effort at a loss. Thus, to counter such opportunistic motive, the principal increases the transfer for sufficiently inefficient states of nature, reducing the loss subject to those states. It can also be seen as an reward for not acquiring information to avoid loss in the most inefficient states of nature, as \( q(e^*, \theta) < q(e(\tilde{\theta}), \tilde{\theta}) \) would have been avoided if the agent had acquired information.

Part 2 captures the other opportunistic motive for the agent to acquire information off the equilibrium path: to discover a relatively efficient state of nature to extract maximal rent. It would be clearer if we think of states of nature as discrete states, so that the density is the probability distribution. The principal is unable to judge directly whether a certain realization of output is produced by an uninformed or an informed agent. If an output level is more likely to be realized by an agent who opportunistically acquired information, \( f(\theta') > f(\theta) \), it is optimal for the principal to punish the agent for such realization relative to the second best contract, and if it is more likely to be realized by an agent who did not acquire information, \( f(\theta) > f(\theta') \), it is then optimal for the principal to reward the agent for such realization more than the second best would have.

Proposition 7 is derived without imposing monotonicity on the transfer scheme. Part 3 indicates that, even if the second best transfer is monotonically increasing, the binding constraint to deter information acquisition creates non-monotonicity to the optimal contract. Thus, imposing non-decreasing transfer, there are some non-contingency of transfer on outputs at least near \( q(e(\tilde{\theta}), \tilde{\theta}) \).
Corollary 3. If \( f_\theta(\theta) = 0 \forall \theta \in [0, \bar{\theta}] \), \( t^{DI}(q) = t^{SB}(q) \) for \( q(e^*, \theta) \geq q(e(\bar{\theta}), \bar{\theta}) \); if \( f_\theta(\theta) > 0 \forall \theta \in [0, \bar{\theta}] \), \( t^{DI}(q) < t^{SB}(q) \) for \( q(e(\theta^0), \theta^0) > q(e^*, \theta) \geq q(e(\bar{\theta}), \bar{\theta}) \) and \( t^{DI}(q) \geq t^{SB}(q) \) for \( q(e^*, \theta) \geq q(e(\bar{\theta}), \bar{\theta}) \); if \( f_\theta(\theta) < 0 \forall \theta \in [0, \bar{\theta}] \), \( t^{DI}(q) > t^{SB}(q) \) for \( q(e(\theta^0), \theta^0) > q(e^*, \theta) \geq q(e(\bar{\theta}), \bar{\theta}) \) and \( t^{DI}(q) \leq t^{SB}(q) \) for \( q(e^*, \theta) \geq q(e(\theta^0), \theta^0) \), where \( \theta^0 \) is such that \( e^* = e(\theta^0) \).

The corollary indicates that, in the case of accepting the contract, if the density of state of nature is increasing, upon observing a sufficiently low realization of output, the principal believes that it is more likely to be produced by an informed agent, and reward the agent less than what he would have been rewarded in the second best; otherwise, she rewards him more than what he would have been rewarded in the second best. The optimal contract to deter information acquisition offers a higher-powered (lower-powered) incentive than offered in the second best if the density of state of nature is increasing (decreasing). This serves as a complement to the literature on information acquisition with the presence of moral hazard mentioned in the literature review, which attributes a higher-powered incentive to inducing information acquisition that generates a mean preserving signal of the random noise. I argue that, deterring information acquisition does not necessarily rely on a lower-powered incentive. It depends on the density of the state of nature.

References


