Extensions

- In this lecture, we extend the basic framework in two directions.
- Purpose: account for more facts.
- Capital utilization
The Labor Force, Employment, and Unemployment

Figure 9.9: Cyclical Behavior of U.S. Real GDP and Average Weekly Hours

- Proportionate deviation from trend
- Average weekly hours (blue)
- GDP (red)
The Labor Force, Employment, and Unemployment

• Basic Concepts and Empirical Patterns
  • \( u = \frac{\text{number unemployed}}{\text{labor force}} \)
  \[ = \frac{\text{labor force} - \text{number employed}}{\text{labor force}} \]
  \[ = 1 - \frac{\text{number employed}}{\text{labor force}} \]
  \[ = 1 - \text{employment rate} \]
  • \( \text{employment rate} = 1 - u \)
The Labor Force, Employment, and Unemployment

Figure 9.7: Cyclical Behavior of U.S. Real GDP and the Labor Force

Proportionate deviation from trend


Labor force GDP
The Labor Force, Employment, and Unemployment

Cyclical Behavior of U.S. Real GDP and the Employment Rate

The figure shows the deviation from trend of both the employment rate and GDP over time from 1960 to 2000. The employment rate is represented by the blue line, and the GDP is represented by the red line. The graph illustrates how both variables fluctuate together, indicating a relationship between economic growth and labor market conditions.
• The equilibrium business-cycle model is probably satisfactory for understanding fluctuations in the labor force and hours worked per worker.

• The real wage rate, \( w \) adjusts to equate the quantity of labor supplied, \( L^s \), to the quantity demanded, \( L^d \).

• However, this approach leaves unexplained the most important factor—the fluctuations in the employment rate or, equivalently, in the unemployment rate.
Search Models

Nobel Prize in Economics 2010, for Peter Diamond, Dale Mortensen and Christopher Pissarides
We used to assume that the capital stock is fixed in the short run (the shock period) and therefore the capital services supply curve is vertical.

This assumption simplifies the analysis greatly. Now let’s add one more realistic feature to the RBC model.

**Capital utilization rate**
- The fraction of the capital stock used in production.
- The Federal Reserve computes capacity utilization by expressing a sector’s output of goods as a percentage of the estimated “normal capacity” of each sector to produce goods.
- The Cyclical Behaviour of Capacity Utilization
Capital Input

Figure 9.6
Cyclical Behavior of U.S. Real GDP and Capacity Utilization

Proportionate deviation from trend


Capacity utilization  GDP
Capital Input

- Let’s incorporate the utilization rate $\kappa$ into our analysis
  - $\kappa$ (the Greek letter kappa) represent the utilization rate for the capital stock, $K$.
  - We have actually **exogenously** assumed that $\kappa = 1$!
  - Now $\kappa$ is a variable instead of constant number.
  - $Y = A \cdot F(\kappa K, L)$

- Given $K$, $\kappa K$, rises with the utilization rate, $\kappa$.
  - It is still true that $K_t = \bar{K}$ is constant in the shock period.
  - But now $\kappa$ can be variable chosen by **households**!
  - We need to understand how $\kappa$ behave during the business cycles.

- Let’s look at the supply and demand sides ···
The Demand for Capital Services
Firms maximize real profit

\[ \pi = A \cdot F[(\kappa K)^d, L^d)] - w \cdot L^d - R \cdot (\kappa K)^d \]

Demand curve ...
Capital Input

Figure 9.1 Demand for Capital Services

Demand curve for capital services
Capital Input

**Figure 9.2** Effect of an Increase in the Technology Level on the Demand for Capital Services

The diagram illustrates the relationship between MPK (Marginal Product of Capital) and the demand for capital services. The curves MPK (A) and MPK (A') show the change in the demand for capital services before and after an increase in technology. The point (K^d) indicates the demand for capital services at a certain level of capital input, and (K^d)' shows a new level of demand after the technology increase. The graph helps to understand how an increase in technology affects the demand for capital services.
The Supply of Capital Services

- For a given stock of capital, $K$, owners can supply more or less capital services per year by varying $\kappa$.
- We have so far assumed for simplicity that $\delta$ is constant over time.
- Since we introduce the utilization rate, we also need to modify this assumption.
- Assumption: $\delta'(\kappa) > 0$ and $\delta''(\kappa) > 0$
- One reason to set the utilization rate, $\kappa$, below its maximum is that increases in $\kappa$ tend to raise the depreciation rate, $\delta$. 
The Supply of Capital Services

- Net real income from supplying capital services =
  real rental payments − depreciation = $R \cdot \kappa K − \delta(\kappa) \cdot K$

- Net real income from supplying capital services
  $= K \cdot [R \cdot \kappa − \delta(\kappa)]$

- Rate of return from owning capital $= R \cdot \kappa − \delta(\kappa)$
Capital Input

Choosing the Capital Utilization Rate

- Rental income and depreciation (per unit of K)
- \( \kappa^* \) (capital utilization rate)
- \( (R/P) \cdot \kappa \)
- \( \delta(\kappa) \)
• Owners of capital (households) select the utilization rate, $\kappa^*$, that maximizes

$$\max_{\kappa} R \cdot \kappa - \delta(\kappa)$$

$$\downarrow$$

$$R = \delta'(\kappa^*)$$

• What happens, if $R$ increases?
Capital Input

**Figure 9.4**
Effect of an Increase in the Real Rental Price on the Capital Utilization Rate

- Rental income and depreciation (per unit of $K$)
- $\kappa^*$ (capital utilization rate)
- $\kappa^*$ (capital utilization rate)
- $(R/P) \cdot \kappa$
- $(R/P) \cdot \kappa$
- $\delta(\kappa)$
- $(R/P)' \cdot \kappa$
Market Clearing and Capital Utilization

- Thus, an increase in the real rental price raises the capital utilization rate; the higher real rental price makes it worthwhile to raise $\kappa$ despite the resulting increase in the depreciation rate, $\delta(\kappa)$.
- The supply curve slopes up because an increase in the real rental price, $R$, motivates a higher capital utilization rate, $\kappa$.
- The technology level, $A$, does not affect the choice of the capital utilization rate, $\kappa$. 
Figure 9.5  Clearing of the Market for Capital Services

$\frac{R}{P}$

$[(R/P)^*]'$

$(R/P)^*$

$(\kappa K)^s$

$(\kappa K)^s$

$(\kappa K)^d (A')$

$(\kappa K)^d (A)$

$(\kappa K)^d, (\kappa K)^s$
Market Clearing and Capital Utilization
- When the technology level rises to $A$, the demand curve for capital services shifts to the right, and the supply curve does not shift.
- Therefore the market clears at the higher real rental price, $R^*$, and the larger quantity of capital services, $[(κK)^*]$.

Interest rate: Pro-cyclical
- Rate of return on bonds = rate of return on ownership of capital
  - $i = R \cdot κ - δ(κ)$
- Increase in the technology level, $A$, raises the rate of return from owning capital, the interest rate, $i$, increases. The interest rate is still pro-cyclical in the model.
• We now have three reasons why real GDP rises in a boom and falls in a recession.
  - First a high or low technology level, $A$, causes real GDP to be correspondingly high or low.
  - A high or low $A$ causes $L$ to be correspondingly high or low.
  - A high or low $A$ causes the capital utilization rate, $\kappa$, and, thereby, the quantity of capital services, $\kappa K$, to be correspondingly high or low.