

MFIN 7003 – Module 1/2

Mathematical Techniques of Finance I

Session A: Aug 24, 09 – Oct 10, 09

Session B: Oct 12, 09 – Nov 28, 09

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Course Description

There are three main approaches to mathematical finance: the tree approach, the martingale approach and the partial differential equation approach. This course will present these three approaches and their applications to pricing and hedging financial derivatives. The corresponding numerical methods of the three approaches are lattice method, Monte Carlo simulation method, and finite difference method. We might briefly introduce them. Along the lectures, we will also review necessary mathematics, such as calculus, partial differential equation, applied probability and stochastic calculus. After taking this course, students should be able to fully understand no-arbitrage theory, risk-neutral probability, martingale, and Black-Scholes equation. The purpose of this course is to lay down a solid mathematical foundation for students to learn more advanced topics in financial engineering and risk management, such as exotic options, interest rate derivatives and credit risk models.

Prerequisites

- MFIN 6003 Derivative Securities

Grading

- | | |
|-------------------------------|-----------|
| • Four Homework (individual) | 52% |
| • One final exam | 40% |
| • <u>Course participation</u> | <u>8%</u> |
| • Total | 100% |

Course Intended Learning Outcomes (CILOs)

Upon completing this course, students should be able to:

- CILO01 Distinguish among three main approaches to mathematical finance and their applications to pricing and hedging financial derivatives. .
- CILO02 Understand no-arbitrage theory, risk-neutral probability, martingale, and Black-Scholes equation.
- CILO03 Develop a solid mathematical foundation to learn more advanced topics in financial engineering and risk management.

Alignment of program ILOs and course ILOs

Program ILOs	Course ILOs
Acquisition and internalization of knowledge of economics & finance	CILO01, CILO02, CILO03
Application and integration of knowledge	CILO01, CILO02, CILO03
Developing global outlook	CILO03
Mastering communication skills	CILO03
Inculcating leadership	CILO03

Teaching and Learning Activities (TLA)

TLA1	Lecture	Instructor will give lectures on major concepts and issues.
TLA2	Consultation	Instructor will provide consultation to students' questions via emails, phones and face-to-face meetings.

Alignment Among Course Intended Learning Outcome, Teaching and Learning Activities and Assessment Tasks:

Learning Outcome	Teaching and learning activity (TLA)	Assessment Tasks
CILO01	TLA1, TLA2	Homework, Course participation, Final Exam
CILO02	TLA1, TLA2	Homework, Course participation, Final Exam
CILO03	TLA1, TLA2	Homework, Course participation, Final Exam

Standards of Assessment

Grade	Grade Definition	Description
A+, A, A-	High distinction 80%-100%	Strong evidence of superb ability to fulfill the intended learning outcomes of the course at all levels of learning: describe, apply, evaluate, and synthesis.
B+, B, B-	Distinction 70%-79%	Strong evidence of the ability to fulfill the intended learning outcomes of the course at all levels of learning: describe, apply, evaluate, and synthesis.
C+, C, C-	Credit pass 60%-69%	Evidence of adequate ability to fulfill the intended learning outcomes of the course at low levels of learning such as describe and apply but not at high levels of learning such as evaluate and synthesis
D	Pass 50%-59%	Evidence of basic familiarity with the subject.
F	Fail <50%	Little evidence of basic familiarity with the subject.

Academic Conduct

- The University Regulations on academic dishonesty will be strictly enforced! Please check the University Statement on plagiarism on the web: <http://www.hku.hk/plagiarism/>
- Academic dishonesty is behavior in which a deliberately fraudulent misrepresentation is employed in an attempt to gain undeserved intellectual credit, either for oneself or for another. It includes, but is not necessarily limited to, the following types of cases:
 - a. Plagiarism - The representation of someone else's ideas as if they are one's own. Where the arguments, data, designs, etc., of someone else are being used in a paper, report, oral presentation, or similar academic project, this fact must be made explicitly clear by citing the appropriate references. The references must fully indicate the extent to which any parts of the project are not one's own work. Paraphrasing of someone else's ideas is still using someone else's ideas, and must be **acknowledged**.
 - b. Unauthorized Collaboration on Out-of-Class Projects - The representation of work as solely one's own when in fact it is the result of a joint effort. Where a candidate for a degree or other award uses the work of another person or persons without due acknowledgement:
 - 1. The relevant Board of Examiners may impose a penalty in relation to the seriousness of the offence;
 - 2. The relevant Board of Examiners may report the candidate to the Senate, where there is *prima facie* evidence of an intention to deceive and where sanctions beyond those in (1) might be invoked.

Reference books

- Baxter, Martin, and Andrew Rennie, 1996, Financial calculus: an introduction to derivative pricing, Cambridge University Press.
Call #: [332.63221 B3](#)
- Hull, John, 2006, Options, Futures, & Other Derivatives, 6th edition, Prentice Hall.
Call #: [LB 332.632 H91](#)
- Ross, Sheldon M., 2003, An elementary introduction to mathematical finance: options and other topics, 2nd edition, Cambridge University Press.
Call #: [332.60151 R826 e](#)
- Wilmott, Paul, 2006, *Paul Wilmott on Quantitative Finance*, Volume One (3 volumes in total), 2nd edition, John Wiley & Sons.
Call #: [332.64 W7 p](#) v.1

Course Topics

Class meets on Monday and Wednesday evenings, 6:45pm-9:45pm, at B4 Town Center or United Conference Center (UCC).

Tentative Course Outline (subject to change)

Lecture 1 Aug 24, 2009 Mon.	<p>A: Introduction Level Test</p> <hr/> <p>B: The Tree Approach I -- Binomial Branch: Construction of a replicating portfolio; The law of one price; No arbitrage; True probability measure; Risk-neutral probability measure; Martingale Lecture Note 1</p> <p>Reference: chapter 2.1 of the book by Baxter, Martin, and Andrew Rennie</p> <p>Homework 1 It is due on Wed. Sept 2, 2009.</p> <p>Handwritten notes. You need to click shift+ctrl+minus to rotate the view.</p>
Lecture 2 Aug 26, 2009 Wed.	<p>In-class Exercise 1</p> <p>The Tree Approach II -- Binomial Tree: Binomial/trinomial tree model; Recombinant and non-recombinant trees; Backwards induction; Path probability; General rule of derivatives pricing by binomial tree (risk neutral valuation); Pricing European options, American options and exotic options (lookback and Asian options) by binomial tree; Early exercise; Monte Carlo simulation; Kolmogorov's strong law of large numbers</p> <p>Lecture Note 2</p> <p>Reference: chapter 2.2 of the book by Baxter, Martin, and Andrew Rennie</p> <p>Handwritten notes.</p>

<p>Lecture 3 Aug 31, 2009 Mon.</p>	<p>Normal distribution; Lognormal distribution; Taylor Expansion; Review of Calculus; Review of probability</p> <p>Lecture Note 3 Math Review 1 Math Review 2</p> <p>Homework 2 It is due on Wed. Sept 9, 2009.</p> <p>Handwritten notes. You need to click shift+ctrl+minus to rotate the view.</p>
<p>Lecture 4 Sept 2, 2009 Wed.</p>	<p>Homework 1 is due today.</p> <p>In-class Exercise 2 (answers) Exercises on Differentiation Rules (answers)</p> <p>Stochastic Differential Equations: The quantitative finance timeline; Brownian motion; Geometric Brownian motion; Ito's lemma; Ito's integration; Modeling Stock Prices</p> <p>Lecture Note 4 Article: The quantitative finance timeline by Paul Wilmott Supplement to Lecture 4</p> <p>Handwritten notes. You need to click shift+ctrl+minus to rotate the view.</p>
<p>Lecture 5 Sept 7, 2009 Mon.</p>	<p>The Martingale Approach (Risk-neutral Pricing Method) I: Black-Scholes economy; Martingale revisit; Change of measure; General rule of derivatives pricing by martingale approach</p> <p>Lecture Note 5A</p> <p>Article: The father of FE_PeterCarr_BloombergMarkets.pdf</p> <p>Handwritten notes. You need to click shift+ctrl+minus to rotate the view.</p>
<p>Lecture 6 Sept 9, 2009 Wed.</p>	<p>Homework 2 is due today.</p> <p>Tradable and non-tradable; Pricing Foreign Exchange; Pricing Equities with Dividends</p> <p>Lecture Note 5B</p> <p>Homework 3 It is due on Mon. Sept 21, 2009.</p> <p>Handwritten notes. You need to click shift+ctrl+minus to rotate the view.</p>

<p>Lecture 7 Sept 14, 2009 Mon.</p>	<p>For those that are not familiar with ordinary integration, please try to read Section 4.1 – Section 4.2 (from pages 52 to 56) of Math Review 3 and try to do Exercise on Integration Rules before class if you have time. Answers to Exercise on Integration Rules</p> <p>The Martingale Approach II -- Deriving Black-Scholes Formula by Martingale Approach: BS formula for vanilla call and vanilla put; Put-call parity; Review of Riemann Integration</p> <p>Lecture Note 6</p> <p>Handwritten notes. You need to click shift+ctrl+minus to rotate the view.</p>
<p>Lecture 8 Sept 16, 2009 Wed.</p>	<p>In-class Exercise 3</p> <p>The Black-Scholes and Merton Approach (The Partial Differential Equation (PDE) Approach) I -- Deriving Black-Scholes Partial Differential Equation: Market price of risk; Delta-neutral portfolio; Correlation; Perfect hedging; Portfolio analysis; Capital Asset Pricing Model (CAPM)</p> <p>Lecture Note 7 An Article by Black</p> <p>Summary of risk neutral valuation</p> <p>Homework 4 It is due on Mon. Sept 28, 2009.</p> <p>Handwritten notes. You need to click shift+ctrl+minus to rotate the view.</p>
<p>Lecture 9 Sept 21, 2009 Mon.</p>	<p>Homework 3 is due today.</p> <p>PDE Approach II -- Deriving Black-Scholes Formula from Black-Scholes PDE: Heat equation; Delta function; Green's function; Transformation method in solving PDE</p> <p>Lecture Note 8 LUO Xingguo's PPT</p> <p>Handwritten notes. You need to click shift+ctrl+minus to rotate the view.</p>
<p>Lecture 10 Sept 23, 2009 Wed.</p>	<p>Asymptotic Analysis of the Black-Scholes Formula and Implied Volatility; A brief discussion on the question of butterfly spread in Group Project 2.</p> <p>Lecture Note 9</p> <p>Handwritten notes. You need to click shift+ctrl+minus to rotate the view.</p>

Lecture 11 Sept 28, 2009 Mon.	Homework 4 is due today. Deriving Greeks and hedging with Greeks Lecture Note 10 Handwritten notes. You need to click shift+ctrl+minus to rotate the view.
Lecture 12 Sept 30, 2009 Wed.	Exam Review and Q&A Good Luck on Exams! Handwritten notes. You need to click shift+ctrl+minus to rotate the view.

Final exam

- **Venue: TBA**
- **4:30 – 6:30 pm on Oct 10, 2009 Saturday for session A and Nov 28 2009 Saturday for session B**
- **Open books and open notes**