

Exercise 1: Adverse Selection (Due: Jan./29 in class)

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Seller

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- In this exercise, we deal with the opposite case.

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where $\theta_i \in \{\theta_1, \theta_0\}$ and $\theta_1 > \theta_0$.

- Only the seller knows the realization θ_i , and the buyer does not know.

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- Only the seller knows the realization θ_i , and the buyer does not know.
- Economically, θ_i reflects the price of inputs the seller pays, and it can be either high (i.e., θ_1) or low (i.e., θ_0).

Buyer

- The payoff to the buyer is

$$b(x_i) - w_i,$$

where $b(x_i) \equiv 2x_i^{\frac{1}{2}}$ is the utility generated from the consumption of x_i units of the good, and w_i is the payment the buyer makes to the seller.

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- Suppose $\theta_1 = 2$ and $\theta_0 = 1$.
- Suppose the probability with which θ_0 is realized is $p(=0.5)$, and the probability with which θ_1 is realized is $1 - p(=0.5)$.
- Suppose the outside option for both type of the seller is zero.

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- Principal: buyer; Agent: seller (different from the lecture).

Question One: First Best

- 1 Suppose both the buyer and the seller know the realization of θ_i (i.e., no private information), what is the objective function of the buyer? What is the constraint the buyer faces?
- 2 What is the optimal contract (i.e., $(w_i^{FB}, x_i^{FB})_{i \in \{0,1\}}$)? Which type of agent has incentives to lie?

Question Two: Second Best

- 1 Now, suppose only the seller knows the realization of θ_i . Write down the optimization problem and the constraints the seller faces.
- 2 Which constraints do not bind in equilibrium? Which constraints bind in equilibrium?
- 3 How can we rewrite the optimization problem after eliminating the constraints?
- 4 What is the optimal contract (i.e., $(w_i^{SB}, x_i^{SB})_{i \in \{0,1\}}$)?
- 5 Which type of the agent receives information rents? Which output level (i.e., x_i^{SB}) is distorted or different from the first best level? Why?