

Exercise 2: Moral Hazard (Due: Jan./29 in class)

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- Cost of implementing a project is $I = 5$, and there are two possible outcomes for the project: S (*uccess*) and F (*ailure*).
- Return is $r_S = 30$ in the case of S and zero otherwise (i.e., $r_F = 0$).

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- Suppose there are an investor and an entrepreneur.
- Cost of implementing a project is $I = 5$, and there are two possible outcomes for the project: S (*uccess*) and F (*ailure*).
- Return is $r_S = 30$ in the case of S and zero otherwise (i.e., $r_F = 0$).
- The entrepreneur can choose between working hard (a_h) and shirking (a_l):
 - ▶ If it is a_h : The probability with which S happens is $p_h = 0.6$; the utility cost for the entrepreneur is $d_h = 2$.
 - ▶ If it is a_l : The probability with which F happens is $p_l = 0.2$; the utility cost for the entrepreneur is $d_l = 0$.

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- Entrepreneur is risk averse, and his utility function is $u(x) = 2x^{1/2} - 2$. This leads to the expected utility as

$$U = p_j * u(w_S) + (1 - p_j) * u(w_F) - d_j - 2$$

or

$$U = p_j * 2 * w_S^{1/2} + (1 - p_j) * 2 * w_F^{1/2} - d_j - 2,$$

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- Remember investor's and entrepreneur's utilities depend on the action taken by the entrepreneur (i.e., a_h or a_l).
- The Entrepreneur's outside option is zero.

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- Which action does the principal want the entrepreneur to choose? What is the optimal contract?
- Is it profitable to implement this project (for the investor)?
- Explain your findings using economics intuitions.

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- What is the optimal contract?
- Which action does the investor want the entrepreneur to choose?
- Is it profitable to implement this project? What happens, if the investment cost $I = 13$.
- Explain your findings using economics intuitions.

Question Three: Second Best and Maximum Punishment

- Suppose the investor *cannot* observe the entrepreneur's action. However, the realization of the project is observable and verifiable.
- Now, suppose that p_h now becomes one. Namely, the success of the project is assured, when the entrepreneur works hard.

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- Now, suppose that p_h now becomes one. Namely, the success of the project is assured, when the entrepreneur works hard.
- What is the best thing the investor can do in order to incentivize her entrepreneur to work hard? Can the first best outcome be achieved?