Solution to Exercise 2: Moral Hazard

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Question One: First Best

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- If the principal wants the agent to choose a_l (i.e., shirking):
  - The constraint is

$$0.2 * 2w_S^{1/2} + 0.8 * 2w_F^{1/2} - 0 - 2 \geq 0, \quad (PCl)$$

    and the solution is $w_S = w_F = 1$ (remember the agent is risk averse)
  - Payoff to the principal is

$$V = 0.2 * (30 - 1) + 0.8 * (0 - 1) = 5.$$

Question One: First Best (Cont.)

- If the principal wants the agent to choose a_h (i.e., working hard):
  - The constraint is

$$0.6 * 2w_S^{1/2} + 0.4 * 2w_F^{1/2} - 2 - 2 \geq 0, \quad (PCh)$$

    and the solution is $w_S = w_F = 4$ (remember the agent is risk averse)
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- Since the investment cost is $I = 5(< 14)$, it is profitable to implement the project.
- First Best (FB) is achieved, since there is no information problem.
  - The agent is perfectly ensured (no welfare loss due to risk aversion). FB effort choice (i.e., a_h) is chosen. The project is implemented.

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- Now we have two constraints (PC and IC), since the action is unobservable. The optimization problem is

$$\max_{w_S, w_F} \quad 0.6(30 - w_S) + 0.4(0 - w_F)$$

$$0.6 * 2w_S^{1/2} + 0.4 * 2w_F^{1/2} - 2 - 2 \geq 0, \quad (PCh)$$

$$0.6 * 2w_S^{1/2} + 0.4 * 2w_F^{1/2} - 2 - 2$$

$$\geq 0.2 * 2w_S^{1/2} + 0.8 * 2w_F^{1/2} - 0 - 2, \quad (ICh)$$

where *ICh* says that the agent is *worse off* by choosing a_h than choosing a_l.

Question Two: Second Best with a_h (Cont.)

- Observations:
  - $w_S > w_F$; PCh holds with equality; ICh holds with equality.
- The optimal contract is

$$2w_S^{1/2} - 2 = d_h + \frac{1-p_h}{p_h - p_l}(d_h - d_l) = 4$$

and

$$2w_F^{1/2} - 2 = d_h - \frac{p_h}{p_h - p_l}(d_h - d_l) = -1$$

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$$w_S = 9; w_F = 1/4.$$

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- Substituting values of p_h, p_l, d_h and d_l into above equations, we arrive at

$$w_S = 9; w_F = 1/4.$$

- Net payoff to the principal is

$$V = 0.6 * (30 - 9) + 0.4 * (0 - 1/4) = 12.5 (> 5).$$

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& \geq 0.6 * 2w_S^{1/2} + 0.4 * 2w_F^{1/2} - 2 - 2, \quad (ICl)
\end{aligned}
$$

- Under perfect insurance (i.e., $w_S = w_F$), *ICl* is met trivially *rightarrow* Optimal contract is the same as in the FB case:

$$w_S = w_F = 1; \;\; V = 0.2 * (30 - 1) + 0.8 * (0 - 1) = 5(< 12.5).$$

Question Two: Conclusion

- Optimal choice is to incentivize the agent to work hard and set $w_S = 9$ and $w_F = 1/4$.
- It is profitable to implement the project, since $V(a_h) = 12.5 > I = 5$. Actually, if the principal incentivizes the agent to choose a_l, she can only break even.

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- Compared with the FB case, the principal's payoff goes down from 14 to 12.5.
  - The payment in case of S has to be higher than the payment in case of L: Incentive concern.
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- Compared with the FB case, the principal's payoff goes down from 14 to 12.5.
  - The payment in case of S has to be higher than the payment in case of L: Incentive concern.
  - Since the agent is risk averse and earns zero payoff in equilibrium, the payoff to the principal is lower in the SB case.
- If the investment cost $I = 13$, then the project is not profitable due to the information problem, since $12.5 < 13$. However, it is profitable in the FB case, since $14 > 13$.
- Market is closed, and there is welfare loss.

Question Three: Second Best and Maximum Punishment

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- Furthermore, there is no loss for the principal, since the agent receives a fixed payment when working hard (remember $p_h = 1$ now).

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- Optimal contract:
  - $w_S = 4$: PCh is satisfied since

    $$p_h * 2 * w_S^{1/2} + (1 - p_h) * 2 * w_F^{1/2} - d_h - 2 = 1 * 2 * (4)^{1/2} - 2 - 2 = 0.$$
  - $w_F = 0$: ICh is met since

    $$\begin{aligned} & p_h * 2 * w_S^{1/2} + (1 - p_h) * 2 * w_F^{1/2} - d_h - 2 = 0 \\ > \quad & p_l * 2 * w_S^{1/2} + (1 - p_l) * 2 * w_F^{1/2} - d_l - 2 = -1.2. \end{aligned}$$

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- Maximum punishment: Realization of F is possible, only when agent shirks. Therefore, principal wants to punish agent when F is realized.