
Suppose there are two plants needed for the production of a good whose market price is $P$. These two plant are led by two managers. There are two possible organization structure: non-integration and integration. We discuss these two cases one by one in what follows.

(1) We consider the non-integration case first. In this case, the total profit from completing and selling the good is

$$[1 - (a - b)^2]P,$$  

where $a$ and $b$ are the actions chosen by the managers of the two plants. Here, the difference between $a$ and $b$ measures the loss from coordination failure inside the firm. Specifically, if the two managers choose the the action (i.e., $a = b$), there is no loss from coordination failure. Suppose the payoff to plant manager $A$ who chooses action $a$ is

$$s[1 - (a - b)^2]P - (1 - a)^2,$$  

while the payoff to plant manager $B$ who chooses action $b$ is

$$(1 - s)[1 - (a - b)^2]P - b^2.$$  

There is a managerial cost paid by manager $A$, $(1 - a)^2$, if he chooses an action different from 1. Similarly, there is a managerial cost paid by manager $B$, $b^2$, if he chooses an action different from 0. In additional, each manager only receives a fraction of the profit as his income (i.e., $s$ and $1 - s$ and $0 < s < 1$).

Now, let us solve the model for the non-integration case. First, what are the optimal actions chosen by the two managers? Second, what are the realized total profit and the payoffs going to managers $A$ and $B$?

Answer: The objective function of manager $A$ is

$$\max_a s[1 - (a - b)^2]P - (1 - a)^2,$$

and the objective function of manager $B$ is

$$\max_b (1 - s)[1 - (a - b)^2]P - b^2.$$  

Therefore, the optimal actions are

$$a^* = 1 - s\frac{P}{1+P}$$

and

$$b^* = (1 - s)\frac{P}{1+P}.$$
The realized total profit is

\[ [1 - (a^* - b^*)^2]P = P \left[1 - \left(\frac{1}{1+P}\right)^2\right]. \]

The payoff to manager A is

\[ s[1 - (a^* - b^*)^2]P - (1 - a^*)^2 = sP \left[1 - \left(\frac{1}{1+P}\right)^2\right] - s^2 \left(\frac{P}{1+P}\right)^2, \]

and the payoff to manager B is

\[ (1 - s)[1 - (a^* - b^*)^2]P - b^*^2 = (1 - s)P \left[1 - \left(\frac{1}{1+P}\right)^2\right] - (1 - s)^2 \left(\frac{P}{1+P}\right)^2. \]

(2) We still consider the non-integration case and the optimization problem defined above. What is the total surplus generated by the production (hint: the managerial costs \((1 - a)^2\) and \(b^2\), need to be taken into account when you calculate the total surplus) What happens to the coordination failure (i.e., \(a - b\)) and the realized total profit, when the price of the good, \(P\), increases? Why?

Answer: The total surplus generated by the production is

\[ [1 - (a^* - b^*)^2]P - (1 - a^*)^2 - b^*^2 = P \left[1 - \left(\frac{1}{1+P}\right)^2\right] - \left[s^2 + (1 - s)^2\right] \left(\frac{P}{1+P}\right)^2. \]

The degree of the coordination failure, \(a - b = 1/(1 + P)\), which decreases in the market price \(P\). The realized total profit equals

\[ P \left[1 - \left(\frac{1}{1+P}\right)^2\right], \]

which increases in the market price \(P\).

The economic intuition is the following. Since the cost of coordination for both managers (i.e., \((1 - a)^2\) and \(b^2\)) does not depend on the price of the good, an increase in \(P\) makes both managers care about the total profit (i.e., \([1 - (a - b)^2]P\)) more. As a result, the degree of the coordination failure decreases. The total profit increases, since both the degree of the coordination failure decreases and the market price, \(P\), increases.

(3) We consider the integration case now. In this case, there is a manager in the headquarters (i.e., manager \(H\)) who chooses the actions of \(a\) and \(b\). The total profit from completing and selling the good is still

\[ [1 - (a - b)^2]P. \] (4)

Suppose the fraction of the final profit that goes to the manager in the headquarters is \(s_H\) which is very small. The share of the final profit that go to managers \(A\) and \(B\) are \(s_A\) and \(s_B\). Obviously, we have \(s_H + s_A + s_B = 1\). The managerial costs, \((1 - a)^2\) and \(b^2\), are still paid by the two plant managers (i.e., manager \(A\) and manager \(B\)). Furthermore, we assume that managers \(A\) and \(B\) are financially constrained and cannot make side payments to manager \(H\) in order to influence manager \(H\)'s decisions.
Now, let us solve the model for the integration case. First, what are the optimal actions (i.e., $a$ and $b$) chosen by manager $H$? Second, what does manager $H$ choose, if he wants to maximize her payoff and minimize the total managerial cost at the same time (i.e., $(1 - a)^2 + b^2$)? What is the realized final profit?

Answer: Since manager $H$’s payoff is

$$s_H[1 - (a - b)^2]P,$$

she chooses $a = b$. Furthermore, if she want to minimize the total managerial cost at the same time, then the objective function is

$$\min_{a,b} (1 - a)^2 + b^2 \quad s.t. \quad a = b,$$

which leads to the solution that $a^I = b^I = \frac{1}{2}$. The The realized final profit is

$$[1 - (a^I - b^I)^2]P = P.$$

(4) We still consider the integration case and the optimization problem defined above. What is the total surplus generated by the production (hint: the managerial costs, $(1 - a)^2$ and $b^2$, need to be taken into account when you calculate the total surplus)? What happens to the coordination failure (i.e., $a - b$) and the realized total profit, when the price of the good, $P$, increases? Why?

Answer: The total surplus is

$$[1 - (a^I - b^I)^2]P - [(1 - a^I)^2 + b^I^2] = P - \frac{1}{2},$$

which increases in price, $P$. The coordination failure is

$$a^I - b^I = 0,$$

which is independent of $P$. Coordination failure is independent of $P$, since manager $H$ only considers the total profit and always makes the two actions perfectly be aligned with each other. Since, the coordination failure is not affected by the price, $P$, the total profit increases one-to-one with the price.

(5) Now, we compare between the above two cases. Suppose the share of the total profit that go to the two managers in the case of non-integration are $s = 0.2$ and $1 - s = 0.8$. What is the condition on $P$ that makes the total surplus in the case of integration be bigger than that in the case of non-integration? Can you explain why?

Answer: When $s = 1 - s = 0.5$, the total surplus in the case of non-integration is

$$S^{NI} = P\left[1 - \left(\frac{1}{1+P}\right)^2\right] - 0.68\left(\frac{P}{1+P}\right)^2,$$

while the total surplus in the case of integration is

$$S^I = P - \frac{1}{2}.$$
Simple calculation shows that $S^I > S^{NI}$ if and only if

$$0.36 P^2 > 1$$

or

$$P > \frac{5}{3}.$$ 

The intuition is the following. The benefit of integration is that the loss (in the final profit) due to coordination failure goes to zero, and the cost of integration is that the total managerial cost is too high due to over-coordination (i.e., $a^I = b^I = 0.5$). An increase in the price, $P$, does not affect the cost of integration and increases the benefit of integration. Therefore, the total surplus in the case of integration is more likely to be bigger than that in the case of non-integration, when the price is high.