Lecture Three: Adverse Selection

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Motivation

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  - *Information asymmetry* is everywhere.

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  - The insurance market.
  - Dropouts of classes after the exam.
  - Others?
- Why do we care about it?
  - *Information asymmetry* is everywhere.
  - Neoclassical economics and frictions. (Pareto improvement and the *mechanism design*).
  - Market efficiency and welfare (e.g., *market shutdown*).

An Example

- Adverse selection: Hidden information.
- A bilateral contracting: an employer (the principal) and an employee (the agent).
  - The employee has one unit of time.
  - Output equals $\theta_i(1 - l_i)$, where $i \in H, L$ ($\theta_H > \theta_L$ two types).
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  - The employer's utility is

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    and the employee's utility is

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- No asymmetric information$\rightarrow$ $\theta_i = t_i$ and $l_i = 0$ for all i (First-Best or FB). (Derivation?)
- Not *incentive compatible*: L type guy wants to mimic H type guy. (Why?)

Mathematical Formulation

- Maximize the principal's payoff under constraints:

$$\begin{aligned} \max_{l_j, t_j} \quad & p_L U(\alpha\theta_L(1 - l_L) - t_L) + p_H U(\alpha\theta_H(1 - l_H) - t_H) \\ s.t. \quad & u(\theta_L l_L + t_L) \geq u(\theta_L), \\ & u(\theta_H l_H + t_H) \geq u(\theta_H), \\ & u(\theta_H l_H + t_H) \geq u(\theta_H l_L + t_L), \\ & u(\theta_L l_L + t_L) \geq u(\theta_L l_H + t_H). \end{aligned}$$

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& u(\theta_L l_L + t_L) \geq u(\theta_L l_H + t_H).
\end{aligned}
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- Revelation Principle.
- First two inequalities: Individual Rationality (IR) or Participation Constraints (PC).
- Last two inequalities: Incentive Compatibility (IR) constraints.

Basic Principles

- One option: $l_H = l_L = 0$ and $t_H = t_L = \theta_H$ (a simple contract).
  - L type agent: information rents (monopoly power).
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- In general, the tradeoff between *information rents* and *allocative efficiency*.
  - The agent that has incentive to mimic (L type): information rents.
  - The agent that is mimicked (H type): allocative distortion.
- How to solve the problem?
  - If the uninformed party makes the contract: screening. (i.e., Mirrlees (1971), Myerson (1979), Stiglitz and Weiss (1981))
  - If the informed party makes the contract: signalling. (i.e., Spence (1973, 1974))

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  - The seller's payoff: $\pi = T - cq$.
  - The buyer's payoff: $u(\theta_i, q, T) = \theta_i v(q) - T \geq \bar{u}$ where $i \in \{H, L\}$.
  - We have $\theta_H > \theta_L$.
  - Asymmetric information: seller does not know θ_i. $Prob(\theta = \theta_L) = \beta$.

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  - We have $\theta_H > \theta_L$.
  - Asymmetric information: seller does not know θ_i. $Prob(\theta = \theta_L) = \beta$.
- FB (no information friction):

$$\max_{T_i, q_i} \quad T_i - cq_i$$
$$s.t. \quad \theta_i v(q_i) - T_i \geq \bar{u}.$$

Solution (type-specific two-part tariffs): $\theta_i v^{'}(\tilde{q}_i) = c$ and $\theta_i v(\tilde{q}_i) = \tilde{T}_i + \bar{u}$.

Incentive Compatibility

- However, this is not incentive-compatible for H type. I.e., H type wants to mimic L type:

$$\tilde{T}_L = \theta_L v(\tilde{q}_L) - \bar{u}$$

If H type takes L type's contract, he receives

$$\theta_H v(\tilde{q}_L) - \tilde{T}_L = (\theta_H - \theta_L) v(\tilde{q}_L) + \bar{u} > \bar{u}.$$

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- On the other hand, payment to H type is

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- Incentive compatible for L type. If L type takes H type's contract, he receives

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- Intuition: Transfer to employer is too much for H type, since its willingness to pay is high.

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- Let

$$D(P) = \beta D_L(P) + (1 - \beta) D_H(P)$$

and

$$S(P) = \beta S_L(P) + (1 - \beta) S_H(P).$$

Two Types: Linear Pricing (Cont.)

- Maximization problem for seller:

$$\max_{P}(P-c)D(P),$$

where $\theta_i v'(D_i(P)) = P$.

- Solution:

$$P_m = c - \frac{D(P)}{D'(P)}.$$

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- Rents for both types of buyers:

$$S_L(P) > 0; \quad S_H(P) > 0.$$

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- Inefficiently low consumption:

$$\theta_i v^{'}(q) = P > c.$$

Two Types: Single Two-Part Tariff

- Single two-part tariff: $(Z,\ P)$ (one type of contract)
- Z: fixed fee. P: unit price.
  - H type always participates.

$$\max_{P}\ Z + (P - c)D(P),$$

where $Z = S_L(P)$ and $S_L(P) = \theta_L v[D_L(P)] - PD_L(P)$: buyer's net surplus.

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  - Solution:$P_d = c - \frac{D(P)+S_L'(P)}{D'(P)}$ and $S_L'(P) = -D_L'(P) > 0$.
- Extract all rents from L type and leave some rents to H type.
- Single two-part tariff is better than linear pricing, and $P_m > P_d > P_c = c$.

Graphical Representation

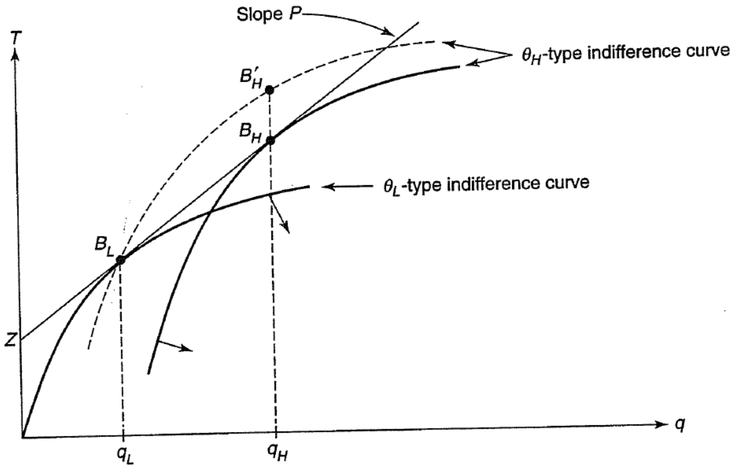

Figure 2.1
Two-Part Tariff Solution

Two Types: IC Contracts

- Second-Best outcome (SB) and nonlinear pricing (remember the revelation principle):

$$
\begin{aligned}
\max_{T_i, q_i} \quad & \beta[T(q_L) - cq_L] + (1-\beta)[T(q_H) - cq_H] \\
s.t. \quad & \theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L, \quad (ICH) \\
& \theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H, \quad (ICL) \\
& \theta_H v(q_H) - T_H \geq 0, \quad (IRH) \\
& \theta_L v(q_L) - T_L \geq 0. \quad (IRL)
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\end{aligned}
$$

- First two inequalities: IRs.
- Last two inequalities: ICs.

IC Contracts: Two Binding Constraints

- *IRH is redundant*, because
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- *ICH* + *ICL* $\rightarrow q_H \geq q_L$ (*monotonicity*):
  $\theta_H[v(q_H) - v(q_L)] \geq T_H - T_L \geq \theta_L[v(q_H) - v(q_L)] \rightarrow q_H \geq q_L.$

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  $\theta_H[v(q_H) - v(q_L)] \geq T_H - T_L \geq \theta_L[v(q_H) - v(q_L)] \rightarrow q_H \geq q_L.$
- *ICH* holds with equality + $(q_H \geq q_L) \rightarrow$ *ICL is redundant.*

$$\begin{aligned} & \theta_H[v(q_H) - v(q_L)] = T_H - T_L \\ \rightarrow \quad & \theta_L[v(q_H) - v(q_L)] \leq T_H - T_L \\ \rightarrow \quad & \theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H. \end{aligned}$$

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- *ICH* holds with equality + $(q_H \geq q_L) \rightarrow$ *ICL is redundant*.

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- *ICL* is redundant $\rightarrow$ *IRL* should hold with equality.
  - If not, principal can *increase* t_L until *IRL* holds with equality.

IC Contracts: Simplified Problem

- Reduced problem:

$$\max_{q_L, q_H} \beta[\theta_L v(q_L) - cq_L] + (1-\beta)[\theta_H v(q_H) - (\theta_H - \theta_L)v(q_L) - cq_H],$$

where $(\theta_H - \theta_L)v(q_L)$ captures information rents.

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  where $(\theta_H - \theta_L) v(q_L)$ captures information rents.
- Solution:
  - No distortion at the top: $\theta_H v^{'}(q_H^*) = c$.
  - Downward distortion at the bottom (due to information rents): $\theta_L v^{'}(q_L^*) = \frac{c}{1 - \left(\frac{1-\beta}{\beta} \frac{\theta_H - \theta_L}{\theta_L}\right)} > c$.
- Tradeoff between allocative efficiency and rent reduction.