Lecture Four: Moral Hazard

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Motivation

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  - Corporate governance and separation of ownership and control inside the firm (Jensen and Meckling, 1876; Fama, 1980)
  - Bonus and incentive pay for employees.
  - Sharecropping

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  - Sharecropping
  - Grades and student performance.
  - Tenure system adopted in the US and elsewhere.
- Why do we care about it?
  - Moral hazard problem is everywhere.
  - Neoclassical economics and frictions. (Pareto improvement and *mechanism design*).
  - Market efficiency and welfare (first-best and second-best).

Digressions: Organizational Failure is Everywhere

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- Case four: 1994 Black Hawk shootdown incident (coordination): https://en.wikipedia.org/wiki/1994_Black_Hawk_shootdown_incident

An Example

- Moral hazard: Hidden action.
- A bilateral contracting: an employer (the principal) wants to incentivize her employee (the agent) to work.
    - Employee's effort: a.
    - Output $q \in \{0, 1\}$ (binary).
    - $Pr(q = 1|a) = p(a)$; $p^{'}(a) > 0$ with $p^{''}(a) < 0$ (a concave function).

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  - $Pr(q = 1|a) = p(a)$; $p^{'}(a) > 0$ with $p^{''}(a) < 0$ (a concave function).
  - Utility functions:
    1. $V(q - w)$ for the principal
    2. $u(w) - \psi(a)$ for the agent.
  - $\psi(a)$: cost of exerting effort (standard assumptions)

First-Best

- F-B case (no information asymmetry):

$$\max_{a,w_i} \quad p(a)V(1-w_1) + [1-p(a)]V(-w_0)$$

$$s.t. \quad p(a)u(w_1) + [1-p(a)]u(w_0) - a \geq \bar{u}. \qquad (PC)$$

- Normalize outside option $\bar{u}$ to zero.

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- Normalize outside option $\bar{u}$ to zero.
- λ: Lagrange multiplier for PC.
- Borch rule (Borch, 1962):

$$\frac{V^{'}(1-w_1)}{u^{'}(w_1)} = \lambda = \frac{V^{'}(-w_0)}{u^{'}(w_0)}$$

- Two cases:
  - Risk-neutral principal ($V(x)=x$): $u(w^*)=a^*$ and $p^{'}(a^*)=\frac{1}{u^{'}(w^*)}$.
  - Risk-neutral agent ($u(x)=x$): $w_1^*-w_0^*=1$ and $p^{'}(a^*)=1$.

Second-Best

- Formalize problem:

$$\begin{aligned} \max_{a,w_i} \quad & p(a)V(1-w_1)+[1-p(a)]V(-w_0) \\ s.t. \quad & p(a)u(w_1)+[1-p(a)]u(w_0)-a \geq \bar{u}; \qquad (PC) \\ & a \in \underset{\hat{a}}{\operatorname{argmax}}\; p(\hat{a})u(w_1)+[1-p(\hat{a})]u(w_0)-\hat{a}. \quad (IC) \end{aligned}$$

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$$a \in \operatorname*{argmax}_{\hat{a}} \; p(\hat{a})u(w_1)+[1-p(\hat{a})]u(w_0)-\hat{a}. \quad (IC)$$

- The FOC for agent:

$$p^{'}(a)[u(w_1)-u(w_0)]=1.$$

- We can use FOC to replace IC (not true in general)

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- Answer: Yes, if there are no resource constraints.
  - Set $w_1^* - w_0^* = 1 \to p'(a^*) = 1$ (i.e., selling the firm to manager).
  - Choose a small (and negative) enough w_0^* such that the PC of agent becomes an equality.
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  - Choose a small (and negative) enough w_0^* such that the PC of agent becomes an equality.
  - Since $\bar{u} = 0$, w_0^* must be negative.
- Answer: No, if there are resource constraints (i.e., $w_i \geq 0$).
  - Suppose it is possible $\rightarrow w_1 - w_0 = 1$ and $p^{'}(a) = 1$.
  - Agent's payoff$=p(a)u(w_1) + [1 - p(a)]u(w_0) \geq p(a)w_1 \geq p(a)$, since $w_0 \geq 0$ and $w_1 \geq 1$.
  - We know $p^{'}(a) = 1 \rightarrow p(a) - a > 0$ (remember the shape of $p(a)$).

Second-Best with Resource Constraints (Cont.)

- Conclusion: Principal *cannot* implement the first best effort level and extract all the payoff from agent, if there *are* resource constraints.

Second-Best with Resource Constraints (Cont.)

- Conclusion: Principal *cannot* implement the first best effort level and extract all the payoff from agent, if there *are* resource constraints.
- Optimization problem becomes

$$\max_{a} \quad p(a)(1-w_1)$$
$$s.t. \quad p^{'}(a)w_1 = 1. \qquad (IC)$$

- Solution:

$$p^{'}(a) = 1 - \frac{p(a)p^{''}(a)}{[p^{'}(a)]^2} > 1$$

- Under-provision of effort (i.e., $a < a^*$): tradeoff between providing incentive and extracting rents from agent.

Managerial Incentive Schemes

- Composition:
  - Main parts: Wage (“safe” transfer); bonus (short-term incentive component); stock option (long-term incentive component)
  - Other parts: Pension rights and severance pay (“golden parachutes")

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  - More than managerial effort: risk-taking, efficient cost cutting, adequate payout provisions, empire building, etc.
  - Compensation committee is often appointed by CEO.

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  - Compensation committee is often appointed by CEO.
  - Managers are often generously rewarded even when their company is doing poor. (Is this true for your company?)
  - Some studies show that a $10,000$ USD increase in profit leads to a $8-10$ USD increase in CEO's pay (surprising?).

A Model of Managerial Remuneration

- Profit: $q = a + \epsilon_q$.
- Stock price: $P = a + \epsilon_P$.
- Both ϵ_q and ϵ_P are normally distributed (variance: σ_q^2 and σ_P^2 and covariance: σ_{qP}).

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- Utility (CARA):

$$u(w, a) = -e^{-\eta[w - \psi(a)]}.$$

- Absolute Risk Aversion: $\frac{-u''(c)}{u'(c)}$.
- Relative Risk Aversion: $\frac{-u''(c)*c}{u'(c)}$.
- Effort cost: $\psi(a) = \frac{1}{2}ca^2$.

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- Effort cost: $\psi(a) = \frac{1}{2}ca^2$.
- Compensation scheme: $w = t + sq + fP$.

A Model of Managerial Remuneration (Cont.)

- Optimization problem:

$$\max_{a,t,s,f} \quad E(q-w)$$
$$s.t. \quad E(-e^{-\eta[w-\psi(a)]}) \geq -e^{-\eta\bar{w}}; \qquad (PC)$$
$$a \in \arg\max_a E(-e^{-\eta[w-\psi(a)]}). \qquad (IC)$$

- Optimal effort chose by manager: $a = \frac{s+f}{c}$.

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- Transformed problem:

$$\max_{t,s} \quad (1-s-f)\frac{s+f}{c} - t$$
$$s.t. \quad (s+f)\frac{s+f}{c} + t - \frac{1}{2}\eta[s^2\sigma_q^2 + 2sf\sigma_{qP} + f^2\sigma_P^2] - \frac{1}{2}\left(\frac{s+f}{c}\right)^2$$
$$= \bar{w}. \qquad (PC)$$

First part: expected income; Second part: loss due to risk aversion; Final part: effort cost.

A model of Managerial Remuneration (Cont.)

- Solution:

$$s^* = \frac{\sigma_P^2 - \sigma_{qP}}{\sigma_P^2 - 2\sigma_{qP} + \sigma_q^2} \frac{1}{1 + \eta c \Sigma}$$

and

$$f^* = \frac{\sigma_q^2 - \sigma_{qP}}{\sigma_P^2 - 2\sigma_{qP} + \sigma_q^2} \frac{1}{1 + \eta c \Sigma},$$

where

$$\Sigma = \frac{\sigma_P^2 \sigma_q^2 - \sigma_{qP}^2}{\sigma_P^2 - 2\sigma_{qP} + \sigma_q^2}.$$

A model of Managerial Remuneration (Cont.)

- Case one: No correlation (i.e., $\sigma_{qP} = 0$)

$$s^* = \frac{\sigma_P^2}{\sigma_P^2 + \sigma_q^2 + \eta c \sigma_P^2 \sigma_q^2};$$

$$f^* = \frac{\sigma_q^2}{\sigma_P^2 + \sigma_q^2 + \eta c \sigma_P^2 \sigma_q^2}.$$

Incentive power, $s^* + f^*$, goes to one, if η goes to zero (Why?).

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- Case two: if $\epsilon_P = \epsilon_q + \zeta$, then

$$f^* = 0 \quad s^* = \frac{1}{1 + \eta c \sigma_q^2}.$$

  Information on stock price is redundant (value of information), and information of output is a *sufficient statistic*. Never use redundant information, as agent is risk averse (Holmstrom's contribution which won Nobel prize).

Digressions: Prof. Van Reenen and HKU

- Prof. Van Reenen gave a public lecture at HKU last year: `https://www.youtube.com/watch?v=wqMksaJ6smM&t=160s`

Debt Financing and Moral Hazard

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- Insight two: if higher effort is associated with higher profit $\rightarrow$ fixed payment when performance is high and all paid to creditor when performance is low (maximize incentive to manager)

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- Insight two: if higher effort is associated with higher profit $\rightarrow$ fixed payment when performance is high and all paid to creditor when performance is low (maximize incentive to manager)
- Output: q; effort: a; conditional distribution of output: $F(q|a)$; payment: $r(q)$.
- Creditor's payoff: $\int_0^{\bar{q}} r(q)f(q|a)dq$; debtor's payoff: $\int_0^{\bar{q}} [q - r(q)]f(q|a)dq - \psi(a)$. $\psi(a)$: cost to exert effort.

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- Two assumptions:
  1. Limited liability: $0 \leq r(q) \leq q$.
  2. Monotonicity: $0 \leq r^{'}(q)$.

Optimization Problem

- Optimization problem:

$$
\begin{aligned}
\max_{r(q),a} \quad & \int_0^{\bar{q}} [q - r(q)] f(q|a) dq - \psi(a) - I \\
s.t. \quad & \int_0^{\bar{q}} [q - r(q)] f_a(q|a) dq = \psi^{'}(a); \qquad (IC) \\
& \int_0^{\bar{q}} r(q) f(q|a) dq = I; \quad (IR) \\
& 0 \le r(q) \le q. \quad (LL)
\end{aligned}
$$

Lagrangean

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$$
\begin{aligned}
L &= \int_0^{\bar{q}} [q - r(q)] f(q|a) dq - \psi(a) \\
&+ \mu \Big[\int_0^{\bar{q}} [q - r(q)] f_a(q|a) dq - \psi'(a) \Big] + \lambda \Big[\int_0^{\bar{q}} r(q) f(q|a) dq - I \Big].
\end{aligned}
$$

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&+\mu[\int_0^{\bar{q}} [q - r(q)] f_a(q|a) dq - \psi'(a)] + \lambda[\int_0^{\bar{q}} r(q) f(q|a) dq - I].
\end{aligned}
$$

- We can rearrange it to

$$
\begin{aligned}
L &= \int_0^{\bar{q}} r(q) \left[\lambda - \mu \frac{f_a(q|a)}{f(q|a)} - 1\right] f(q|a) dq \\
&+ \int_0^{\bar{q}} q \left[1 + \mu \frac{f_a(q|a)}{f(q|a)}\right] f(q|a) dq - \psi(a) - \mu\psi'(a) - \lambda I.
\end{aligned}
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- Solution: $r^*(q) = q$ if $\lambda > 1 + \mu \frac{f_a(q|a)}{f(q|a)}$ and $r^*(q) = 0$ if $\lambda \leq 1 + \mu \frac{f_a(q|a)}{f(q|a)}$.

MLRP

- Monotone likelihood ratio property (MLRP):

$$\frac{d}{dq}\left[\frac{f_a(q|a)}{f(q|a)}\right] \geq 0.$$

- Higher output $\rightarrow$ higher pay.

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$$\frac{d}{dq}\left[\frac{f_a(q|a)}{f(q|a)}\right] \geq 0.$$

- Higher output $\rightarrow$ higher pay.
- MLRP is satisfied $\rightarrow$ $r^*(q) = 0$ if $q > Z$ and $r^*(q) = q$ if $q < Z$.

Summary

- Highly non-monotonic. In reality, we probably have
  - $r_D(q) = D$ if $q > D$ and $r_D(q) = q$ if $q \leq D$.
  - $\int_0^D qf(q|a^*)dq + [1 - F(D|a^*)]D = I$ and $\int_D^{\bar{q}} (q - D)f_a(q|a^*)dq = \psi'(a^*)$.

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- Why? We want to maximize incentive power.

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- Why? We want to maximize incentive power.
- Key assumption: risk neutrality.
- Subsequent work: Dewatripont, Legros and Matthews (2003): dynamic setting with renegotiation.

Graphical Representation

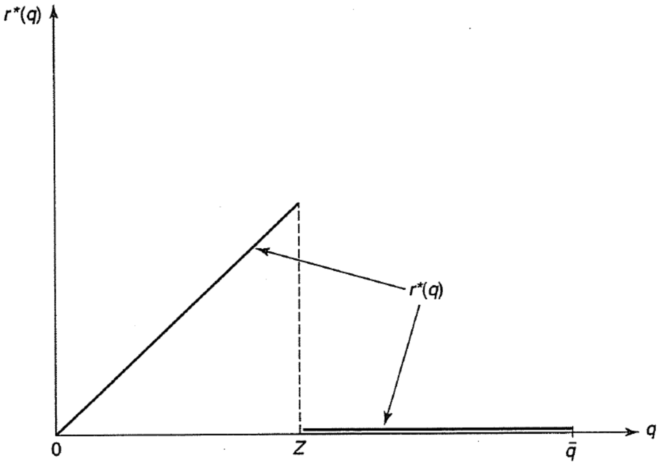

Figure 4.3
Optimal Nonmonotonic Contract

Debt Financing and Adverse Selection

- References: Townsend (1979) and Gale and Hellwig (1985).

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- Big insight: debt financing is still optimal (minimization of auditing cost).

Debt Financing and Adverse Selection

- References: Townsend (1979) and Gale and Hellwig (1985).
- Suppose it is costly to observe information on profit π. Cost: K (e.g., auditing).
- Big insight: debt financing is still optimal (minimization of auditing cost).
- Revelation principle: contacts conditional on π.
- True profit: π; reported profit: $\hat{\pi}$.
- Maximum punishment: $r(\hat{\pi}, \pi) = \pi$ whenever $\hat{\pi} \neq \pi$.
- Random auditing: not allowed (i.e, $p(\pi) \in \{0, 1\}$). Maybe realistic.

Optimization Problem

- Optimization problem:

$$
\begin{aligned}
&\min_{p(\pi),r(\pi),r_a(\pi)} K\int_0^\infty p(\pi)f(\pi)d\pi \\
s.t.\quad & r_a(\pi_1) \leq r(\pi_2);\ \forall\ \pi_1 \neq \pi_2 \\
& such\ that\ p(\pi_1) = 1\ and\ p(\pi_2) = 0;\ (IC1) \\
& r(\pi_1) = r(\pi_2) = r;\ \forall \pi_1 \neq \pi_2 \\
& such\ that\ p(\pi_1) = 0 = p(\pi_2) = 0;\ (IC2) \\
& \int_0^\infty p(\pi)[r_a(\pi) - K]f(\pi)d\pi \\
& + \int_0^\infty [1 - p(\pi)]r(\pi)f(\pi)d\pi \geq I;\ (IR) \\
& r(\pi) \leq \pi \ \ r_a(\pi) \leq \pi. \quad (LL)
\end{aligned}
$$

Incentive Compatibility

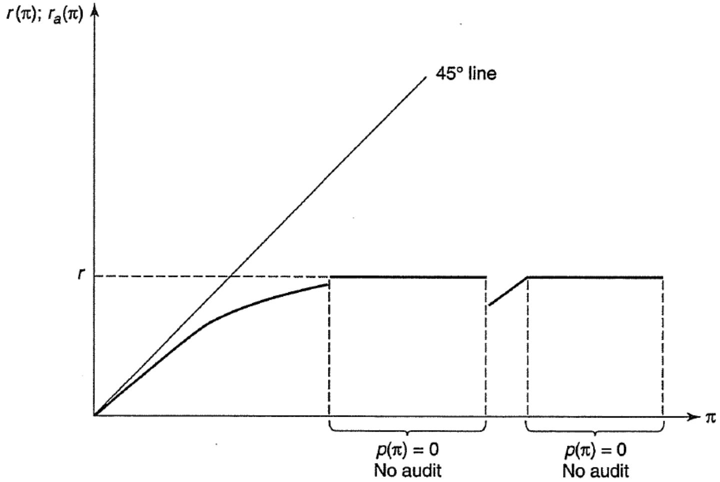

Figure 5.1
An Incentive-Compatible Repayment Schedule

Observations

- Three observations:
  1. $\pi = 0$ is in the audit set.

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  2. $r_a(\pi) = \min\{\pi, r\}$. (Fig. 5.2)

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  2. $r_a(\pi) = \min\{\pi, r\}$. (Fig. 5.2)
  3. Any contract with a disconnected audit subset $[0, \hat{\pi}] \cup [\pi_0, \pi_1]$ would be inefficient. (Fig. 5.3 and 5.4)

Observation Two

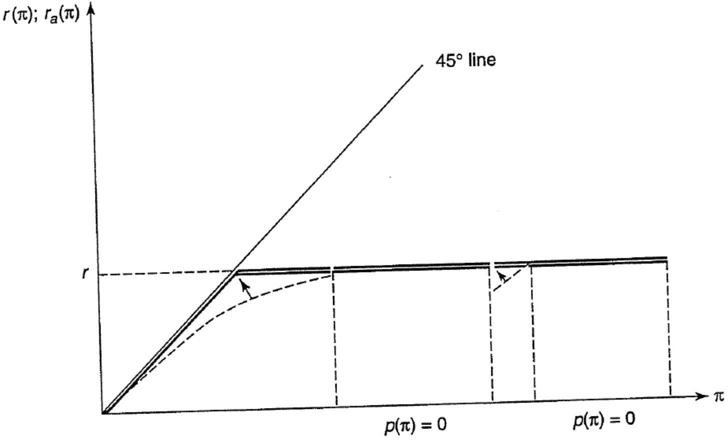

Figure 5.2
Incentive-Compatible Repayment Schedule with $r_a(\pi) = \min\{\pi, r\}$

Observation Three

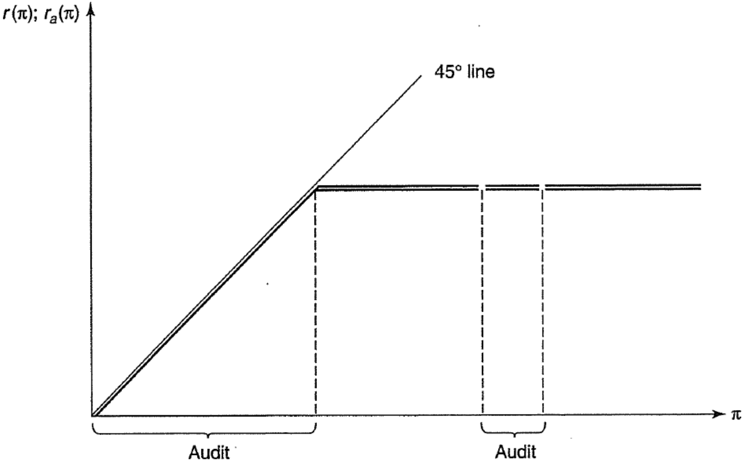

Figure 5.3
Inefficient Audit Region

Observation Three (Cont.)

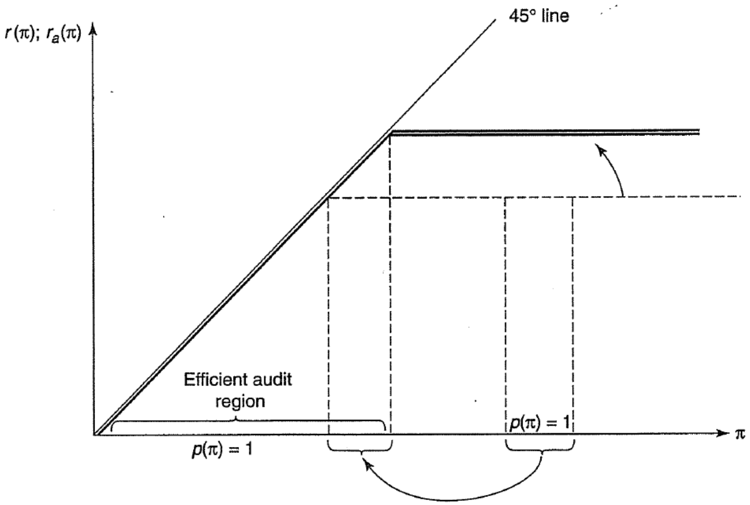

Figure 5.4
Efficient Audits

Optimal Contract

- Standard debt contract: $r_a(\pi) = \pi$ for $\pi \leq \bar{\pi}$ $r = \bar{\pi}$ for $\pi > \bar{\pi}$.
- PC of creditor:

$$\int_0^{\bar{\pi}} (\pi - K) f(\pi) d\pi + [1 - F(\bar{\pi})] r = I$$

and expected cost of auditing: $F(\bar{\pi})K$.

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- Problem: not renegotiation-proof or subgame perfect (commitment problem).
- Several assumptions:
  1. Risk neutrality (Gale and Hellwig, 1985);
  2. No random auditing (Mookherjee and Png, 1989);
  3. Multiple projects and financiers (Gale and Hellwig, 1989 and Winton, 1995).