Foreign direct investment and international trade in a continuum Ricardian trade model

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Abstract

We develop a continuum Ricardian trade model to capture both North–South trade and technology transfer via foreign direct investment (FDI) by multinational enterprises (MNEs). We show that there is a unique range of products produced in the South by MNEs. In the case of an infinitely elastic supply of expatriates, if the ability of Southern workers in absorbing Northern technology increases, then (a) the range of MNE production increases, (b) Northern workers’ welfare and Southern workers’ welfare change in opposite directions, and (c) the world aggregate welfare increases under certain conditions. We explore issues such as North–South wage gaps, FDI policies and the product cycle. We also derive results under a general supply of expatriates.

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1. Introduction

Foreign direct investment (FDI) is investment that gives an investor control over the firms that operate in foreign countries. It has become increasingly important in the world...
economy. According to the UNCTAD (2002), the total stock of FDI in the world increased from US$763.4 billion in 1985 to US$4015.3 billion in 1998, and the total FDI stock as a percentage of the world’s total GDP increased from 6.7% in 1985 to 14% in 1998. In developing countries, the percentage increased from 9.1% in 1985 to 20% in 1998.

FDI is traditionally regarded as an organic amalgamation of capital, technology, and management. With the increasing integration of the global capital market and the development of domestic capital markets in many host countries of multinational enterprises (MNEs), capital movement from the home countries of MNEs to the host countries seems to have become the least important ingredient of FDI. In contrast, technology and managerial talent have become the key ingredients of FDI (see, e.g., Root, 1994, p. 591). As trainers and managers, expatriate employees of MNEs play a crucial role in the process of technology transfer, even though their number is small in relative terms. They are relatively more expensive and limited in supply, and are often supplemented by nationals of the host economies who are educated in the advanced economies (e.g., Hieneman et al., 1985; Stewart and Nihei, 1987, p. 74, and Dunning, 1993, p. 373).

In light of these facts, we regard FDI as synonymous to technology (and managerial skill) transfer. In this paper we develop a continuum Ricardian trade model that features technology transfer via FDI to explore the implications of technology transfer that is brought about by MNEs. The implications include (a) the division of labor between local firms and MNEs; (b) North–South wage gaps; (c) regional and aggregate welfare; and (d) government incentives to promote or discourage technology transfer via MNEs. Using this model, we can also examine the pattern of product cycles as technology transfer becomes easier over time.

There is a substantial economic literature on FDI and MNEs. The central question is why a firm establishes and maintains production in two or more countries. There are two lines of explanation. First, MNEs make FDI to take advantage of international factor-price differences. Researchers along this line, for example, Helpman (1984), incorporate MNEs into general equilibrium trade models based on “headquarter services” that can be used to support both local plants and subsidiaries set up abroad. On the one hand, production of headquarter services, which include management, distribution, marketing, and product-

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1 For some evidence about the lack of a strong positive correlation between total capital flows and FDI flows, see Krugman (1998). As reported by Blomstrom and Kokko (1994), the main reason for FDI by Swedish firms has been technological advantages. Swedish MNEs expanded FDI despite limitation on financing such investment with funds raised in Sweden.

2 In Bai et al. (2004) study of 200 foreign joint ventures in China (whose average size of investment is US$12 million), 95% of the foreign partners provide patent, design, trademark and equipment, 56% provide technical training and 49% provide technical and management support.

3 According to Stewart and Nihei’s (1987, Table 6-1) study of Japanese subsidiaries and joint ventures in Indonesia and Thailand, the Japanese expatriates accounted for 10–75% of total professional and managerial employment, but only 0.5–3.0% of total employment.

4 The “factor price equalization theorem” in international trade theory predicts that international trade leads to a convergence of the factor prices of the trading economies. In this paper we investigate how technology transfer via FDI affects the wage gap between Northern and Southern workers.

5 See Markusen (2002) for a complete review of the literature.
specific R&D, enjoy economies of scale. Thus MNEs have an incentive to concentrate the production of these services in a single location where physical and human capital are relatively abundant. On the other hand, international differences in factor endowments and technologies provide an incentive to locate the production of final goods in countries with lower unskilled-labor costs. MNEs and FDI arise if the relative factor endowments are sufficiently different across countries so that international trade alone does not lead to factor price equalization.\(^6\) Vertical FDI is driven by such a motive.

The second line of explanation says that in order to sell products to other countries’ markets, MNEs make FDI to overcome transportation and trade barriers. Markusen (1984), Brainard (1993) and Horstmann and Markusen (1992) develop models that highlight this feature. They find that MNEs are more likely to arise when firm-level fixed costs (like R&D) are large, tariffs or transportation costs are high, plant-level scale economies are not large, and countries are large. Horizontal FDI is driven by such a motive.

Influenced by a large descriptive literature on MNEs and empirical studies of the industry characteristics and geographical location of MNEs, existing theoretical studies in the literature of MNEs focus on the case where MNEs arise from *imperfectly* competitive markets as a result of increasing returns to scale or product differentiation.\(^7\)

Our paper differs from other research in the literature in two important ways. First, we explain that MNEs and FDI arise due to technological differences between two regions. Difference in technologies leads to factor price differentials. Second, our model is based on perfect competition in the product markets with neither transportation costs nor trade barriers. We show that due to technological differences between countries, technology transfer via FDI that requires the use of a specific resource can occur even in the absence of imperfect competition.\(^8\) Theoretically, technology transfer by competitive MNEs is novel. Empirically, it has support from FDI in China and other developing countries. Evidence of FDI in industries characterized by perfect competition can be easily found. For instance, a large percentage of Hong Kong’s FDI in Mainland China’s manufacturing sector in early 1990s was in the textiles and clothing industries, which can be regarded as industries with perfect competition.\(^9\)

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\(^6\) Ethier (1986) also incorporates FDI into a general equilibrium factor endowment trade model. By emphasizing the internalization decision of MNEs, he obtains results that are contrary to those of Helpman (1984): MNEs and FDI can arise when relative factor endowments are very similar. Moreover, FDI may either substitute or complement trade.

\(^7\) In the aforementioned studies, Markusen (1984) and Helpman (1984) use the Dixit-Stiglitz model and so there is product differentiation and monopolistic competition, while Brainard (1993) and Horstmann and Markusen (1992) consider Duopoly and Cournot competition.

\(^8\) Thus, this paper is similar in spirit to Davis’ (1995) attempt to explain intra-industry trade under perfect competition and constant returns to scale, as opposed to the usual models of intra-industry trade based on imperfect competition and increasing returns to scale.

\(^9\) Due to historical reasons, the textiles and clothing industries were two of the key industries in Hong Kong. Hong Kong has advanced technologies in these industries. FDI liberalization taking place in many Asian countries in the 1980s and 1990s attracted Hong Kong textiles and clothing firms to relocate their production base to those counties, particularly China. As a result, according to Hong Kong Industry Department’s publication *Hong Kong Industries* (various issues), the number of firms remaining in Hong Kong has been declining over time from 9792 in 1993 to 4502 in 1999. Total employment also dropped from 210,000 in 1993 to 75,000 in 1999. Similar changes have occurred in the electronics industry and the watches and clocks industry.
Oftentimes there is a natural tendency to associate MNEs with huge firms, e.g., IBM and GE. In many competitive industries, there are indeed small MNEs. While large MNEs with proprietary technology tend to exploit host countries’ domestic markets, small MNEs tend to exploit host countries’ low cost production of relatively standardized products. But MNEs are defined as firms that operate in more than one nation, and there is no presumption about their size. Our analysis is complementary to those papers that deal with oligopolistic industries, because by definition firms in those industries are big and few in number.

Unlike the aforementioned studies of MNEs, we employ the continuum Ricardian trade model developed by Dornbusch et al. (1977, hereafter DFS for short), which highlights international differences in technologies as a basis of international trade. Krugman (1985) uses this model with a given “technology gap” to study the changes in production pattern, wage rates and welfare as a result of technological progress in the advanced region that widens the technology gap as well as technological progress in the less advanced region that narrows the gap. Complementing Krugman’s analysis, we investigate the impact of technology transfer made possible by the existence of given technology gaps.

It has long been recognized that FDI by MNEs is by far the most important channel of technology transfer from advanced economies to developing economies (e.g., Quinn, 1969; Stewart and Nihei, 1987, pp. 8–12). Many researchers of MNEs have emphasized the importance of training managers, workers and engineers of the technology recipient countries (e.g., Quinn, 1969; Hieneman et al., 1985; Stewart and Nihei, 1987, pp. 10–12 and pp. 74–75; Sakakibara and Westney, 1992; Root, 1994, p. 590). Teece (1976, p. 36–37 and p. 44) and Mansfield et al. (1982, p. 69–71) have identified four groups of transfer costs, which all involve technical and operational personnel training. The average transfer costs were found to be 19 percent of total project costs.

Based on the empirical facts about technology transfer by MNEs and consistent with the transaction costs theory of technology transfer (see Markusen, 1995), we assume that MNEs are the only vehicle of technology transfer between a technologically advanced region, the “North”, and a technologically backward region, the “South”, and that technology transfer requires the use of specific resources. These specific resources consist of expatriate managers and technicians who possess the knowledge needed in managing a foreign subsidiary and in adopting advanced production technology in the backward region. For simplicity we assume that these resources, to be called “expatriates”,

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10 The four groups of transfer costs are (i) the cost of pre-engineering technological exchanges; (ii) the engineering costs associated with transferring the process design and product design; (iii) the cost of R&D personnel during all phases of the transfer project for the purpose of solving unexpected transfer problems and adapting the technology, and (iv) pre-start-up training costs and learning and debugging costs during the start-up phase.

11 We emphasize the resource required by the technologically advanced region to transfer its technology to the technologically backward region. In contrast, Keller (1996) and Glass and Saggi (1998) emphasize that a technologically backward country must have a sufficiently high level of human resources to effectively transfer/imitate the advanced technology from abroad. However, there is no disagreement that resources are required for international technology transfer.
are only available in the North and cannot be used for production. We examine welfare implications of technology transfer in this type of model.\textsuperscript{12}

The paper is organized as follows. We set up the basic model in Section 2. We analyze the equilibrium and welfare under the assumption that the supply of expatriates is infinitely elastic in Section 3. In Section 4, we perform a similar analysis by endogenously determining expatriates’ wage rate under the assumption that the supply of expatriates is upward sloping. Section 5 concludes the paper.

2. A continuum Ricardian trade model with technology transfer

2.1. Goods and technology

As in the standard Ricardian trade model, there are two regions (the North and the South) but only one factor of production (labor) in each region. The supply of labor in the South and North is fixed at $L$ and $L^*$, respectively, and labor is internationally immobile. Unlike the standard two-good model, there are infinitely many goods modelled as real numbers along a line interval. Following DFS’s notation, let $a(z)$ be the amount of Southern labor, and $a^*(z)$ the amount of Northern labor that is needed to produce one unit of good $z$, where $z$ lies within the unit interval, i.e., $z \in [0,1]$. For notational simplicity, we set $a^*(z) = 1$ for all $z$ by choosing an appropriate unit of measurement for each good. With appropriate indexing of the goods, $a(z)$ can be made an increasing function of $z$, implying that the South’s comparative advantage in good $z$ decreases as $z$ increases. Given this indexing convention, $z$ may be loosely interpreted as the level of technological sophistication of the product. For convenience of our analysis, we further assume that $a(z)$ is continuously differentiable.

Assume that the North is superior to the South in the production technology of all goods, i.e., $a(z) > 1$. Let Southern workers be the numeraire (i.e., wage of Southern workers is equal to 1) and denote the relative wage of Northern workers by $w^*$. Assume that there are no transportation costs or other trade barriers. Since $a(z) > 1$ for all $z$, the equilibrium $w^*$ must exceed unity.

2.2. Technology transfer

As discussed in the Introduction, we assume that MNEs are the only vehicle of technology transfer between North and South and that technology transfer requires the use of “expatriates”, which are only available in the North and cannot be used for production. Let $k(z)$ denote the number of expatriates required for each unit of good $z$ produced in the South using Northern technology. We assume that the resource cost of technology transfer is higher the more sophisticated is the product, i.e., $k(z)$ is an

\textsuperscript{12} Markusen and Rutherford (2002) also emphasize the importance of foreign expertise for developing countries’ economic growth and trade. They model producer service, which include “expatriates”, as an intermediate input for certain goods production. In contrast, we focus on the dependence of technology transfer on “expatriates”.\textsuperscript{12}
increasing function of \( z \). We assume that \( k(z) \) is continuously differentiable.\(^{13}\) In addition to expatriates, \( e \) units of Southern workers are required to produce one unit of output using the Northern technology. We use this value to capture the efficiency loss of technology transfer due to incomplete absorption of Northern technology by Southern workers, or barriers to technology transfer such as communication difficulty and cultural differences.\(^{14}\) We have \( e \geq 1 \) because even if Southern workers are as efficient as Northern workers, one unit of Southern workers is required to produce the product as in the North. Moreover, a decrease in the value \( e \) represents increased efficiency of technology transfer. With the above specification of inputs, the unit cost of producing good \( z \) by MNEs in the South is equal to

\[
b(z) = e + rk(z),
\]

where \( r \) is wage rate for expatriates.\(^{15}\)

2.3. Preferences and welfare

Demand for goods depends on consumer preferences. We assume that preferences of all consumers are identical and are represented by the continuum version of the Cobb–Douglas utility function with a uniform weight, i.e.,

\[
\log u(d) = \int_0^1 \log x(z) dz.
\]

Under this assumption, each consumer devotes the same fraction of his/her income to the consumption of each good \( z \). If the representative consumer’s income is \( y \), then demand for good \( z \) is given by \( x(z) = y/p(z) \), where \( p(z) \) is the market price of \( z \). Therefore, the log of indirect utility or welfare is given by

\[
\log y - \int_0^1 \log p(z) dz.
\] \( (1) \)

3. Equilibrium analysis with exogenous \( r \)

In this section, we confine our analysis to the case where \( r \) is fixed, i.e., the supply of expatriates is perfectly elastic. The case of endogenous \( r \) and an upward sloping supply curve of expatriates will be analyzed in Section 4. The case of fixed \( r \) can be viewed as a

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\(^{13}\) Both Keller (1996) and Glass and Saggi (1998) have stressed the point that the effectiveness of technology transfer depends on the technology gap between the advanced and backward regions. We make the same assumption here.

\(^{14}\) We make a similar assumption in Cheng et al. (2001).

\(^{15}\) Alternatively, we can also consider the following case: each unit of technology transfer requires just one expatriate, but efficiency loss from technology transfer is an increasing function of \( z \), denoted by \( h(z) \). Then, the unit cost of producing product \( z \) by the MNEs with technology transfer is \( h(z) + r \).
first approximation of reality if the pool of expatriates in the North is very large relative to the amount required for technology transfer in the South.

3.1. The benchmark model

Suppose there is no technology transfer from the North to the South. Then, we have the standard DFS model, in which the equilibrium consists of \((\tilde{z}, \tilde{w}^*)\) and is determined by

\[
a(\tilde{z}) = \tilde{w}^* \quad \text{and} \quad (1 - \tilde{z})(L + \tilde{w}^*L^*) = \tilde{w}^*L^*.
\]

It follows that the equilibrium value of \(\tilde{z}\) satisfies

\[
a(\tilde{z}) = \frac{(1 - \tilde{z})l}{\tilde{z}},
\]

where \(l = L/L^*\) is the ratio of the labor supplies between the two countries.

The South produces products in the range \([0, \tilde{z}]\) and the North produces products in the range \([\tilde{z}, 1]\). The dividing point \(\tilde{z}\) increases with \(l\), and so does the equilibrium relative wage rate of Northern workers \(\tilde{w}^*\).

3.2. Equilibrium with technology transfer

In our model with technology transfer, good \(z\) can also be produced by MNEs in the South at the unit cost \(b(z)\). Since goods are produced only in sites and in ways that offer the lowest unit costs, good \(z\) is produced by

\[
\left\{ \begin{array}{ll}
\text{Southern firms} & a(z) \leq \min\{b(z), \tilde{w}^*\} \\
\text{MNEs} & b(z) < \min\{a(z), \tilde{w}^*\} \\
\text{Northern firms} & \tilde{w}^* \leq \min\{a(z), b(z)\}.
\end{array} \right.
\]

We make the following assumption.

**Assumption (A1) (Single-Crossing Property).** \(a(z) - b(z)\) crosses zero only once strictly from below.

An alternative statement for (A1) is that there exists a unique \(z_0 \in (0, 1)\) satisfying

\[
a(z_0) = b(z_0)
\]

and that it is cheaper (more expensive) for MNEs to produce good \(z > z_0\) \((z < z_0)\) than Southern firms. With reference to Fig. 1, this assumption implies that the curve \(b(z)\) is flatter than \(a(z)\) near \(z = z_0\), but does not have to be flatter for the whole range of \(z\). Moreover, if the Southern firms produce \(z'\); then they also produce all \(z < z'\). In particular, it is cheaper to produce product 0 by the Southern firms than by the MNEs. If \(r\) is very large, there will be no crossing at all, i.e., no technology transfer. Thus, Assumption (A1) puts an upper bound on \(r\).
We now turn to another condition that characterizes the border between MNEs and Northern production

\[ w^* = b(z_1). \]  

First, given the positive slopes of \( a(z) \) and \( k(z) \), if \( z' \) is produced in the North, then all products \( z > z' \) will also be produced in the North. Second, at least some products must be produced in the North since otherwise the Northern production workers will be unemployed; \( w^* \) adjusts to ensure that all Northern workers are employed. Hence, there exists a unique \( z_1 \); as defined in Eq. (4), such that products with \( z \geq z_1 \) are produced in the North but products with \( z < z_1 \) are produced in the South either by Southern firms or by MNEs.

Clearly, a necessary condition for technology transfer to occur is \( z_0 < z_1 \). In that case, the Southern firms produce goods in \([0, z_0]\), MNEs produce goods in \((z_0, z_1)\) and Northern firms produce goods in \([z_1, 1]\).

Because of perfect competition in the goods markets, we obtain the goods prices as

\[
p(z) = \begin{cases} 
  a(z) & \text{for } z \in [0, z_0] \\
  b(z) & \text{for } z \in [z_0, z_1] \\
  w^* & \text{for } z \in (z_1, 1]. 
\end{cases}
\]  

Let \( Y \) be world income, which includes the wage income of the Southern workers, the Northern workers and the expatriates, i.e.,

\[ Y = L + w^*L^* + rT, \]
where $T$ is the equilibrium demand and supply of expatriates. Hence, we have another condition to determine wage rate $w$:

\[(1 - z_1)Y = w^*L^* , \tag{6}\]

where the LHS represents world demand (in value terms) for goods produced in the North while the RHS represents the total supply (in value terms) of goods by the North.

Finally, we need to determine the total demand for expatriates $T$. The demand for expatriates by industry $z$ is

\[T_D(r; z) = \begin{cases} 
  k(z)D(r; z) & \text{for } z \in (z_0, z_1) \\
  0 & \text{otherwise,}
\end{cases}\]

where $D(r; z)$ is the total demand for good $z$ at the good’s price $b(z)$ and is given by $Y/b(z)$.

Hence, the total demand for expatriates by all industries is given by

\[T_D(r) = \int_{z_0}^{z_1} T_D(r; z) dz = YM,\]

where $M$ is defined as

\[M = \int_{z_0}^{z_1} \left( \frac{k(z)}{b(z)} \right) dz. \tag{7}\]

We combine the above demand equation with the world income equation to solve for the world income $Y$ as

\[Y = \frac{L + w^*L^*}{1 - rM}. \tag{8}\]

To summarize, the equilibrium $(z_0, z_1, w^*)$ is determined by three conditions, Eqs. (3), (4) and (6), where $Y$ is given in Eq. (8).

The following proposition characterizes the equilibrium in our model.

**Proposition 1.** Suppose $(A1)$ holds. Then there exists a unique equilibrium in which technology transfer occurs if and only if $z_0 < z$, where $z$ and $z_0$ are determined by Eqs. (2) and (3), respectively.

**Proof.** There are two cases to be considered. In the first case, $z_0 \geq z$, and it can be easily verified that the equilibrium coincides with that in the DFS benchmark model where technology transfer does not occur.

In the second case, $z_0 < z$. The equilibrium is a triplet $(z_0, z_1, w^*)$ with $z_0 \leq z_1$; which is determined by Eqs. (3), (4) and (6).

To determine the equilibrium values of $z_1$ and $w$, we write condition (6) as

\[\frac{l + w^*}{w^*} = \frac{1 - rM}{1 - z_1},\]
Taking into account the equilibrium condition for $z_1$ and the definition of $M$, we rewrite the above equation as

$$b(z_1) = \frac{(1 - z_1)l}{z_0 + \int_{z_0}^{z_1} \frac{e}{b(z)} \, dz}. \tag{9}$$

Notice that the LHS strictly increases with $z_1$ while the RHS strictly decreases with $z_1$. Moreover, the LHS is less than the RHS at $z_1 = z_0$ because $b(z_0) = a(z_0) < (1 - z_0)l/z_0$ and clearly the LHS exceeds the RHS at $z_1 = 1$. Therefore, there exists a unique pair of $z_1$ and $w^*$ that satisfies the equilibrium conditions (4) and (6).

Proposition 1 provides a necessary and sufficient condition for technology transfer to take place. Note that an increase in $l$ increases $\hat{z}$ but does not affect $z_0$. Similarly, a decrease in $e$ or $r$ reduces $z_0$ but does not change $\hat{z}$. These properties imply the following result.

**Corollary 1.** An increase in $l$ or a decrease in $e$ or $r$ makes technology transfer more likely.

Proposition 1 and Corollary 1 together imply that for any given $a(z)$ and $l$, which determines $\hat{z}$, technology transfer takes place if and only if $e + rk(\hat{z}) < a(\hat{z})$. This also implies that for any given $r$, there exists a critical point $e^*$ defined by $e^* = a(\hat{z}) - rk(\hat{z})$ such that technology transfer takes place if and only if $e < e^*$. Similar critical points exist for $l$ and $r$. These findings are intuitive because the cost of technology transfer depends positively on $e$ and $r$. An increase in the relative population of the South increases the range of products that it will produce in the absence of technology transfer, thus opening more opportunities for profitable technology transfer.

Another feature of the equilibrium with technology transfer is that the equilibrium value of $z_1$ has lower and upper bounds described in the following corollary.

**Corollary 2.** In equilibrium, $z_1$ is bounded below by $\hat{z}$ and above by $l/(e + l)$.

**Proof.** See Appendix A.

We know that $w^*$ adjusts to ensure that workers in the North are fully employed, which implies that $z_1 < 1$. But Corollary 2 goes further and provides the upper bound of $z_1$.

### 3.3. Comparative statics

In this subsection we examine how the equilibrium $(z_0, z_1, w^*)$ changes with respect to labor endowment ratio $l$, the technology absorption parameter $e$ and the fixed wage rate for expatriates $r$.

**Proposition 2.** Suppose (A1) holds. Then, in equilibrium, (i) $z_0$ is independent of $l$ but increases with both $e$ and $r$ and (ii) $z_1$ increases with $l$ but decreases with both $e$ and $r$.

**Proof.** See Appendix A.

The effect of labor ratio $l$ on the equilibrium is similar to that in the basic DFS model. A higher $l$ increases the borderline good $z_1$ above which the Northern firms supply. However,
as long as technology transfer takes place, the range of the products produced by Southern firms is independent of \( l \). A higher ratio of Southern labor supply to Northern labor supply only expands the range of the products produced by MNEs.

The proposition also implies that as \( e \) or \( r \) increases, the range of products produced by MNEs, \([z_0, z_1] \), shrinks and the ranges of products produced by both Southern and Northern firms expand.

Technology transfer moves production of goods in the range \([\hat{z}, z_1] \) from the North to the South. Proposition 2 says that with an increase of Southern workers’ ability to absorb Northern technology or a decrease of the cost of employing expatriates to transfer technology, the range of products “going south” will increase. To contribute to the product cycle literature pioneered by Vernon’s (1966), our model identifies factors that help to speed up a product cycle.

Since \( z_1 \) increases with \( l \), it follows that the equilibrium relative wage for Northern workers \( w^* \) also increases with \( l \). However, it is generally ambiguous how \( w^* \) varies with \( e \) or \( r \). To see this, note that

\[
\frac{dw^*}{de} = 1 + rk'(z_1) \frac{dz_1}{de}.
\]

An increase in \( e \) has a direct positive effect and an opposite indirect effect on \( w^* \). The direct, positive effect occurs since an increase in \( e \) makes the transfer of technology by MNEs more costly and reduces the range of products produced by MNEs in the South. This puts upward pressure on Northern relative wage \( w^* \). The indirect, negative effect arises since a reduction of expatriates’ income due to a reduced range of goods produced by MNEs decreases demand for Northern goods, thus putting downward pressure on \( w^* \). An increase in \( r \) units has similar opposing effects. In the extreme case where \( k(z) \) is independent of \( z \), the indirect effect disappears and the equilibrium wage always increases with \( e \) and \( r \).

To further understand the trade-off between the two effects, we consider a situation in which MNEs in equilibrium are initially indifferent between transferring technology and not transferring technology (i.e., \( e = e^* \) or \( r = r^* \)). A small reduction in \( e \) units will induce MNEs transfer technology to the South via FDI. The following proposition shows that the change in equilibrium wage depends on the relative slopes of \( a(z) \) and \( b(z) \). For the theoretical treatment see Grossman and Helpman (1991).

**Proposition 3.** Suppose initially the equilibrium is such that there is no difference between transferring technology and not transferring technology (i.e., \( e = e^* \)). Then a small reduction in \( e \), which induces technology transfer, reduces (increases) the equilibrium wage \( w \) if \( 2d(\hat{z})/bl(\hat{z}) > (<)l/(e^* + l) \).

**Proof.** See Appendix A. \( \square \)

Note that at \( e = e^* \), \( z_1 = z_0 = \hat{z} \) and \( b(\hat{z}) = a(\hat{z}) = (1 - \hat{z})l/\hat{z} \). It follows that \((e^* + l)\hat{z} < l \). Hence, the proposition states that if the slope of \( a(z) \) at \( z = \hat{z} \) is greater than the slope of \( b(z) \) at \( \hat{z} \) by a positive amount, then a small reduction of \( e \) decreases the equilibrium Northern wage. On the other hand, if the two slopes are sufficiently close, then a small reduction of \( e \) will increase the equilibrium Northern wage.
Given $e$, there is a critical point $r^*$ below which technology transfer takes place. Similar to Proposition 3, it can be easily shown that starting with $r^*$, a small reduction in $r$ decreases (increases) the equilibrium relative wage $w^*$ if

$$\frac{\hat{z}a'(\hat{z})}{b'(\hat{z})} > (<) \frac{l}{l + e}.$$ 

The intuition behind the proposition can be described simply as follows. Starting from the initial situation where $z_1 = z_0 = \hat{z}$, a small decrease in $e$ or $r$ will result in a small range of $z \in [z_0, z_1]$ within which MNEs produce goods in the South. Given $a(z)$, both $z_0$ and $z_1$ depend on the slope of $b$. If $b'(\cdot)$ becomes smaller, then at the original wage rate $\hat{w}$, both $z_0$ and $z_1$ would increase, decreasing the demand for Northern workers while increasing the demand for Southern workers. That is to say, the bigger is the difference between $a'(\hat{z})$ and $b'(\hat{z})$, the more negative (or the less positive) pressure there is on $w^*$. A limiting case is given by $b'(z) = 0$. In this case, at the original wage rate $z_0 = \hat{z}$, but $z_1 = 1$, leaving no goods to be produced in the North. Thus, the only possible way for Northern workers to be employed at all after technology transfer becomes profitable (as a result of a decrease in $e$ and $r$) is a reduction in $w^*$. Indeed, the equilibrium wage $w^*$ unambiguously increases with both $e$ and $r$.

Another extreme case arises if both $a'(z)$ and $b'(z)$ are equal to a constant. In this case, $z_0 = 0$ and $z_1 = \hat{z}$, i.e., the Northern workers continue to produce the same range of goods $(z_1, 1)$ but Southern workers are all employed by MNEs. However, the real income generated in the South has gone up because the more efficient MNEs replace all the Southern firms, resulting in an increase in GDP there. But this increase in GDP also results in an increase in demand for Northern products, which in turn implies that $w^*$ must rise to restore equilibrium. However, the impact of $e$ and $r$ on $w^*$ may change sign over the whole range of $e$ and $r$.

### 3.4. Regional welfare analysis

Let us examine the welfare of Northern and Southern workers (per capita). Each Northern worker’s welfare after technology transfer is

$$W_N = \log w^* - \int_0^{z_0} \log a(z)dz - \int_{z_0}^{z_1} \log b(z)dz - (1 - z_1)\log w^*$$

$$= z_1 \log w^* - \int_0^{z_0} \log a(z)dz - \int_{z_0}^{z_1} \log b(z)dz.$$ 

Each Southern worker’s welfare after technology transfer is

$$W_S = \log 1 - \int_0^{z_0} \log a(z)dz - \int_{z_0}^{z_1} \log b(z)dz - (1 - z_1)\log w^*.$$
It can be shown that

\[
\frac{\partial W_N}{\partial t} = z_1 \frac{\partial \log w^*}{\partial t} \quad \text{and} \quad \frac{\partial W_S}{\partial t} = -(1 - z_1) \frac{\partial \log w^*}{\partial t}
\]

(10)

for \( t = e \) or \( r \). Thus, changes in \( e \) and \( r \) affect regional welfare only through the equilibrium relative wage. Therefore, the discussions on the equilibrium relative wage in the previous subsection immediately apply here. Moreover, the impact on Southern welfare is opposite in sign to that on Northern workers’ welfare.

**Corollary 3.** Starting from a situation of barely no technology transfer, a small reduction in \( e \) increases (decreases) Southern welfare but decreases (increases) Northern workers’ welfare if \( \tilde{z}d(\tilde{z})/b'(\tilde{z}) > (e^*/e^* + l) \). Given technology transfer, the impact of changes in \( e \) and \( r \) on Northern workers’ welfare is always opposite to that on Southern workers’ welfare.

### 3.5. Aggregate welfare analysis

We next examine how technology transfer affects the aggregate (or world) welfare which includes the welfare of Southern workers, Northern workers and expatriates. Note from Eq. (1) that the aggregate welfare is

\[
W = \log Y - \int_0^1 \log p(z) \, dz,
\]

where \( Y \) is the aggregate income and \( p(z) \) is the price given by Eq. (5).

We would expect the aggregate welfare to decrease as \( e \) increases. However, this is not obvious since an increase in \( e \) has ambiguous effect on the equilibrium wage. The following proposition provides a sufficient condition for the aggregate welfare to decrease with \( e \).

**Proposition 4.** Suppose (A1) holds and \( b(z)/z \) decreases with \( z \). Then the aggregate welfare strictly decreases with \( e \) as long as technology transfer occurs.

**Proof.** See Appendix A.

As \( e \) increases, technology transfer becomes more costly. As long as \( b(z) \) does not increase with \( z \) rapidly, the aggregate welfare always decreases with \( e \) until it is not worthwhile to transfer technology, i.e., we end up with the standard DFS model without technology transfer.

### 3.6. South’s FDI policies

Like trade policies, various types of policies to affect FDIs are often adopted by governments of the host and source countries. According to the UN’s World Investment Report 1998, there were a total of 151 major regulatory changes made in 1997 by 76 countries, of which 89% were in the direction of creating more favorable conditions for
FDI, and 11% in the opposite direction. The favorable policy changes included liberalization measures as well as new incentives. In general, we can classify the main incentives for FDI into two types, fiscal incentives (e.g., reduction of the standard corporate income-tax rate, investment and reinvestment allowances, tax holidays, accelerated depreciation, exemptions from import duties) and financial incentives (e.g., government grants, subsidized credits, government equity participation, government insurance at preferential rates, and preferential market access).

In this section, we show how our model can be used to analyze FDI policies. To this end, let us focus on the host country’s (the South in our model) subsidy policies to see how they affect the equilibrium technology transfer, relative wage and welfare.

3.6.1. Subsidizing MNEs’ use of local labor

Suppose the Southern government provides financial incentives to MNEs for creating jobs in the South. Specifically, a MNE receives subsidy \( s(x) \) from the Southern government for employing \( x \) Southern workers. Consequently, the cost of production by the MNE that produce product \( z \) is \( b(z) = e - s(e) + r(k) \). It is natural to assume \( s(e) < e \), but the analysis does not depend on such an assumption. A special form of this financial incentive is wage subsidy. Setting \( s(e) = \alpha e \), where \( \alpha \in (0, 1) \), then \( \alpha \) is the subsidy rate. Note that an increase in \( s(e) \) has the same qualitative effects on \( b(z) \) as a decrease in \( e \). Hence, the comparative statics results obtained in Section 3.3, in particular Propositions 2 and 3, with regard to a decrease in \( e \) hold here for an increase in \( s(e) \). Specifically, an increase in the subsidy enlarges the range of products produced by the MNEs (reducing \( z_0 \) and increasing \( z_1 \)). However, the effect of increasing the subsidy on the Northern wage \( w^* \) is ambiguous.

The welfare analysis is somewhat different from Section 3.4 because we need to subtract the subsidy payment to MNEs from Southern welfare. From Eq. (8) we obtain the subsidy payment as

\[
S = s(e)Y \int_{z_0}^{z_1} \frac{1}{b(z)} \, dz = s(e) \int_{z_0}^{z_1} \frac{e}{b(z)} \, dz = \frac{s(e)}{e} (L + w^* L^*).
\]

Thus, the total income for the South is given by

\[
L - S = \left( 1 - \frac{s(e)}{e} \right) L - \frac{s(e)}{e} w^* L^* ,
\]

where \( s(e) \) is chosen such that \( L - S > 0 \). The aggregate Southern welfare can be expressed as

\[
W_S = \log \left( \left( 1 - \frac{s(e)}{e} \right) L - \frac{s(e)}{e} w^* L^* \right) - \int_{z_0}^{z_1} \log a(z) \, dz - \int_{z_0}^{z_1} \log b(z) \, dz - (1 - z_1) \log w^*.
\]
Note that
\[
((e - s(e))l - s(e)w^*) \frac{\partial W_S}{\partial s(e)} = -(l + w^*)w^* \\
- [(e - s(e))(1 - z_1)l + s(e)z_1w^*] \frac{\partial w^*}{\partial s(e)}.
\]

Therefore, a necessary but not sufficient condition for a subsidy to improve Southern welfare is that the subsidy reduces the relative Northern wage, or equivalently the price of Northern products relative to the price of products produced by Southern firms goes down. If the subsidy increases the equilibrium relative Northern wage, then it will necessarily reduce Southern welfare.

3.6.2. Providing training to local labors employed by MNEs

Suppose the worker training cost for bringing down \( e \) by \( e \) is \( ec(z) \). Then, we have
\[
b(z) = (1 - z)e + rk(z).
\]
It can be seen that such a policy has the same qualitative effect as job subsidy: it encourages FDI and enlarges the range of products produced by the MNEs (reducing \( z_0 \) and increasing \( z_1 \)), but the effect of providing training to local laborers employed by MNEs on the Northern wage \( w^* \) is ambiguous.

Because of the training cost, there is a reduction in the Southern welfare equal to
\[
S = ec(z)Y \int_{z_0}^{z_1} \frac{1}{b(z)} dz = c(z)(L + w^*L^*)
\]
Thus, the total income for the South is given by
\[
L - S = (1 - c(z))L - c(z)w^*L^*,
\]
where \( z \) is chosen such that \( L - S > 0 \). The aggregate Southern welfare can be expressed as
\[
W_S = \log((1 - c(z))L - c(z)w^*L^*) - \int_0^{z_0} \log a(z) dz - \int_{z_0}^{z_1} \log b(z) dz
\]
\[
-(1 - z_1)\log w^*.
\]
It is clear that the discussion on the welfare effect of increasing job subsidy obtained earlier also applies to the case of increasing provision of training to Southern workers.

4. Equilibrium analysis with endogenous \( r \)

In Section 3, we derive the equilibrium \((z_0, z_1, w^*)\) and conduct comparative statics analysis under the assumption that \( r \) is an exogenously given constant. Now we turn to the case in which \( r \) is determined endogenously with an upward sloping supply curve of \( T \). Given the comparative statics results of equilibrium \( z_0, z_1 \) and \( w^* \) with respect to \( r \) discussed in Section 3.3, we can derive the demand for \( T \). The intersection between this derived demand curve and an exogenously given upward sloping supply curve yields the equilibrium \( r \) and \( T \).
For any given \( r \), let \((z_0, z_1, w^*)\) be the equilibrium outcomes when technology transfer takes place. The induced demand for \( T \) is given by
\[
T_D(r) = YM, \tag{11}
\]
where \( M \) and \( Y \) are given in Eqs. (7) and (8), respectively. Note that from Eqs. (3) and (9) when \( r = 0 \), \( z_0 = a^{-1}(e) \) and \( z_1 = L / (L + eL^*) \) so that
\[
T_D(0) = \left( \frac{L}{e} + L^* \right) \int_{a^{-1}(e)}^{L/(L+eL^*)} k(z)dz.
\]

In our model, for technology transfer to take place at all, it must be the case that \( e + rk(\tilde{z}) < a(\tilde{z}) \) for some \( r \), where \( \tilde{z} \) is determined by \( a(\tilde{z}) \) and \( l \) in Eq. (2). We analyze the impact of technology transfer and hence make the following assumption.

**Assumption (A2).** \( e < a(\tilde{z}) \).

Assumption (A2) implies that \( a^{-1}(e) < L / (L + eL^*) \). Thus, when \( r = 0 \), the demand for expatriates is positive.

On the other hand, \( T_D(r) = 0 \) when \( z_0 = z_1 = \tilde{z} \), or \( a(\tilde{z}) = e + rk(\tilde{z}) \), i.e.,
\[
\tilde{r} = \frac{a(\tilde{z}) - e}{k(\tilde{z})}.
\]

If \( r \) is sufficiently large, i.e., \( r \geq \tilde{r} \), there will be no technology transfer, so the demand for expatriates is zero. The following proposition shows that the demand for expatriates is strictly decreasing in \( r \) whenever it is positive.

**Proposition 5.** Suppose (A1) and (A2) hold. Then the demand for expatriates \( T_D \) is continuous and strictly decreasing in \( r \) for all \( r \leq \tilde{r} = [a(\tilde{z}) - e] / k(\tilde{z}) \).

**Proof.** See Appendix A. \( \square \)

We now describe the restrictions on the supply function \( T_S(r) \). Let \( \bar{r} = \max \{ r \mid T_S(r) = 0 \} \).

We make the following assumption.

**Assumption (A3).** (i) \( T_S \) is continuously differentiable and strictly increasing in \( r \) for \( r \geq \bar{r} \); and (ii) \( \tilde{r} < \bar{r} \).

Assumptions (A2) and (A3), together with Proposition 5, imply that the demand and supply curves for expatriates intersect once. It then follows from Proposition 1 that there exists a unique equilibrium in our model. We state this result formally.

**Proposition 6.** Suppose (A1)–(A3) hold. Then there exists a unique equilibrium profile \((z_0, z_1, w^*, r)\) satisfying Eqs. (3), (4), (6) and \( T_D = T_S \).

We next turn to comparative static analysis. First, it can be easily verified that the demand for expatriates decreases uniformly with \( e \) and \( L^* \) but increases with \( L \). Proposition 5 then implies that the equilibrium \( r \) and \( T \) decrease with \( e \) and \( L^* \) but increase with \( L \). Similarly, when the supply curve shifts outward, the equilibrium \( r \) decreases but the equilibrium \( T \) increases.

Second, we study how a change in \((e, L, L^*)\) or a shift in supply of expatriates may affect other equilibrium variables \((z_0, z_1, w^*)\). Following a two-step approach, we consider direct and indirect effects. The direct effect is shown in Proposition 2. The indirect effect is
through the change in the equilibrium $r$. In the case of changes in $L$ and $L^*$ on $z_0$, for instance, the direct effect is zero and the indirect effect implies that an increase in $L$ or a decrease in $L^*$ increases $z_0$. In the case of an increase in $e$, the direct effect is that $z_0$ increases but $z_1$ decreases, whereas the indirect effect is that $r$ decreases, pushing $z_0$ down while pushing $z_1$ up. In the case of shifting the supply curve outward, there is only an indirect effect which decreases $z_0$ and increases $z_1$. We summarize these comparative static findings in the following proposition.

**Proposition 7.** Suppose (A1)–(A3) hold. Then in equilibrium the following hold.

(i) As $L$ increases ($L^*$ decreases), $z_0$, $r$ and $T$ increase (decrease).
(ii) As $e$ increases, $r$ and $T$ decrease.
(iii) As the supply of expatriates expands, $z_0$ and $r$ decreases but $T$ and $z_1$ increase.

In other cases, the direct and indirect effects work in opposite directions so that the net effect is often ambiguous. To further understand the impact of the opposite effects, we now consider a limiting case along the line of Proposition 3. Suppose $e = \bar{e}$ so that in equilibrium MNEs are indifferent between transferring technology and not transferring technology. This is the situation in which the demand and supply curves for expatriates intersect at $r = \bar{r}$ with $\bar{r} > 0$, which implies that $\bar{e} = a(\bar{z}) - \bar{r}k(\bar{z})$. A small reduction in $e$ will encourage the MNEs to transfer technology to the South via FDI and hence shift the demand curve for expatriates upward. We evaluate how such a change affects the equilibrium and welfare. The results are presented in the following proposition.

**Proposition 8.** Suppose initially the equilibrium is such that there is no difference between transferring technology and not transferring technology (i.e., $e = \bar{e}$ and $r = \bar{r}$). Then a small reduction in $e$, which induces technology transfer,

(i) reduces $z_0$, but increases $z_1$ and $r$;
(ii) reduces (increases) $w^*$ and Northern workers’ welfare and increases (reduces) Southern workers’ welfare if and only if $\bar{z}d(\bar{z})/[\bar{r}k(\bar{z})] > (<) l/(\bar{e} - l)$; and
(iii) increases the aggregate welfare.

**Proof.** See Appendix A.

Results (i) and (iii) are intuitive and as expected because technology transfer increases global productivity via MNE production in the South. The welfare results (ii) depend critically on the effect of $e$ on $w^*$. The intuition behind changes in $w^*$ in relation to $a'$ and $b'$ is the same as that under Proposition 3.

5. Concluding remarks

We have extended the continuum Ricardian trade model to feature both international trade and technology transfer via FDI by MNEs. We have shown that there is a unique
range of products produced by MNEs in the South using Northern technology. Comparative static analysis shows that in the case of infinitely elastic supply, (a) an increase in the ratio of the Southern to Northern labor supplies, an increase in the efficiency of technology transfer, or decrease in the wage rate for the expatriates increases the range of MNE production, (b) an increase in the efficiency of technology transfer will affect Northern and Southern workers’ welfare in opposite directions, and (c) an increase in the efficiency of technology transfer increases aggregate welfare if the unit cost of MNE products increases with product sophistication less than linearly. In the case of a general supply curve of expatriates, an increase in the South’s absorption ability raises the wage rate for expatriates. An expansion of the supply of expatriates expands the range of products produced by MNEs. An increase in the South’s labor force increases the range of products produced by Southern firms except when the supply of expatriates is horizontal. An increase in the North’s labor force increases the range of products produced in the North.

If a way can be found to ascertain the impact of an increase in the supply of resources that are essential to technology transfer or in improvement in Southern workers’ absorptive ability on the share of MNE production, then the above predictions can be empirically tested.

We have found that in the case of an infinitely elastic supply of expatriates the Southern government has an incentive to subsidize MNEs’ technology transfer only if the subsidy lowers the relative wage of Northern workers, or if the price of products produced by Southern firms rises relative to the price of products produced by Northern firms.

To address the issue of income distribution, it would be useful for future research to allow for more than one factor of production. In the present model, MNEs may or may not produce simultaneously in both the advanced and backward regions, so a second direction of future research is to consider vertically related production processes. A third direction of research is to extend the technology transfer model to many countries in order to capture the cascading pattern of FDI and trade, namely, that advanced countries invest in all economies while newly industrializing economies invest in the developing economies.

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16 A partial equilibrium model developed by us (Cheng et al., 2001) featuring two kinds of labor (skilled and unskilled) is used to explain vertical fragmentation of production between Hong Kong and South China with and without FDI.
Appendix A

Proof of Corollary 2. Proposition 1 implies that when technology transfer occurs, \( z_0 < \tilde{z} \). It follows from Assumption (A1) that \( b(\tilde{z}) < a(\tilde{z}) = (1 - \tilde{z})l/\tilde{z} \). In contrast, Eq. (9) implies that \( b(z_1) > (1 - z_1)/z_1 \). The above two inequalities imply that \( \tilde{z} < z_1 \). Again from Eq. (9) we have

\[
(1 - z_1)l = b(z_1) \left( z_0 + \int_{z_0}^{z_1} \frac{e}{b(z)} \, dz \right) \geq b(z_1)z_0 + e(z_1 - z_0)
\]

since \( b(z) \) is an increasing function of \( z \). As \( b(z_1) \geq e \), the above inequality implies that \( z_1 \leq l/(e + l) \). \( \Box \)

Proof of Proposition 2. The monotonicity of \( z_0 \) and \( z_1 \) with respect to \( l \) is straightforward. The monotonicity of \( z_0 \) with respect to \( e \) and \( r \) follows immediately from Assumption (A1).

We now prove the monotonicity of \( z_1 \) with respect to \( e \) and \( r \). Differentiating both sides of the balanced trade equation with respect to \( e \) by taking into account the fact that \( z_0 \) depends on \( e \) yields

\[
\left( \frac{1 + e/l}{1 - z_1} + \frac{b'(z_1)}{b(z_1)} \right) \frac{\partial z_1}{\partial e} + \frac{1}{b(z_1)} + \frac{b(z_1)}{l(1 - z_1)} \times \left( \frac{rk(z_0)}{b(z_0)(a'(z_0) - b'(z_0))} + \int_{z_0}^{z_1} \left( \frac{rk(z)}{b(z)^2} \right) \, dz \right) = 0.
\]

It follows immediately that \( \partial z_1 / \partial e < 0 \). Similarly, differentiating both sides of the balanced trade equation with respect to \( r \) by taking into account the fact that \( z_0 \) depends on \( r \) yields

\[
\left( \frac{1 + e/l}{1 - z_1} + \frac{b'(z_1)}{b(z_1)} \right) \frac{\partial z_1}{\partial r} + \frac{k(z_1)}{b(z_1)} + \frac{b(z_1)}{l(1 - z_1)} \times \left( \frac{rk(z_0)^2}{b(z_0)(a'(z_0) - b'(z_0))} - \int_{z_0}^{z_1} \left( \frac{ek(z)}{b(z)^2} \right) \, dz \right) = 0.
\]

It suffices to show that

\[
k(z_1) \frac{b(z_1)}{b(z_1)} > \frac{b(z_1)}{l(1 - z_1)} \int_{z_0}^{z_1} \left( \frac{ek(z)}{b(z)^2} \right) \, dz
\]

which, by the balanced trade condition, is equivalent to

\[
k(z_1) \left( z_0 + \int_{z_0}^{z_1} \left( \frac{e}{b(z)} \right) \, dz \right) > b(z_1) \int_{z_0}^{z_1} \left( \frac{ek(z)}{b(z)^2} \right) \, dz.
\]
The last inequality is implied by the following inequality

$$\int_{z_0}^{z_1} \left( \frac{k(z_1)}{b(z)} \right) dz - \int_{z_0}^{z_1} \left( \frac{b(z_1)k(z)}{b(z)^2} \right) dz = \int_{z_0}^{z_1} \left( \frac{k(z_1)}{b(z)} \frac{k(z_1) - k(z)}{b(z)} \right) dz \geq 0,$$

which holds since $k(z)/b(z)$ increases with $z$. The claim follows.

**Proof of Proposition 3.** Note that

$$\frac{dw}{de} = 1 + rk'(z_1) \frac{dz_1}{de}$$

and differentiating both sides of Eq. (6) yields

$$\left( \frac{(1 + e/l)z_1}{1 - z_1} + z_1b'(z_1) \right) \frac{dz_1}{de} + z_1z_1b(z_1) + \frac{z_1b(z_1)}{l(1 - z_1)} \left( \frac{rk(z_0)}{b(z_0)(a'(z_0) - b'(z_0))} \right) + \int_{z_0}^{z_1} \left( \frac{rk(z)}{b(z)^2} \right) dz = 0. \quad (A1)$$

Letting $e = e^*$ and using $b(\tilde{z}) = a(\tilde{z}) = (1 - \tilde{z})l/\tilde{z}$ yield

$$(l + e + \tilde{z}b'(\tilde{z})) \frac{dz_1}{de} + \tilde{z} + \frac{rk(\tilde{z})}{a'(\tilde{z}) - b'(\tilde{z})} = 0.$$

It follows that

$$\left. \frac{dw}{de} \right|_{e=e^*} = 1 - rk'(\tilde{z}) \frac{\tilde{z} + rk(\tilde{z})/(a'(\tilde{z}) - b'(\tilde{z}))}{l + e^* + \tilde{z}b'(\tilde{z})} = \frac{l + e^*}{l + e^* + \tilde{z}b'(\tilde{z})} - \frac{rk(\tilde{z})}{l + e + \tilde{z}b'(\tilde{z})} a'(\tilde{z}) - b'(\tilde{z}) > 0$$

if and only if

$$(l + e^*) \left[ a'(\tilde{z}) - b'(\tilde{z}) \right] > rk(\tilde{z})b'(\tilde{z})$$

or equivalently

$$a'(\tilde{z}) \frac{1 + e^* + rk(\tilde{z})}{1 + e^*} = \frac{l + a(\tilde{z})}{l + e^*} = \frac{l}{(l + e^*)\tilde{z}}.$$

The claim follows.

**Proof of Proposition 4.** Note that

$$W = \log Y - \int_{0}^{z_0} \log a(z) dz - \int_{z_0}^{z_1} \log b(z) dz - (1 - z_1) \log w^*.$$
The balanced trade condition implies that
\[ Y = \frac{w^*L^*}{1 - z_1} \]
so that
\[ \log Y = \log L^* + \log w^* - \log (1 - z_1). \]
Integration by parts yields
\[ \int_0^{z_0} \log a(z) \, dz = z_0 \log a(z_0) - \int_0^{z_0} \left( \frac{z_0 a'(z)}{a(z)} \right) \, dz \]
and
\[ \int_{z_0}^{z_1} \log b(z) \, dz = z_1 \log b(z_1) - z_0 \log b(z_0) - \int_{z_0}^{z_1} \left( \frac{z_0 b'(z)}{b(z)} \right) \, dz. \]
Using the above expressions and the equilibrium conditions \( a(z_0) = b(z_0) \) and \( w^* = b(z_1) \), we can rewrite \( W \) as
\[ W = \log L^* - \log (1 - z_1) + \int_0^{z_0} \left( \frac{z_0 a'(z)}{a(z)} \right) \, dz + \int_{z_0}^{z_1} \left( \frac{z_0 b'(z)}{b(z)} \right) \, dz. \]
Since
\[ \frac{\partial W}{\partial z_1} = \frac{1}{1 - z_1} + \frac{z_1 b'(z_1)}{b(z_1)}, \]
\[ \frac{\partial W}{\partial z_0} = \frac{z_0 a'(z_0)}{a(z_0)} - \frac{z_0 b'(z_0)}{b(z_0)} = \frac{z_0}{b(z_0)} (a'(z_0) - b'(z_0)), \]
\[ \frac{\partial W}{\partial e} = - \int_{z_0}^{z_1} \left( \frac{b'(z)}{b(z)^2} \right) \, dz, \]
and
\[ \frac{\partial z_0}{\partial e} = \frac{1}{a'(z_0) - b'(z_0)}, \]
it follows that
\[ \frac{dW}{de} = \frac{\partial W}{\partial z_1} \frac{\partial z_1}{\partial e} + \frac{\partial W}{\partial z_0} \frac{\partial z_0}{\partial e} + \frac{\partial W}{\partial e} = \left( \frac{1}{1 - z_1} + \frac{z_1 b'(z_1)}{b(z_1)} \right) \frac{\partial z_1}{\partial e} + \frac{z_0}{b(z_0)} \]
\[ - \int_{z_0}^{z_1} \left( \frac{b'(z)}{b(z)^2} \right) \, dz. \]
It suffices to show that the first two terms above together are negative.
Indeed, differentiating both sides of the modified Eq. (6) yields Eq. (A1), which implies that
\[
\left(\frac{1 + e/l}{1 - z_1} + \frac{z_1 b'(z_1)}{b(z_1)}\right) \frac{\partial z_1}{\partial e} + \frac{z_1}{b(z_1)} < 0,
\]
or
\[
\left(\frac{1}{1 - z_1} + \frac{z_1 b'(z_1)}{b(z_1)}\right) \frac{\partial z_1}{\partial e} < \frac{1 - (1 + e/l)z_1}{1 - z_1} \frac{\partial z_1}{\partial e} - \frac{z_1}{b(z_1)}.
\]
It follows that
\[
\left(\frac{1}{1 - z_1} + \frac{z_1 b'(z_1)}{b(z_1)}\right) \frac{\partial z_1}{\partial e} + \frac{z_0}{b(z_0)} < \frac{1 - (1 + e/l)z_1}{1 - z_1} \frac{\partial z_1}{\partial e} + \frac{z_0}{b(z_0)} - \frac{z_1}{b(z_1)}.
\]
Now, Corollary 2 implies that
\[
1 \geq \left(1 + \frac{e}{T}\right)z_1.
\]
By assumption,
\[
\frac{z_0}{b(z_0)} \leq \frac{z_1}{b(z_1)}.
\]
The claim follows from Proposition 4 since \(\frac{\partial z_1}{\partial e} \leq 0\).

**Proof of Proposition 5.** First note that the equilibrium condition (6) and the definition \(Y = L + w^*L^* + rMY\) together imply
\[
w^*L^* = \frac{(1 - z_1)L}{z_1 - rM}.
\]
It follows that
\[
\frac{T_D(r)}{L} = \frac{M}{z_1 - rM} = \frac{\int_{z_0}^{z_1} f_r(z) dz}{z_1 - r \int_{z_0}^{z_1} f_r(z) dz},
\]
where \(f_r(z) = k(z)/b(z)\).

Second, notice that
\[
\frac{\partial T_D(r)/L}{\partial z_1} (z_1 - rM)^2 = z_1 f_r(z_1) - \int_{z_0}^{z_1} f_r(z) dz
\]
\[
= z_0 f_r(z_1) + \int_{z_0}^{z_1} [f_r(z_1) - f_r(z)] dz > 0.
\]
Since \(f_r(z)\) is an increasing function,
\[
\frac{\partial T_D(r)/L}{\partial z_0} (z_1 - rM)^2 = - z_1 f_r(z_0) < 0,
\]
and
\[ \frac{\partial T_D(r)}{\partial r} \frac{L}{1 - rM} (z_1 - rM)^2 = -z_1 \int_{z_0}^{z_1} f_r(z) dz + \left( \int_{z_0}^{z_1} f_r(z) dz \right)^2. \]

We want to show the above expression is negative. To that end, define
\[ g(x) = -x \int_{z_0}^{x} f_r(z) dz + \left( \int_{z_0}^{x} f_r(z) dz \right)^2 \]
for all \( x \in (z_0, 1] \). Note that \( g(z_0) = 0 \) and
\[ g(1) = - \int_{z_0}^{1} f_r(z) dz + \left( \int_{z_0}^{1} f_r(z) dz \right)^2 < 0. \]
Furthermore,
\[ g'(x) = - \int_{z_0}^{x} f_r(z) dz - xf_r(x) + 2f_r(x) \left( \int_{z_0}^{x} f_r(z) dz \right) \]
which implies \( g'(z_0) < 0 \), and
\[ g''(x) = 2f'_r(x) \left( \int_{z_0}^{x} f_r(z) dz - xf_r(x) \right) < 0. \]
That is, \( g(x) \) is strictly concave and initially decreasing. It is therefore globally decreasing in \( x \).

Finally, note that
\[ \frac{dT_D(r)}{dr} = \frac{\partial T_D(r)}{\partial z_1} \frac{\partial z_1}{\partial r} + \frac{\partial T_D(r)}{\partial z_0} \frac{\partial z_0}{\partial r} + \frac{\partial T_D(r)}{\partial r}. \]
Since the equilibrium \( z_1 \) (given \( r \)) decreases with \( r \) and \( z_0 \) increases with \( r \), all the three terms above are negative. The claim follows.

**Proof of Proposition 8.** Differentiating the equilibrium conditions with respect to \( e \) and taking into account the fact that \( z_0 = z_1 = \bar{z} \) at \( e = \bar{e} \) yield
\[
\begin{align*}
\left\{ a'(\bar{z}) - \bar{r}k'(\bar{z}) \right\} \frac{dz_0}{de} - k(\bar{z}) \frac{dr}{de} &= 1 \\
\bar{r}k(\bar{z}) \frac{dz_0}{de} + \left\{ l + \bar{e} + \bar{r}k'(\bar{z}) \bar{z} \right\} \frac{dz_1}{de} + k(\bar{z}) \bar{z} \frac{dr}{de} &= -\bar{z} \\
- \frac{Lk(\bar{z})}{\bar{z}a(\bar{z})} \frac{dz_0}{de} + \frac{Lk(\bar{z})}{\bar{z}a(\bar{z})} \frac{dz_1}{de} - T'_S(\bar{r}) \frac{dr}{de} &= 0.
\end{align*}
\]
The solution is given by

\[
\frac{dz_0}{de} = \frac{l + \bar{\epsilon} + \bar{r} k'(\bar{z})\bar{z}}{\bar{r}(\bar{z}) + \bar{z}(a'(\bar{z}) - \bar{r}k'(\bar{z}))} T'_S(\bar{r}) > 0
\]

\[
\frac{dz_1}{de} = -\frac{T'_S(\bar{r})}{k(\bar{z})} < 0
\]

\[
\frac{dr}{de} = -\frac{L}{\bar{z}a(\bar{z})} \frac{\bar{r}k(\bar{z}) + \bar{z}a'(\bar{z}) + l + \bar{\epsilon}}{\bar{r}(\bar{z}) + \bar{z}(a'(\bar{z}) - \bar{r}k'(\bar{z}))} < 0,
\]

where

\[
\Lambda = \left( \frac{Lk(\bar{z})}{\bar{z}a(\bar{z})} + T'_S(\bar{r}) \frac{a'(\bar{z}) - \bar{r}k'(\bar{z})}{k(\bar{z})} \right) \frac{l + \bar{\epsilon} + \bar{r} k'(\bar{z})\bar{z}}{\bar{r}k(\bar{z}) + \bar{z}(a'(\bar{z}) - \bar{r}k'(\bar{z}))} + \frac{Lk(\bar{z})}{\bar{z}a(\bar{z})} > 0.
\]

Moreover, it can be shown that

\[
\frac{dw^*}{de} = \frac{T'_S(\bar{r})}{k(\bar{z})} \left\{ a'(\bar{z}) - \bar{r}k'(\bar{z}) \right\} \frac{(l + \bar{\epsilon}) - \bar{r}^2(\bar{z})k'(\bar{z})}{\bar{r}k(\bar{z}) + \bar{z}(a'(\bar{z}) - \bar{r}k'(\bar{z}))},
\]

which is positive if and only if

\[
\frac{\bar{z}a'(\bar{z})}{\bar{r}k'(\bar{z})} > \frac{l}{l + \bar{\epsilon}}.
\]

The results on welfare can be easily verified. \(
\square
\)

References


