Qing Liu, Larry D. Qiu

Cross-Country Externalities of Trade and FDI Liberalization

Abstract We develop a three-country heterogeneous-firm model and show that FDI liberalization in one foreign country (F1) results in the following: (i) some firms from the home country switch from export to FDI in F1; (ii) skilled labor’s wage rate drops in the home country; (iii) wage inequality between the skilled and unskilled labor decreases; and (iv) some firms from the home country switch from FDI to export to another foreign country (F2). The effects from trade liberalization are just the opposite, but the effects from education improvement are qualitatively the same as FDI liberalization. The cross-country externalities work through the domestic labor market.

Keywords export, FDI, firm heterogeneity, cross-country externalities, wage inequality, skill training, contractual friction

JEL Classification F12, F14, F16, F23, J24, J31, L11

1 Introduction

Foreign direct investment (FDI) is playing an increasingly more important role in globalization and the world economic development. In this paper, we investigate the global patterns of export and FDI in different countries. We also explore how FDI and trade liberalization in one host country affects the source country’s labor market and FDI flows to another host country.
The above issues can be best studied using a modified Helpman, Melitz, and Yeaple (HMY) (2004) model with three countries and two types of heterogeneity: heterogeneous firms and heterogeneous countries. In particular, we consider the case where firms from country $H$ (the home country) produce differentiated goods and contemplate serving two segmented foreign markets, $F_1$ and $F_2$, via export or FDI. The two foreign countries can be different in market size, education level, or economic development level (country heterogeneity). Firms are different in their total factor productivity (firm heterogeneity). Production requires both skilled labor and unskilled labor. There are both skilled and unskilled labors in the home country, but all workers in the foreign countries are unskilled. If a home firm undertakes FDI in a foreign country, it needs to provide training to the workers in the host country to perform the skilled labor’s job. The untrained workers (without skills) cannot perfectly substitute the trained workers (with skills); thus, the trained workers have bargaining power and engage in ex post bargaining with the firm over the surplus. This generates contractual frictions in the labor market.

The equilibrium analysis of such a model with multidimensional heterogeneity is inevitably complicated, but we are able to derive clean, interesting and empirically testable results. This is achieved by taking an analytical approach that focuses first on firm heterogeneity, and then encompasses country heterogeneity. Our firm-heterogeneity focus examines the export-FDI decisions in the same foreign market by heterogeneous firms from the same industry. In this case, our model resembles the well-known HMY (2004) model and yields the usual sorting pattern: the most efficient firms undertake FDI, the median efficient firms choose export, and the less efficient firms stay in the home market. We then show how the export-FDI cutoffs are affected by the host country’s characteristics. In the second step of our analysis, we analyze country heterogeneity, which is more complicated because the two foreign countries can be different in many aspects. We first focus on single-dimensional heterogeneity by comparing the export-FDI sorting pattern between the two foreign countries when the two countries are different only in their market size, or education level. The results are simple and intuitive: The market with a smaller size or a lower education level is tougher in the sense that the export-FDI cutoff is higher. We then explore the cases where the two foreign countries are different in more than one dimension. The most interesting case is that one country (e.g., a more developed one) has a higher wage rate and a higher education level than the other (i.e., a less developed one). We show that for any single industry, the cutoff is lower (higher) in the more developed country than in the less developed country if the industry’s skill intensity is high (low).

After characterizing and comparing the export-FDI sorting patterns in individual countries, we show how the export-FDI cutoffs are linked and intervene with each other. Suppose that $F_1$ has FDI liberalization characterized by a reduction in fixed cost of FDI. This reform makes FDI in $F_1$ more attractive and the marginal exporters in all industries switch to FDI in $F_1$, which reduces labor demand in $H$. As a result,
the skilled labor’s wage rate drops, and the wage gap narrows down in $H$. Moreover, production in $H$ becomes more profitable, and the marginal firms in all industries that used to undertake FDI in $F_2$ find it now more profitable to switch to export. These results indicate cross-country externalities or spillovers. The implications are important. First, there exists complementarity between FDI in one country and export in another country. Second, there is inter-country FDI competition (FDI substitution). These cross-country externalities work through the changes in the source country’s labor market.

Our paper is related to the FDI literature. As summarized in the knowledge-capital framework, there are market access motive for FDI and comparative advantage motive for FDI. The former describes the proximity-concentration tradeoff for horizontal FDI, which predicts more FDI (substituting for export) if trade costs (trade barriers, transport costs, etc.) are high or plant-level scale economies are low. This prediction has received larger empirical support using country-level data (e.g., Brainard, 1999). The comparative advantage motive explains multinationals’ investment in a foreign country that has abundant endowment in production factors. Although empirical studies based on country-level data generally do not give strong support to this second motive, comparative advantage has been shown to be also an important factor in determining FDI using industry-level data (e.g., Yeaple, 2003). Recently, a new explanation for FDI has emerged. HMY (2004) introduce firm heterogeneity into the traditional proximity-concentration model and show that even for firms from the same industry of the same country and serving the same foreign market, the most productive firms choose FDI whereas median productive firms choose export. They also show that there could be more FDI (relative to export) in industries with high dispersion of productivity. Accordingly, Chen and Moore (2010) examine how heterogeneous firms make export-FDI decisions in different foreign markets differently. They show, both theoretically and empirically, that firms choosing FDI in tougher markets (e.g., smaller market size or higher fixed cost of FDI) are on average more productive than those choosing FDI in easier markets.

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1 See Markusen and Maskus (2003) for a survey of models relevant to the knowledge-capital framework.
2 Helpman (1994) is the best example of models with comparative advantage motive for vertical FDI.
3 The public and policy makers have voiced concerns about FDI competition. In Asia, the emergence of China has caused the fear that China is adversely affecting FDI flows into their economies. For example, in November 2002, the then Singaporean Deputy Prime Minister, Lee Hsien Loong, commented that “Southeast Asian countries are under intense competitive pressure, as their former activities, especially labor-intensive manufacturing, migrate to China. One indicator of this massive shift is the fact that Southeast Asia used to attract twice as much foreign direct investment as Northeast Asia, but the ratio is reversed.” (China Online, November 14, 2002).
Our paper contributes to this literature in two important ways. First, we emphasize the cross-country linkage or externality of FDI and export, which is important but has been neglected in the literature. The commonly perceived cross-country linkage of FDI is implicitly based on the traditional market access models: when policies in one country make FDI in that country more attractive, they divert FDI from other countries. This FDI diversion argument assumes resources constraints on the multinationals: When the multinationals undertake new FDI in a country, they have to reduce their FDI in other countries. Hence, there is direct competition for FDI between host countries. We show that competition for FDI can also be indirect: FDI in one country affects the source country’s economic condition, which in turn affects FDI in another country. The issue of cross-country linkage of market entry has recently captured the attention of some researchers. Albornoz et al. (2010) find that when a firm exports to a foreign market and finds it profitable, it is more likely that it will export to another foreign market later if market demands across countries are positively correlated. Cherkashin et al. (2010) find that when EU lowers trade barriers, it does not divert Bangladeshi export away from the US but actually raises Bangladeshi exports to both EU and the US markets. Although these two studies have the feature of cross-country externalities, the mechanism that links various foreign countries in our model is very different from theirs. Albornoz et al. (2010) assume that demands in foreign markets are positively correlated, and a firm’s successful export to one market leads to entry to another market. Cherkashin et al. (2010) find that trade liberalization in one market raises export profitability in that market, which induces more firms in the home country to enter the industries; these new entrants also export to other markets. In our paper, FDI liberalization in one foreign country induces some domestic firms to switch from export to FDI in that country. This reduction in export reduces labor demand and consequently wage rate drops in the home country. As a result, all firms use domestic labor for production benefit, which raises export profits; thus, some firms substitute exports for FDI in the other foreign country. While all three papers have cross-country externalities in market entry and the other two papers are about export externalities only, our paper is about externalities between foreign countries in both FDI and export. We have also derived the cross-country externality results based on trade liberalization and education improvement.

Our mechanism for cross-country externalities relies on the effect of FDI on the source country’s employment or wage rate. Not only is this effect clear in our model but it is also consistent with some empirical findings. Two empirical studies are most relevant in this regard. The first is Harrison and McMillan (2006), which finds that horizontal outward FDI from the US reduces employment of the parent firms in the US. The second is Debarera et al. (2010), which finds that for South Korean firms

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4 Other multicountry models (Chen and Moore, 2010; HMY, 2004) do not have cross-country linkage.
having FDI in countries with lower per capita income than South Korea, their parent firms’ employment in South Korea grows more slowly than those that do not invest abroad.\(^5\) These employment effects of FDI can translate to wage decrease if the wage rate is flexible, and therefore lend support to our prediction that home wage rate drops when there is FDI liberalization in a foreign country.

Our second contribution is to the debate on the effects of globalization on income inequality. On one hand, although we confirm the result obtained in most studies in the literature that trade liberalization worsens income distribution, we further show that FDI liberalization reduces income inequality.\(^6\) On the other hand, as the change in the trade-FDI pattern in the liberalizing country \((F_1)\) is always associated with the opposite change in the other country \((F_2)\), the latter change impacts the domestic \((H)\) labor market in the opposite way as compared with the former change; thus, the effects of trade/FDI liberalization on wage rate and wage inequality are less significant when we consider cross-country externalities than when we do not. These two points provide a support to Krugman’s view in the trade-cum-wage inequality debate.\(^7\)

Our paper is also related to the labor economics literature. Although most papers in international trade take factor endowment as given, researchers in labor economics pay much attention to skill training, which results in changes in factor endowment. Relevant research questions include who (firms or workers) should finance the training and whether there is underprovision for training. The answers depend on whether the trained skills are general or firm specific. See Becker (1964) for the first analysis on skill training and Acemoglu (1999) for a survey of some related studies. In this paper, we consider firm-specific skill training, and firms pay the training costs. Incorporating skill training into our trade-FDI model alone is not very interesting because it simply increases the cost of FDI. Realize the incomplete contracting nature of labor training is

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\(^5\) In fact, the empirical evidence on the employment effects of FDI based on country-level data is mixed. See a literature survey by Debaera et al. (2010). However, the effects become much clearer when the types of FDI are classified into different groups by industry nature, e.g., horizontal or vertical FDI (as in Harrison and McMillan, 2006) or host country’s characteristics (as in Debaera et al., 2010). The two cases mentioned above are most closely related to our model.

\(^6\) Our prediction with regard to wage gap is different from those by Markusen and Venables (1997). The models are very different. We emphasize the cross industry differences in skill intensity in production, whereas they focus on the skill-intensity differences in various phrases of the production chain: firm-level entry, plant setup, production stage, etc. Moreover, we consider a foreign country’s FDI liberalization effects on the source country’s labor market and wage gap, but they consider FDI liberalization in all countries and their impacts on wage gap. The empirical findings on trade and FDI’s effects on income gap are mixed. For references, see Feenstra and Hanson (1997) and Slaughter (2000).

\(^7\) The two sides of the debate are represented by Edward Leamer and Paul Krugman. While Leamer thinks trade has played an important role in wage inequality, Krugman thinks it has not.
important. In the presence of some labor market imperfections [e.g., information about the amount of training investment (Katz and Ziderman, 1990) or about the training level (Chang and Wang, 1996)], contractual friction is inevitable. As argued by Hart and Moore (1994) with regard to the inalienability of human capital, because human capital is associated with the trainees once they are trained, it is inevitable that the trained workers will renegotiate with the firms to split the surplus generated from the training. This contractual friction discourages investment in skill training. The degrees of this friction vary from country to country. It is larger in a country with a lower education level because in the case of a negotiation breakdown, the firm has to hire unskilled workers to perform the skilled labor’s job in production, resulting in a larger loss if the unskilled workers’ education level is lower. Thus, our paper allows us to investigate how a change in one foreign country’s education level, which influences contractual friction in that country, affects export and FDI in that country and in the other country as well (through cross-country externalities).

There is an increasing body of studies incorporating the imperfect labor market in models of trade with heterogeneous firms. Helpman and Itskhoki (2009) and Helpman, Itskhoki, and Redding (2010) introduce labor searching and bargaining between firms and workers in the Melitz (2003) model to study the effects of trade on unemployment and wage inequality. The feature of those models with regard to the labor market is searching and matching between firms and workers, but that of our model is labor training. Moreover, the objective of our paper is also very different from theirs: we study cross-country export and FDI patterns in the presence of imperfect labor market.\(^8\)

The paper is organized as follows. We describe the model in Section 2. We perform the equilibrium analysis of export and FDI by heterogeneous firms in Section 3. In Section 4, we analyze how a host country’s conditions determine the export-FDI pattern. In Section 5, we show the cross-country externalities of export and FDI. Concluding remarks are presented in Section 6.

## 2 Model

Consider a world with three countries: the home country, \(H\), and two foreign countries, \(F_1\) and \(F_2\). There is a continuum of industries represented by \(\eta \in (0, 1)\). Each of the industries produces a continuum of differentiated products in \((0, \infty)\). To focus our study on the equilibrium choices between export and FDI by the firms from the same country, we assume that only firms from \(H\) have the technology to produce the

\(^8\) The recent paper by Liu and Qiu (2012) also includes labor training with contract friction. However, Liu and Qiu (2012) focus on the pattern of FDI cross industries with different degrees of skilled-labor intensity.
industries’ differentiated products. The two foreign countries can produce a numeraire good.

The production of differentiated goods require both skilled and unskilled labor. We assume that $H$ is endowed with $L$ skilled labors and $L_U$ unskilled labors. The production of numeraire good uses unskilled labor only, and thus we normalize unskilled worker’s wage rate in $H$ to 1. Wage rate for skilled labor in $H$, denoted by $w$, is endogenously determined in equilibrium by labor demand and supply. In the foreign countries, all labors are initially unskilled, and wage rates are exogenously given (determined by their productivity of numeraire good production), which are denoted by $w_1$ and $w_2$, respectively.

Consumers in all countries have identical preferences over the goods. In particular, we assume that in each country, a representative consumer derives utility from consuming the goods as (CES preference):

$$U = Q_0 + \int_0^1 \frac{1}{\alpha} \log \left( \int_{v \in V_\eta} x_\eta(v)^{\alpha} dv \right) d\eta, \quad 0 < \alpha < 1,$$

where $Q_0$ is the consumption of the numeraire good, $x_\eta(v)$ is the consumption quantity of variety $v$ in industry $\eta$, $V_\eta$ is the product set of industry $\eta$, and $\alpha$ captures the elasticity of substitution across varieties in the same industry. For the sake of convenience, in what follows, we drop the industry index $\eta$ whenever we do not need to distinguish the industries. Utility maximization results in the following demand for each variety in any given industry and country:

$$x = A_i p^{-\epsilon}, \quad \text{where} \quad \epsilon = \frac{1}{1 - \alpha} > 1,$$

(1) $A_i$ is the industry’s aggregate consumption index in the corresponding market, and $p$ is the price. We use $A_1$ and $A_2$ to denote the demand level in $F_1$ and $F_2$, respectively, and $A$ (without subscript) in $H$. They are assumed to be given exogenously.

Let us now describe entry and exit in all differentiated goods industries in $H$. There are a continuum of industries uniformly distributed in $(0, 1)$. In each industry, there are a continuum of firms. Each firm is assigned a productivity level $\theta$ ($> 0$) which follows a cumulative distribution, $G(\theta)$. In the industry, each of the differentiated varieties is produced by a single firm, and each firm produces only one variety. After receiving the productivity level, each firm decides whether to exit or stay in the industry. If a firm exits, then the game is over for it. If a firm decides to produce, it needs to hire workers to set up a production plant. For simplicity, we assume that the total number of labor required is $f_D$ and half of it is skilled labor. Consequently, the fixed plant set-up cost is equal to $\frac{1}{2}(w + 1)f_D$. The staying firms also need to decide how much to produce for the domestic market, and how to serve the foreign markets, which can be either export or FDI. Both export and FDI incur additional fixed costs. If the firm exports its product to $F_i$, it pays an additional overhead cost $w_if_{Xi}$, where $f_{Xi}$ is the number of workers hired in $F_i$ to set up a distribution network in $F_i$, and bears an
iceberg transport cost: only $\tau \in (0, 1)$ unit of the good reaches the destination per unit of the good shipped. If the firm chooses FDI in $F_i$, it pays an additional overhead cost $w_i f_{Ii}$, where $f_{Ii}$ is the number of workers hired in $F_i$ to set up a plant and a distribution network in $F_i$. Thus, $w_i (f_{Ii} - f_{Xi})$ represents the extra fixed costs of producing in $F_i$ (i.e., FDI) compared with exporting to $F_i$. It is reasonable to assume $f_{Ii} - f_{Xi} > 0$. We also assume that firms cannot switch between industries.

We next describe the production technologies. Labor is the only factor used to produce the goods. Following the labor economics literature, we use the popular canonical model, which assumes that both skilled labor and unskilled labor are used in the production of a good. Specifically, we assume that if firm $\theta$ (i.e., the firm with the drawn productivity $\theta$) uses $s$ skilled workers and $u$ unskilled workers, its output becomes

$$x = \theta \left( \frac{s}{\eta} \right) \eta \left( \frac{u}{1 - \eta} \right)^{1 - \eta}, \quad 0 < \eta < 1,$$

where $\eta$ captures the skilled-labor intensity in production.

We assume that there is no skilled labor in the foreign countries and so if a multinational needs skilled workers in the foreign countries for production, it needs to provide training. However, we assume that there is a sufficient supply of skilled labor in $H$ and so the wage rate $w$ is so low such that hiring skilled workers from the market is always less costly than training unskilled workers.

## 3 Analysis

In this section, we first analyze a firm’s profit from each market, based on which we derive its optimal decision with regard to foreign market entry. For expositional convenience, we use $\Theta = \theta^\alpha \epsilon$, which is proportional to the productivity variable $\theta$. Thus, we also regard $\Theta$ as a productivity variable.

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9 This basic setup is similar to Melitz (2003) with some modifications which have no consequence. First, we omit the entry cost. This will affect the level of labor demand and so wage rate, but not any of the results obtained in this paper. Second, we have skilled and unskilled labor for fixed costs. It will become clear later that our specifications of skilled-versus-unskilled labor and domestic-versus-foreign labor in fixed costs have no impact on the qualitative aspect of the results derived in this study. Only the result on wage gap may be altered in the extreme case, where the fixed FDI cost requires a very large amount of skilled labor from $H$.

10 If one wants to analyze the separation of different tasks, which use different skills of labor and are components of the final goods, then the canonical model is not useful. A more general model is required. Acemoglu and Autor (2010) have a general discussion about this, whereas Grossman and Rossi-Hansberg (2008) have a specific analysis of the task model for offshoring. However, outsourcing and offshoring are not issues in our study; the canonical model is not only a simpler one but also a more appropriate one to use.
3.1 Domestic Market

Each firm faces the given market wage rates when it makes the hiring decision. Suppose that a firm hires \( s \) skilled workers and \( u \) unskilled workers to produce for the domestic market. The firm’s profit from the domestic market is

\[
\pi_D = A^{1-\alpha} \theta^\alpha \left( \frac{s}{\eta} \right)^\alpha \left( \frac{u}{1-\eta} \right)^\alpha (1-\eta) - sw - u - \frac{1}{2}(w+1)f_D.
\]

The firm chooses \( s \) and \( u \) to maximize \( \pi_D \). This gives \( s^* = A \eta \alpha^\epsilon w^{-1-\alpha \eta} \Theta \), and \( u^* = A(1-\eta)\alpha^\epsilon w^{-\alpha \eta} \Theta \). Using this in the profit function to obtain the optimal profit \( \pi_D^* = A(1-\alpha)\alpha^\epsilon w^{-\alpha \eta} \Theta - \frac{1}{2}(w+1)f_D \).

Define \( \Theta_D \) from \( \pi_D^*(\Theta_D) = 0 \). Then,

\[
\Theta_D = \frac{(w+1)u^{\alpha \epsilon} f_D}{2A(1-\alpha)\alpha^\epsilon}.
\]

If the firm does not enter the foreign markets (either export or FDI), it stays in the industry if and only if \( \pi_D^* > 0 \), which is \( \Theta > \Theta_D \).

3.2 Exports

When a firm chooses to export its product, it produces in \( H \) and sells part of the output to the foreign markets. Let \( x_D \) be the quantity sold in the home market, \( x_i \) the quantity sold in market \( F_i \), and \( e_i \) a dummy variable equal to unity if the firm exports to \( F_i \) and zero otherwise. In this subsection, we suppose that the firm does not have FDI in any foreign market. The firm then chooses \( (x_D, x_1, x_2, s, u) \) to maximize the following profit, which is the sum of profits from the domestic and two export markets:

\[
\pi_X = x_D p_D + \sum_{i=1}^{2} (x_i p_i - e_i w_i f_{X_i}) - sw - u - \frac{1}{2}(w+1)f_D
\]

\[
= A^{1-\alpha} x_D^\alpha + \sum_{i=1}^{2} [A_i^{1-\alpha} (\tau x_i)^\alpha - e_i w_i f_{X_i}] - sw - u - \frac{1}{2}(w+1)f_D,
\]

subject to the production constraint \( x_D + x_1 + x_2 = \theta \left( \frac{s}{\eta} \right)^\eta \left( \frac{u}{1-\eta} \right)^{1-\eta} \).

- **Export to One Country Only.** Suppose that a firm exports to only one foreign country, say \( F_i \), in which case, \( x_{j \neq i} = 0, e_i = 1 \) and, \( e_{j \neq i} = 0 \). Profit maximization then implies that the marginal revenue in \( H \) and that in \( F_i \) must be equal, which leads to \( A^{1-\alpha} x_D^{\alpha-1} = A_i^{1-\alpha} \tau^\alpha x_i^{\alpha-1} \) and \( x_i = \frac{A_i}{A} \tau^\alpha x_D \). The firm’s optimization
problem is reduced to
\[
\max_{s,u} \pi_X = Q_i^{1-\alpha} \theta^\alpha \left( \frac{s}{\eta} \right)^{\alpha \eta} \left( \frac{u}{1-\eta} \right)^{\alpha(1-\eta)} - sw - u - \frac{1}{2}(w+1)f_D - w_i f_{X_i},
\]
where \( Q_i \equiv A + A_i \tau^{\alpha \epsilon}. \)

We obtain the optimal solution as \( s^* = Q_i \eta^\epsilon w^{-1-\alpha \eta} \Theta \) and \( u^* = Q_i (1-\eta) \alpha^\epsilon w^{-\alpha \eta} \Theta. \) Consequently, the optimal profit is \( Q_i (1-\alpha) \alpha^\epsilon w^{-\alpha \eta} \Theta - \frac{1}{2}(w+1)f_D - w_i f_{X_i} = \pi_D^* + \pi_{X_i}^* \), where the optimal export profit is
\[
\pi_{X_i}^* = A_i \tau^{\alpha \epsilon}(1-\alpha) \alpha^\epsilon w^{-\alpha \eta} \Theta - w_i f_{X_i}. \quad (3)
\]

Evidently, the firm’s export activity does not affect its optimal domestic profit \( \pi_D^* \).

- **Export to Two Countries.** Suppose that a firm exports to both foreign markets, in which case, \( e_1 = e_2 = 1. \) Profit maximization implies that the marginal revenues from each of the three markets must be equal, which results in \( A_1^{1-\alpha} x_D^{\alpha\epsilon-1} = A_2^{1-\alpha} x_1^{\alpha\epsilon-1} = A_2^{1-\alpha} x_2^{\alpha\epsilon-1}. \) Then, \( x_1 = \frac{A_1}{A} \tau^{\alpha \epsilon} x_D, x_2 = \frac{A_2}{A} \tau^{\alpha \epsilon} x_D. \) With this, the firm’s optimization problem is reduced to
\[
\max_{s,u} \pi_X = Q^{1-\alpha} \theta^\alpha \left( \frac{s}{\eta} \right)^{\alpha \eta} \left( \frac{u}{1-\eta} \right)^{\alpha(1-\eta)} - sw - u - \frac{1}{2}(w+1)f_D - w_1 f_{X_1} - w_2 f_{X_2},
\]
where \( Q \equiv A + (A_1 + A_2) \tau^{\alpha \epsilon}. \)

The optimal solution is \( s^* = Q \eta^\epsilon w^{-1-\alpha \eta} \Theta \) and \( u^* = Q (1-\eta) \alpha^\epsilon w^{-\alpha \eta} \Theta, \) and the optimal profit is \( Q (1-\alpha) \alpha^\epsilon w^{-\alpha \eta} \Theta - \frac{1}{2}(w+1)f_D - w_1 f_{X_1} - w_2 f_{X_2} = \pi_D^* + \pi_{X_1}^* + \pi_{X_2}^*. \) Hence, the firm’s total profit is simply the sum of the optimal profits from each individual markets.

- **Summary.** For a firm with \( \Theta > \Theta_D, \) if it does not take FDI, then its total profit is
\[
\Pi_X^* = \pi_D^* + e_1 \pi_{X_1}^* + e_2 \pi_{X_2}^*,
\]
where \( e_i = 1 \) if \( \pi_{X_i}^* > 0 \) and \( e_i = 0 \) otherwise. In particular, at the firm level, the decision on whether or not to export to one foreign market is not affected by its entry decision in the other foreign market. Define \( \Theta_{X_i} \) from \( \pi_{X_i}^*(\Theta_{X_i}) = 0. \) Then,
\[
\Theta_{X_i} = \frac{w_i^\epsilon \eta w_i f_{X_i}}{A_i \tau^{\alpha \epsilon}(1-\alpha) \alpha^\epsilon}. \quad (4)
\]
Given the other parameters, we have \( \pi_{X_i}^* > 0 \) if and only if \( \Theta > \Theta_{X_i}. \)
To obtain the case where some firms are pure domestic producers, we need to impose the condition $\Theta_D < \Theta_X$, which is\[ \frac{2A}{A_1 \tau^\alpha} > \frac{(w + 1) f_D}{w_i f_X}. \] This is typically true in a model like ours that considers a large developed country as the home country, that is, $A$ is larger than $A_i$; assuming $f_X > \tau^\alpha f_D$ is common in the literature. However, all the main results derived in this study remain unchanged if this condition is violated; thus, there are no pure domestic firms.

3.3 FDI and Labor Training

When a firm chooses FDI to enter a foreign country, it produces the good in the host country. We do not consider export-platform FDI and thus rule out the case where a firm undertakes FDI in $F_1$ and sells the output from its subsidiary to the market in $H$ or $F_2$.\(^{11}\) There are three options for a firm’s FDI decision: (i) a firm undertakes FDI in both $F_1$ and $F_2$, (ii) it undertakes FDI in $F_1$ only, and (iii) it undertakes FDI in $F_2$ only. As for any single firm, entry and production in one country does not affect entry and production in another country, we can investigate a firm’s FDI in each country separately.

Suppose that a firm undertakes FDI in $F_i$. Then in $F_i$, the firm hires local workers to produce. However, because there is no skilled labor in $F_i$, the firm needs to provide training to some workers to acquire the skill.\(^{12}\) For simplicity, we assume that the firm pays the training cost, which is $t_i$ per worker, and workers need not to exert any effort in the learning. As a result, the firm pays both the unskilled workers and the trained workers the market wage rate, $w_i$. If the firm trains $s_i$ workers and hires $u_i$ unskilled workers in production, its output is given from the production function as in (2). However, anticipating that the firm can benefit from getting the trained-workers’ services, the trained workers may bargain with the firm after the training but at the right beginning of the production.\(^{13}\)

What is the disagreement payoff to the trained workers? Following the labor eco-

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\(^{11}\) Antras and Foley (2010) study the implications of a free trade agreement between two foreign countries on FDI in those countries in the presence of export-platform FDI.

\(^{12}\) See Acemoglu and Pischke (1999) for a discussion and literature on skill training. With some labor market imperfections, which we implicitly assume, we can allow trained skill to be general (not firm specific).

\(^{13}\) Contractual frictions exist in the presence of labor market imperfections, such as informational imferctions as analyzed by Acemoglu and Pischke (1999) and some studies cited in their paper. In order not to divert our attention, we do not explore the optimal labor contracts that might help mitigate or even eliminate contractual frictions (see Acemoglu and Pischke (1999), for a discussion).
nomics literature, assume that the (short-term, on-the-job) training is firm specific.\textsuperscript{14} That is, the trained skill provided by the firm for producing a variety is no use for production of another variety. Thus, if the trained workers quit, they would go back to the labor pool and receive the market wage $w_i$.

What is the disagreement payoff to the firm? Due to the inalienability of human capital, the firm and the skilled workers can not contract ex ante upon the trained-workers’ future services.\textsuperscript{15} As there would be no time to train new workers if the trained workers quit, the firm has to hire the unskilled workers to take the skilled jobs, which would inevitably lower the productivity.\textsuperscript{16} Suppose the productivity discount rate is $1 - \delta_i \in (0, 1)$, in the sense that if the firm hires $u_i$ unskilled workers to take the unskilled jobs and $s_i$ unskilled workers to take the skilled jobs, the output becomes

$$x_i = \delta_i \theta \left( \frac{s_i}{\eta} \right)^{\eta} \left( \frac{u_i}{1 - \eta} \right)^{1 - \eta}$$

The degree of productivity loss is determined by the gap between the basic capability of the unskilled labor without training and their capability after training. This basic capability is affected by many factors, including general education. For convenience, we simply consider $\delta_i$ as labor’s education level in $F_i$.

Let us now turn to the bargaining between the firm and the trained workers, assuming that the trained workers act as a union so that they quit or stay with one decision.\textsuperscript{17} If the trained workers stay with the firm, the firm’s profit from market $F_i$ (excluding training cost and fixed cost $w_i f_i$, which are sunk) is $A_i^{1 - \alpha} \theta^\alpha \left( \frac{s_i}{\eta} \right)^{\alpha\eta} \left( \frac{u_i}{1 - \eta} \right)^{\alpha(1 - \eta)} - w_i (s_i + u_i)$. If, however, the trained workers quit, the firm’s profit from the market is $\delta_i^\alpha A_i^{1 - \alpha} \theta^\alpha \left( \frac{s_i}{\eta} \right)^{\alpha\eta} \left( \frac{u_i}{1 - \eta} \right)^{\alpha(1 - \eta)} - w_i (s_i + u_i)$ by hiring unskilled workers to

\textsuperscript{14} An alternative theory based on adverse selection argues that the training could be general. However, due to the information asymmetry on the employee’s ability, if the employee quits the current firm, he/she will suffer a decrease in his/her earning because the new employer does not know his/her productivity; thus, the employee is also locked-in (Acemoglu and Pischke, 1998, 1999).

\textsuperscript{15} Following Hart and Moore (1994), we also suppose that at the beginning they cannot contract upon the future output or revenue either. In the European Chamber’s 2007 survey of European firms in China, 64 percent of the firms find it more difficult to retain engineers in China than in Europe, and the number is 74 percent for sales persons.

\textsuperscript{16} It could also lower the quality of the product, but quality is not a dimension in our model.

\textsuperscript{17} The quitting threat of the unskilled workers is not credible, because they can be replaced by the outside workers without causing a loss in the output.
replace them.\(^{18}\) Hence, the surplus associated with the firm when the trained workers do not quit is

\[
(1 - \delta_i^\alpha) A_i^{1-\alpha} \theta^\alpha \left( \frac{s_i}{\eta} \right)^{\alpha \eta} \left( \frac{u_i}{1 - \eta} \right)^{\alpha (1 - \eta)}.
\]

Recall that the trained workers receive the market wage rate with or without quitting. Thus, the surplus associated with the trained-workers when they do not quit is zero.

Suppose that the firm and the trained workers engage in a Nash bargaining and, without loss of generality, the bargaining power is equally distributed. The firm and the trained workers then equally split the joint surplus. Anticipating this, the firm’s ex ante optimization problem is to maximize the sum of its disagreement payoff and its share of the joint surplus, which is

\[
\max_{s_i, u_i} \pi_{II} = \frac{1}{2} (1 + \delta_i^\alpha) A_i^{1-\alpha} \theta^\alpha \left( \frac{s_i}{\eta} \right)^{\alpha \eta} \left( \frac{u_i}{1 - \eta} \right)^{\alpha (1 - \eta)}
- w_i (s_i + u_i + f_{II}) - t_i s_i.
\]

Solving the optimization problem, we have

\[
s^* = \eta A_i \Theta \left[ \frac{\alpha (1 + \delta_i^\alpha)}{2 w_i} \right]^{\epsilon} \left( \frac{w_i}{w_i + t_i} \right)^{1 + \alpha \epsilon \eta}
\]

and

\[
u^* = (1 - \eta) A_i \Theta \left[ \frac{\alpha (1 + \delta_i^\alpha)}{2 w_i} \right]^{\epsilon} \left( \frac{w_i}{w_i + t_i} \right)^{\alpha \epsilon \eta}.
\]

Following the literature (e.g., Antras and Helpman, 2005), we suppose that ex ante the firm could require a lump-sum transfer \(T\) from the to-be-trained workers’ group, which would allow the firm to grasp all the surplus. As a result, the firm’s optimal profit from FDI in \(F_i\) is

\[
\pi^*_I = A_i \left( \frac{2 - \alpha}{2} \right)^{\alpha \epsilon} \left( \frac{1 + \delta_i^\alpha}{w_i} \right)^{\alpha \epsilon} \left( \frac{w_i}{w_i + t_i} \right)^{\alpha \epsilon \eta} \Theta - w_i f_{II}.
\]

\(^{18}\) Here we have supposed that the output loss \(\delta\) is assumed independent of skill intensities, i.e., the same across industries. This follows Antras and Helpman (2004) to simplify the analysis.

An alternative way to model output loss is to assume \(x_i = \theta \left( \frac{s_i}{\eta} \right)^{\eta} \left( \frac{u_i}{1 - \eta} \right)^{1 - \eta}\), which means that each unskilled worker is equivalent to \(\delta\) skilled worker when performing the skilled job. The main results are not sensitive to such a change in the model. Liu and Qiu (2012) have a discussion on this.

We have also supposed that the positions of the skilled labor are replaced one-by-one with the unskilled labor. We could allow the firm to reoptimize its profit by choosing a new combination of unskilled workers for the unskilled job and those for the skilled job. This will change the disagreement payoff, but not the qualitative aspects of results obtained in the paper.
3.4 Optimal Foreign Market Entry Decisions by Heterogeneous Firms from a Given Industry

As previously shown, a firm’s entry decision in one foreign market (export or FDI) does not affect its optimal decision in the other markets. Hence, we can derive a firm’s optimal decision in each market separately. The firm chooses export to $F_i$ if and only if $\pi_{X_i}^* > \max\{0, \pi_{I_i}^*\}$. The firm chooses FDI to enter $F_i$ if and only if $\pi_{I_i}^* > \max\{0, \pi_{X_i}^*\}$. Let us define $\Theta_i$ from $\pi_{I_i}^*(\Theta_i) - \pi_{X_i}^*(\Theta_i) = 0$, which yields

$$\Theta_i = \frac{w_i(f_{I_i} - f_{X_i})}{A_i \alpha^\alpha \Gamma},$$

where

$$\Gamma \equiv \left(\frac{2 - \alpha}{2}\right) \left(1 + \delta_i^\alpha\right)^{\alpha \epsilon} \left(\frac{w_i}{w_i + t_i}\right)^{\alpha \epsilon \eta} - \tau^{\alpha \epsilon (1 - \alpha)} w^{-\alpha \epsilon \eta}.$$

If we draw the firm’s export profit and FDI profit lines against $\Theta$, we obtain Figure 1. The two lines intersect (at $\Theta_i$) if and only if the slope of $\pi_{I_i}^*$ is steeper than that of $\pi_{X_i}^*$. Note that if $\pi_{I_i}^*$ is too steep, we will have $\Theta_{X_i} < \Theta_i$, resulting in no firm choosing export in $F_i$. To obtain the most interesting case, as shown in Fig. 1, for all industries, we assume

$$\frac{f_{X_i}}{f_{I_i}} < \left(\frac{2 - \alpha}{2}\right) \left(\frac{1 + \delta_i^\alpha}{2w_i}\right)^{\alpha \epsilon} \left(\frac{w_i}{w_i + t_i}\right)^{\alpha \epsilon \eta} - \tau^{\alpha \epsilon (1 - \alpha)} w^{-\alpha \epsilon \eta} < 1.$$  \hspace{1cm} (C1)

![Fig. 1 Sorting of Foreign Market Entry by Heterogeneous Firms](image)

With the above analysis, we can now characterize all firms’ entry decisions in any given industry. The five cutoff points, $\Theta_D$, $\Theta_{X_1}$, $\Theta_{X_2}$, $\Theta_1$ and $\Theta_2$, together partition the whole productivity space and the firms entry decisions are determined by
their individual productivity positions. In turn, the cutoff points are determined by all parameters that characterize the home and foreign countries’ economic conditions. Let

\[ C = \{ A, w, f_D; A_1, w_1, f_{I_1}, t_1, \delta_1; A_2, w_2, f_{I_2}, f_{X_2}, t_2, \delta_2 \} \]

be the parameter space of economic conditions and \( c \in C \) be a specific situation. Suppose that (C1) is satisfied for all \( \eta \). The following sorting pattern emerges: firms with \( \Theta \leq \Theta_D \) exit their industries, firms with \( \Theta \in (\Theta_D, \Theta_X) \) focus on the domestic market, firms with \( \Theta \in (\Theta_X, \Theta_i) \) export to \( F_i \), and firms with \( \Theta > \Theta_i \) undertake FDI in \( F_i \).

3.5 Domestic Labor Market Equilibrium

With the above sorting pattern, we can derive the total labor demand for skilled labor in \( H \), which is the sum of labor demand in production for the local market, labor demand in production for exports, and labor demand in the fixed entry costs and fixed plant setup costs. The home country’s skilled labor market equilibrium is established by equating labor supply \( L \) and labor demand, which is given on the right-hand-side of the following equilibrium condition

\[
L = \int_0^1 \left\{ \int_0^{\Theta_D} \left[ A \eta \alpha^\epsilon w^{1-\alpha \eta} \Theta + \frac{1}{2} f_D \right] dG(\Theta) + \int_{\Theta_D}^{\Theta_1} A_1 \eta \alpha^\epsilon w^{1-\alpha \eta} \Theta dG(\Theta) \right. \\
\left. + \int_{\Theta_1}^{\Theta_X_1} A_1 \tau \alpha^\epsilon \eta \alpha^\epsilon w^{1-\alpha \eta} \Theta dG(\Theta) + \int_{\Theta_X_1}^{\Theta_2} A_2 \alpha^\epsilon \eta \alpha^\epsilon w^{1-\alpha \eta} \Theta dG(\Theta) \right\} d\eta. \tag{7}
\]

Thus, skilled labor’s wage rate, \( w \), is determined by (7).

Note that \( w \) needs to be sufficiently small to satisfy (C1). We assume that skilled labor supply in \( H \) is sufficiently high so that this happens. Note that \( w \) is bounded from 1; thus, the second inequality of (C1) does not impose additional constraint on \( w \).

4 Country Heterogeneity and Foreign Market Entry

In the previous section, we have investigated foreign market entry by heterogeneous firms in a given industry. In this section, we turn our attention to country heterogeneity. In this model, the two foreign countries can be different in many dimensions, that is, market size \( (A_i) \), wage rate \( (w_i) \), education level \( (\delta_i) \), training cost \( (t_i) \), fixed entry costs \( (f_{X_i} \text{ and } f_{I_i}) \); thus, our analysis focuses on some of them.

Aside from focusing on certain important dimensions, we also restrict the parameter space to avoid having too many cases to discuss. Recall that in the previous sections,
we characterized the conditions for $\Theta_D < \Theta_{X_1} < \Theta_i$. In this section, we assume that all those conditions still hold. Then, the central issue is how the sorting patterns are different between the two foreign countries. That is, we are interested in the comparison between $\Theta_{X_1}$ and $\Theta_{X_2}$, that between $\Theta_1$ and $\Theta_2$, and that between $\eta_1^*$ and $\eta_2^*$.

As one central question of this paper is how the presence of labor training and contract friction affects $H$ firms’ foreign market entry, we assume $t_1 + w_1 = t_2 + w_2$ to eliminate the cost difference between the two foreign countries.\textsuperscript{19} Without loss of generality, let $t_1 + w_1 = t_2 + w_2 = 1$. For the same reason, we also assume $w_{1fX_1} = w_{2fX_2}$, and $w_{1fI_1} = w_{2fI_2}$.

### 4.1 Single-Dimensional Heterogeneity

In this subsection, we explore two cases where the two foreign countries are different in one aspect only: market size or education level. Accordingly, we assume $w_1 = w_2$.

- **Market Size Difference**

  Suppose that the two foreign countries are different only in market size. In particular, suppose $A_1 > A_2$, but $w_1 = w_2$ and $\delta_1 = \delta_2$. Then, for any given firm, 
  \[
  \pi^*_X - \pi^*_X = (A_1 - A_2)(1 - \alpha)\tau^{\alpha \epsilon \omega} - \alpha \epsilon \Theta > 0, \text{ and } \pi^*_I - \pi^*_I = \frac{1}{2\epsilon}(2 - \alpha)\alpha \epsilon (A_1 - A_2)\Theta \left(\frac{1 + \delta_1}{w_1}\right)^{\alpha \epsilon} w_1^{\alpha \epsilon \eta} > 0. \]
  Hence, the country with a larger market ($F_1$) is more attractive than the other ($F_2$) in both export and FDI. Although some firms find it not profitable to export to $F_2$, they find it profitable to export to $F_1$. Although some firms find it not profitable to have FDI in $F_2$, they find it profitable to have FDI in $F_1$. This comparison is simple and intuitive, but it does not tell us anything about the difference in entry decision between the two foreign countries. The foreign market entry decision, between export and FDI, is affected by the relative attractiveness of FDI to export. In what follows, we show that the relative attractiveness of FDI to export in the country with a larger market ($F_1$) is greater than that in the other ($F_2$). The reason is that the firm has a lower marginal cost with FDI than with export; thus, it benefits more with FDI than with export in bigger market.

  For firms from the same industry, a direct comparison based on the expression of $\Theta_i$ yields $\Theta_1 < \Theta_2$. \textit{Hence, although some firms $\Theta \in (\Theta_1, \Theta_2)$ find it profitable to have export and FDI in both countries, they choose FDI in $F_1$ but export to $F_2$.} The result

\textsuperscript{19}Although the comparative advantage motive for FDI can be easily analyzed in this framework, we abstract from it to highlight the importance of labor training and general education level. This is achieved in our analysis by assuming that the labor costs (i.e., basic wage and training cost) are identical in the two foreign countries.
that the larger market attracts more FDI is definitely not surprising, and it has been empirically confirmed by many existing studies (e.g., Yeaple (2003)). Our analysis at the firm level provides a theoretical explanation for Yeaple’s (2003) empirical finding at country level: the export/FDI ratio is lower in larger markets.20

We can also see how the industry sorting is different between the two countries. Based on (6) and with \( \frac{\partial \Gamma}{\partial \eta} < 0 \) (see Appendix A), total differentiation yields \( \frac{\partial \eta^*_i}{\partial A_i} > 0 \); thus, \( \eta^*_1 > \eta^*_2 \) because \( A_1 > A_2 \). Thus, for firms with the same productivity level, those in industries with \( \eta \leq \eta^*_2 \) choose FDI in both \( F_1 \) and \( F_2 \), those in industries with \( \eta \geq \eta^*_1 \) choose export to both countries, and those in industries with \( \eta \in (\eta^*_2, \eta^*_1) \) choose FDI in \( F_1 \) but export to \( F_2 \). Therefore, the larger country attracts more firms from skill-intensive industries to undertake FDI than the smaller country. This country-industry pairing has not been derived and tested in the literature. However, some other types of country-industry pairing can be found: for example, using industry level data, Yeaple (2003) finds that countries with abundant skilled labor will attract more FDI from skill-intensive industries.

* Education Level Difference

Suppose that the two foreign countries differ only in education level. Specifically, assume \( A_1 = A_2 \), \( w_1 = w_2 \), but \( \delta_1 > \delta_2 \). A firm does not produce in country \( F_1 \) or \( F_2 \) if it chooses export; thus, the education level of the foreign workers does not affect the firm’s export profit. However, in the case of FDI, the firm hires local workers in the host countries for production. In a host country with a higher education level, the firm’s disagreement payoff is also higher, which results in a higher FDI profit. Hence, the country with the higher education level is always more attractive for FDI than the country with the lower education level. These two comparisons immediately lead to the following result: although some firms find it not profitable to have FDI in \( F_2 \), they find it profitable to have FDI in \( F_1 \). This is formally proved as follows.

If we fix the industry, then from (6), we have \( \Theta_1 < \Theta_2 \) because \( \delta_1 > \delta_2 \). If we fix the level of productivity, then, because \( \frac{\partial \Gamma}{\partial \delta_i} > 0 \) and \( \frac{\partial \Gamma}{\partial \eta} < 0 \), we obtain \( \frac{\partial \eta^*_i}{\partial \delta_i} > 0 \) from (6). Hence, \( \eta^*_1 > \eta^*_2 \). That is, for firms with the same given productivity level, those from industries with \( \eta \in (\eta^*_2, \eta^*_1) \) will choose FDI in \( F_1 \) but export to \( F_2 \), although others have the same foreign market entry decisions in both \( F_1 \) and \( F_2 \). Both proofs show that the country with the higher education level will attract more FDI than the country with the lower education level.

20 According to Yeaple (2003), his finding indicates that firms tend to substitute FDI for export to larger markets. We show that the firms with median level productivity are the ones that make this substitution.
4.2 Multidimensional Heterogeneity

In reality, countries are different in many dimensions. This subsection is devoted to examining how a firm’s foreign market entry decisions are different in the two foreign countries, which are different in more than one dimension. As there are too many cases in which countries are different, let us focus on just one that we think is both realistic and interesting: \( A_1 = A_2 \), but \( w_1 > w_2 \) and \( \delta_1 > \delta_2 \). This captures the situation where a more developed country generally has a higher wage rate and a higher education level. Note that a more developed country may not have a larger market, which is affected not only by the development level but also the population size. Our specifications also imply \( t_1 < t_2 \), which is reasonable because training workers from a more developed country is generally easier than from a less developed country.

Let us first focus on any given industry \( \eta \). As \( A_1 = A_2 \) and \( w_1 f_{X1} = w_2 f_{X2} \), we can easily obtain \( \pi_{\eta X1}^* = \pi_{\eta X2}^* \) from (3). This result of equal export profits is clear because export profit is not affected by the foreign country’s wage rate and education level.

Utilizing \( t_1 + w_1 = t_2 + w_2 = 1 \) in (5), we can rewrite a firm’s FDI profit in \( F_i \) to

\[
\pi_{\eta Ii}^* = A_i 2^{-\epsilon} (2 - \alpha) \alpha \epsilon \Theta (1 + \delta_i^\alpha) \alpha \epsilon w_i^\alpha (\eta - 1) - w_i f_{Ii}.
\]

Given any \( \eta \), we can view \( \pi_{\eta Ii}^* \) as a function of \( \Theta \), which is linear. Note that \( \pi_{I1}^* (\Theta = 0) = -w_1 f_{I1} = -w_2 f_{I2} = \pi_{I2}^* (\Theta = 0) \), and that the profit line \( \pi_{I1}^* (\Theta) \) is steeper than that of \( \pi_{I2}^* (\Theta) \) if and only if

\[
\left( \frac{w_1}{w_2} \right)^{1-\eta} < \frac{1 + \delta_1^\alpha}{1 + \delta_2^\alpha}.
\]

(8)

The following result is straightforward.

**Lemma 1.** In any given industry, if (8) holds, then all firms’ FDI profits in \( F_1 \) are larger than in \( F_2 \). The result is reversed if the inequality in (8) is reversed.

The above result is intuitive. There is also a tradeoff for a firm’s FDI profit: a higher education level reduces the profit loss from contractual friction, but a higher wage rate raises the production cost. If (8) holds, the relative education advantage associated with \( F_1 \) is stronger than the relative cost disadvantage with it; thus, the FDI profit in \( F_1 \) is higher.

5 Wage Rate and Cross-Country Externalities

After conducting the equilibrium analysis in the preceding sections, we are now ready to examine whether a change in economic condition in one foreign country affects \( H \) firms’ entry decisions in the other foreign country. That is, we want to determine whether there exist cross-country externalities in market entry decisions. We will ex-
amine various cases of exogenous condition changes.21

5.1 FDI Liberalization

Suppose that the government in $F_1$ lowers the fixed cost of FDI in its country (e.g., subsidization on plant building) so that $f_{I1}$ decreases.

Based on our earlier analysis, $f_{I1}$ affects $\Theta_1$ directly and affects all other cutoffs indirectly through its effect on $w$. The question is whether the effects are positive or negative and how strong each effect is. Note that $\frac{\partial \Theta_1}{\partial f_{I1}} > 0$. This direct effect is clear: a reduction in $F_1$’s fixed FDI cost encourages the marginal firms in all industries to switch from export to FDI in $F_1$. This switch affects the home country’s labor demand, which will in turn generates the indirect effects on $H$ firms’ market entry decisions in both the home and foreign countries. These changes in entry decision once again affect labor demand in the domestic labor market. To obtain the equilibrium effects, we take the total differentiation in the home country’s labor market equilibrium equation as given by (7). This allows us to obtain (see Appendix B for the detailed steps)

$$\Phi_1 \frac{dw}{df_{I1}} = \Phi_2,$$

(9)

where $\Phi_i$ are some complicated functions as given in the proof, and both are positive. Therefore, we have $\frac{dw}{df_{I1}} > 0$. Although the wage rate drops, we still have $\frac{d\Theta_1}{df_{I1}} > 0$ (see proof in Appendix C). That is, the reduction in wage rate will not offset or reverse the initial switch from export to FDI.

Note from (6), $\frac{d\Theta_2}{dw} < 0$. Hence, $\frac{d\Theta_2}{df_{I1}} = \frac{d\Theta_2}{dw} \frac{dw}{df_{I1}} < 0$.

Clearly we can also have $\frac{dw}{df_{I2}} > 0$ and $\frac{d\Theta_1}{df_{I2}} < 0$.

As FDI liberalization does not affect the domestic wage rate for unskilled workers, FDI liberalization (in either one of the foreign countries) clearly reduces $H$’s wage inequality between skilled and unskilled labor.

21 Antras and Foley (2010) include a multicountry feature in their model and also explore how economic policy changes in the host countries affect FDI. In particular, they examine how the formation of a free trade agreement between two foreign countries affects the home firms entry strategies. However, their focus is very different from ours: We consider one country’s policy change and its effects on the other country, whereas they consider a joint policy change of the two foreign countries. Although they also predict FDI substitution, the reason is very different from that in this study.
We summarize the analysis above in the following proposition.

**Proposition 1.** FDI liberalization in one foreign country (i) induces more FDI and reduces export to this country; (ii) reduces FDI and increases export to the other foreign country; and (iii) lowers the skilled labor’s wage rate and reduces the wage gap between the skilled and unskilled labor in the home country.

There are three important points in Proposition 1. First, it shows the existence of cross-country externalities in firms’ foreign market entry decisions. Second, it emphasizes the channel through which one foreign country’s FDI policy affects FDI flows in another foreign country. The channel is the home country’s labor market. Finally, it predicts the effects of foreign country’s FDI policy on the home country’s wage inequality.

5.2 Trade Liberalization

Suppose that the government in $F_1$ undertakes trade liberalization. There are two types of trade liberalization. On the one hand, the government may lower the fixed cost of other countries’ export to its country (e.g., eliminating some government red tape) so that $f_{X1}$ decreases. On the other hand, in our model, $1 - \tau_i$ can be reinterpreted as the tariff rate imposed by $F_i$ on imports. Another type of trade liberalization by $F_1$’s government is to lower $1 - \tau_1$ (i.e., raise $\tau_1$).

We first examine the effects of a change in $f_{X1}$. In Appendix D, we show

$$\frac{dw}{df_{X1}} < 0.$$  

With this important inequality, we can also derive the effects of $f_{X1}$ on cutoff productivities as shown below (see Appendix D):

$$\frac{d\Theta_D}{df_{X1}} < 0, \quad \frac{d\Theta_{X1}}{df_{X1}} > 0, \quad \frac{d\Theta_1}{df_{X1}} < 0, \quad \frac{d\Theta_{X2}}{df_{X1}} < 0, \quad \frac{d\Theta_2}{df_{X1}} > 0.$$  

We next examine the effects of a change in $\tau_1$. Similarly, in Appendix E, we prove

$$\frac{dw}{d\tau_1} > 0.$$  

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22 Inter-country FDI competition is a common concern, and is also evident. For example, Chantasasawat et al. (2003) find that a 10 percent increase in China’s FDI causes the eastern and southeastern Asian countries’ shares of FDI to Asia to drop by about 2–2.5 percent. However, the channel is traditionally believed to be through resource distribution.

23 Many countries impose technical barriers to trade (TBT), which requires exporters to go through complicated process to show that their products satisfy the standards. Bao and Qiu (2010) show that a total of 106 WTO member countries initiated 9913 TBT notifications during the period 1995–2008 and those TBT increase exporters’ fixed cost of exports.
Then, the effects of $\tau_1$ on cutoff productivities can be derived as follows (also see Appendix D)

$$\frac{d\Theta_D}{d\tau_1} > 0, \quad \frac{d\Theta_X}{d\tau_1} < 0, \quad \frac{d\Theta_1}{d\tau_1} > 0, \quad \frac{d\Theta_X}{d\tau_1} > 0, \quad \frac{d\Theta_2}{d\tau_1} < 0.$$ 

The above analysis shows that both types of trade liberalization produce the same result, which is summarized in the following proposition.

**Proposition 2.** Trade liberalization in one foreign country (i) induces more export and reduces FDI to this country; (ii) reduces export and increases FDI to the other foreign country; and (iii) raises the skilled labor’s wage rate and raises the wage gap between the skilled and unskilled labor in the home country.

Trade liberalization produces the opposite effects compared with FDI liberalization. The cross-country externalities also work through the impacts on the home country’s labor market. When $F_1$ takes trade liberalization, more firms choose export ($\Theta_X$ drops) to this country, substituting FDI ($\Theta_1$ increases). This raises the demand for labor in $H$. The wage rate for skilled labor increases, worsening the wage inequality. This wage hike reduces the profitability of domestic production; thus, increases the cutoffs $\Theta_D$ and $\Theta_X$ but lowers $\Theta_2$; that is, export to $F_2$ drops while FDI to $F_2$ increases.

### 5.3 Education Improvement

Suppose that one foreign country, say $F_1$, significantly improves its labor’s education level. As a result, $\delta_1$ increases. Note that $\frac{\partial \Theta_1}{\partial \delta_1} < 0$. Hence, the immediate effect of education improvement in $F_1$ is that more FDI is attracted to this country and less export is taken by $H$’s firms to enter $F_1$. This reduces the labor demand in $H$, lowers wage rate for skilled labor, and reduces the wage gap. Finally, the decline in wage rate induces more export and less FDI to $F_2$. The equilibrium results of education improvement are the same as those of FDI liberalization. The formal proof is given in Appendix E.

### 5.4 Discussion

Although the key result of this section is the existence of cross-country externalities of trade and FDI liberalization, the following three remarks are worth emphasizing. First, our finding that trade liberalization enlarges the income gap between the skilled and unskilled labor is consistent with most other studies in the literature, for example, Helpman et al. (2010). However, our reason is different. We show that as a result of trade liberalization in $F_1$, some firms in $H$ switch from FDI in $F_1$ to export to $F_1$;
thus, it increases demand for skilled labor in $H$.

Second, while the literature focuses on how trade liberalization affects the labor market, we go further to examine how FDI liberalization and education improvement impact the skilled labor’s wage rate and the wage inequality. These changes have the opposite effects as compared with those of trade liberalization. An important implication is that we must include these factors in the empirical studies of the effects of globalization on income inequality to avoid bias towards the effects of trade liberalization.

Third, as the change in the trade-FDI pattern in the liberalizing country ($F_1$) is always associated with the opposite change in the other country ($F_2$), the latter change impacts the domestic ($H$) labor market in the opposite way as compared with the former change. Hence, the effects of liberalization on wage rate and wage inequality are less significant when cross-country externalities are taken into account than when they are not. One direct implication is that perhaps the effects of trade liberalization on labor income are not as significant as we thought. This gives a support to Krugman’s view in the trade-cum-wage inequality debate.

6 Concluding Remarks

This paper extends the HMY (2004) model to include two factors of production and the two dimensions of heterogeneity: firm heterogeneity and country heterogeneity. In addition to the usual finding in the Melitz (2003) types of models where in a given industry the most efficient firms choose FDI, median efficient firms choose exports, and less efficient firms stay in the home market, we also find that foreign countries have different attractiveness to FDI and export depending on their market size, education level, and economic development level, which are all conducive to FDI. Policy changes in one foreign country affect not only FDI and export in that country, but also FDI and export in the other foreign country, and thus policy exhibits cross-country externalities. The mechanism of such cross-country externalities works through the indirect effects of the policy changes on the source country’s labor market. For example, FDI liberalization on one foreign country induces some firms from the source country in all industries to switch from export to FDI, which reduces the demand for labor and hence wage rate in the source country. As a result, some firms switch from FDI to export to another foreign country. This FDI liberalization reduces the source country’s wage for skilled labor and narrows the wage gap.

To focus on the cross-country externalities, we simplified the domestic labor market by assuming that the unskilled labor’s wage rate is fixed due to a large supply of unskilled labor or the use of unskilled labor in the production of numeraire goods. This is perfectly legitimate, and relaxing this restriction will not affect the main results of the paper. However, one (and only one) of the results need to be reexamined: the FDI liberalization’s effect on wage gap. With FDI liberalization in one foreign country,
the demand for both skilled and unskilled labor drops due to the switch from export to FDI by some firms. In equilibrium, the wage rate for skilled and that for unskilled labor both drop. The question is which drops more. We hypothesize that the wage gap drops, that is, the result obtained when we fix the unskilled labor’s wage rate still holds.

Our paper has produced a number of testable predictions, such as the effects of foreign country’s FDI liberalization, trade liberalization, and education improvement on domestic wage rates, wage inequality, and firms’ entry mode choice into other countries. Although some of these findings are indirectly consistent with those from some existing empirical studies, it would be ideal to conduct an empirical analysis using firm-level data to test all our hypotheses directly. The most interesting hypothesis is the linkage between the changes in FDI (and exports) in two foreign countries via the domestic labor market. This is left for future work.

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Appendix

A. Slope of the Export-FDI Division Line.

Note that under (C1)-(C2), we have

\[
\frac{\partial \Gamma}{\partial \eta} = \left( 2 - \alpha \right) \left( 1 + \delta^\alpha \right) \left( \frac{w_i}{w_i + t_i} \right) \alpha \epsilon \ln \left( \frac{w_i}{w_i + t_i} \right) - \tau^\alpha (1 - \alpha) w^{-\alpha \epsilon \eta} \alpha \epsilon \ln \left( \frac{1}{w} \right) < 0.
\]

As \( \eta^*_i(\Theta) \) is also defined by (6); thus, differentiating the condition with respect to \( \Theta \) yields

\[
\frac{\partial \Gamma}{\partial \eta} \frac{\partial \eta^*_i}{\partial \Theta} + \Gamma = 0.
\]

Hence, \( \frac{\partial \eta^*_i}{\partial \Theta} > 0 \). Similarly, we can have \( \frac{\partial \Theta^*}{\partial \eta} > 0 \). □

B. Proof of (9)

Differentiating (7) with respect to \( f_{I1} \) yields

\[
0 = \int_0^1 \left\{ \int_\Theta^\infty - A \eta^\epsilon w^{-2 - \alpha \eta} \left( \alpha \epsilon \eta + 1 \right) \frac{dw}{df_{I1}} \Theta dG(\Theta) - \left( A \eta^\epsilon w^{-1 - \alpha \eta} \Theta_D + \frac{1}{2} f_D \right) \frac{\partial \Theta_D}{\partial w} \frac{dw}{df_{I1}} \right\}.
\]
We prove clearly, \( C \). Proof of the integrations and reorganize the above equation as (9), where

\[
- \int_{\Theta_{x1}}^{\Theta_1} A_1 \tau^\alpha \eta \alpha^\epsilon w^{-2-\alpha \eta} (\alpha \epsilon \eta + 1) \frac{dw}{df_1} \Theta dG(\Theta) \\
+ A_1 \tau^\alpha \eta \alpha^\epsilon w^{-1-\alpha \eta} \Theta_1 g(\Theta_1) \left( \frac{\partial \Theta_1}{\partial w} \frac{dw}{df_1} + \frac{\partial \Theta_1}{\partial f_1} \right) \\
- A_1 \tau^\alpha \eta \alpha^\epsilon w^{-1-\alpha \eta} \Theta_{x1} g(\Theta_{x1}) \frac{\partial \Theta_{x1}}{\partial w} \frac{dw}{df_1} \\
- \int_{\Theta_{x2}}^{\Theta_2} A_2 \tau^\alpha \eta \alpha^\epsilon w^{-2-\alpha \eta} (\alpha \epsilon \eta + 1) \frac{dw}{df_1} \Theta dG(\Theta) \\
+ A_2 \tau^\alpha \eta \alpha^\epsilon w^{-1-\alpha \eta} \Theta_2 g(\Theta_2) \frac{\partial \Theta_2}{\partial w} \frac{dw}{df_1} \\
- A_2 \tau^\alpha \eta \alpha^\epsilon w^{-1-\alpha \eta} \Theta_{x2} g(\Theta_{x2}) \frac{\partial \Theta_{x2}}{\partial w} \frac{dw}{df_1} \right) d\eta.
\]

As \( w \) is independent of \( \eta \), \( \frac{dw}{df_1} \) is not a function of \( \eta \), and we can take \( \frac{dw}{df_1} \) out of the integrations and reorganize the above equation as (9), where

\[
\Phi_1 \equiv - \int_0^1 \left\{ -\eta (\alpha \epsilon \eta + 1) \alpha^\epsilon w^{-2-\alpha \eta} \left[ \int_{\Theta_D}^\infty A \Theta dG(\Theta) \\
+ \int_{\Theta_{x1}}^{\Theta_1} A_1 \tau^\alpha \Theta \Theta dG(\Theta) + \int_{\Theta_{x2}}^{\Theta_2} A_2 \tau^\alpha \Theta \Theta dG(\Theta) \\
- \left( A \eta \alpha^\epsilon w^{-1-\alpha \eta} \Theta_D + \frac{1}{2} f_D \right) g(\Theta_D) \frac{\partial \Theta_D}{\partial w} \\
+ A_1 \tau^\alpha \eta \alpha^\epsilon w^{-1-\alpha \eta} \left( \Theta_1 g(\Theta_1) \frac{\partial \Theta_1}{\partial w} - \Theta_{x1} g(\Theta_{x1}) \frac{\partial \Theta_{x1}}{\partial w} \right) \\
+ A_2 \tau^\alpha \eta \alpha^\epsilon w^{-1-\alpha \eta} \left( \Theta_2 g(\Theta_2) \frac{\partial \Theta_2}{\partial w} - \Theta_{x2} g(\Theta_{x2}) \frac{\partial \Theta_{x2}}{\partial w} \right) \right\} d\eta,
\]

\[
\Phi_2 \equiv \int_0^1 A_1 \tau^\alpha \eta \alpha^\epsilon w^{-1-\alpha \eta} \Theta_1 g(\Theta_1) \frac{\partial \Theta_1}{\partial f_1} d\eta.
\]

Clearly, \( \frac{\partial \Theta_D}{\partial w} > 0, \frac{\partial \Theta_{x1}}{\partial w} > 0, \frac{\partial \Theta_1}{\partial w} < 0, \) and \( \frac{\partial \Theta_1}{\partial f_1} > 0 \). Hence, \( \Phi_i > 0 \), for both \( i = 1, 2 \), and from (9), we must have \( \frac{dw}{df_1} > 0 \). □

C. Proof of \( \frac{d\Theta_1}{df_1} > 0 \).

We prove \( \frac{d\Theta_1}{df_1} > 0 \) by contradiction. Clearly, \( \frac{d\Theta_D}{df_1} = \frac{d\Theta_D}{dw} \frac{dw}{df_1} > 0 \) and \( \frac{d\Theta_{x1}}{df_1} = \frac{d\Theta_{x1}}{dw} \frac{dw}{df_1} > 0 \). Suppose \( \frac{d\Theta_1}{df_1} \leq 0 \), then from the effects of \( f_1 \) on \( \Theta_D, \Theta_{x1}, \Theta_2 \)
and \( w \), we know that an increase in \( f_{I1} \) will always decrease the domestic labor demand for skilled workers [i.e., the RHS of (7)]; thus, the labor market equilibrium condition (7) can never hold after FDI policy changes. However, as (7) always holds in equilibrium, we know that \( \frac{d\Theta_1}{df_{I1}} \leq 0 \) cannot be true, and we must have \( \frac{d\Theta_1}{df_{I1}} > 0 \). □

D. The Case of Trade Liberalization

- We first analyze the change in \( f_{X1} \). Differentiating (7) with respect to \( f_{X1} \) yields

\[
0 = \int_0^1 \left\{ -A \eta \alpha \epsilon w^{-2-\alpha \eta} (\alpha \epsilon \eta + 1) \frac{dw}{df_{X1}} \Theta dG(\Theta) \\
- \left( A \eta \alpha \epsilon w^{-1-\alpha \eta} \Theta_D + \frac{1}{2} f_D \right) g(\Theta_D) \frac{\partial \Theta_D}{\partial w} \frac{dw}{df_{X1}} \\
- \int_{\Theta_{X1}}^{\Theta_2} A_1 \tau^\alpha \eta \alpha \epsilon w^{-2-\alpha \eta} (\alpha \epsilon \eta + 1) \frac{dw}{df_{X1}} \Theta dG(\Theta) \\
+ A_1 \tau^\alpha \eta \alpha \epsilon w^{-1-\alpha \eta} \Theta_{X1} g(\Theta_{X1}) \left( \frac{\partial \Theta_{X1}}{\partial w} \frac{dw}{df_{X1}} + \frac{\partial \Theta_{X1}}{\partial f_{X1}} \right) \\
- A_1 \tau^\alpha \eta \alpha \epsilon w^{-1-\alpha \eta} \Theta_{X1} g(\Theta_{X1}) \left( \frac{\partial \Theta_{X1}}{\partial w} \frac{dw}{df_{X1}} + \frac{\partial \Theta_{X1}}{\partial f_{X1}} \right) \\
- \int_{\Theta_{X2}}^{\Theta_2} A_2 \tau^\alpha \eta \alpha \epsilon w^{-2-\alpha \eta} (\alpha \epsilon \eta + 1) \frac{dw}{df_{X1}} \Theta dG(\Theta) \\
+ A_2 \tau^\alpha \eta \alpha \epsilon w^{-1-\alpha \eta} \Theta_{X2} g(\Theta_{X2}) \frac{\partial \Theta_{X2}}{\partial w} \frac{dw}{df_{X1}} \\
- A_2 \tau^\alpha \eta \alpha \epsilon w^{-1-\alpha \eta} \Theta_{X2} g(\Theta_{X2}) \left( \frac{\partial \Theta_{X2}}{\partial w} \frac{dw}{df_{X1}} + \frac{\partial \Theta_{X2}}{\partial f_{X1}} \right) \right\} \, d\eta.
\]

\( w \) is independent of \( \eta \); thus, \( \frac{dw}{df_{X1}} \) is not a function of \( \eta \), and we can take \( \frac{dw}{df_{X1}} \) out of the integrations and reorganize to have \( \Delta_1 \frac{dw}{df_{X1}} = \Delta_2 \), where

\[
\Delta_1 \equiv \int_0^1 \left\{ -\eta (\alpha \epsilon \eta + 1) \alpha \epsilon w^{-2-\alpha \eta} \left[ \int_{\Theta_{D}}^{\infty} A \Theta dG(\Theta) \right. \\
+ \int_{\Theta_{X1}}^{\Theta_2} A_1 \tau^\alpha \Theta dG(\Theta) + \int_{\Theta_{X2}}^{\Theta_2} A_2 \tau^\alpha \Theta dG(\Theta) \right] \\
- \left( A \eta \alpha \epsilon w^{-1-\alpha \eta} \Theta_D + \frac{1}{2} f_D \right) g(\Theta_D) \frac{\partial \Theta_D}{\partial w} \\
+ A_1 \tau^\alpha \eta \alpha \epsilon w^{-1-\alpha \eta} \left( \Theta_{X1} g(\Theta_{X1}) \frac{\partial \Theta_{X1}}{\partial w} - \Theta_{X1} g(\Theta_{X1}) \frac{\partial \Theta_{X1}}{\partial f_{X1}} \right) \right\}
\]
+ A_2 \tau^{\alpha \epsilon} \eta^{\epsilon} w^{-1} \eta^{\alpha} \left( \Theta_2 g(\Theta_2) \frac{\partial \Theta_2}{\partial w} - \Theta X_2 g(\Theta X_2) \frac{\partial \Theta X_2}{\partial w} \right) \, d\eta,

\Delta_2 \equiv A_1 \tau^{\alpha \epsilon} \eta^{\epsilon} w^{-1} \eta^{\alpha} \int_0^1 \left( \Theta X_1 g(\Theta X_1) \frac{\partial \Theta X_1}{\partial f X_1} - \Theta_1 g(\Theta_1) \frac{\partial \Theta_1}{\partial f X_1} \right) \, d\eta.

Clearly, \frac{\partial \Theta_1}{\partial w} > 0, \frac{\partial \Theta X_1}{\partial w} > 0, \frac{\partial \Theta_1}{\partial f X_1} < 0, \frac{\partial \Theta X_1}{\partial f X_1} > 0, and \frac{\partial \Theta_1}{\partial f X_1} < 0. Hence, \Delta_1 < 0 and \Delta_2 > 0, and thus \frac{d\Theta}{d f X_1} < 0.

From the expressions of the cutoffs, we immediately have

\frac{d\Theta_D}{d f X_1} = \frac{d\Theta_X}{d f X_1} \frac{d w}{d f X_1} < 0, \quad \frac{d\Theta X_2}{d f X_1} = \frac{d\Theta X_2}{d w} \frac{d w}{d f X_1} < 0,

and \quad \frac{d\Theta_1}{d f X_1} = \frac{d\Theta_1}{d w} \frac{d w}{d f X_1} > 0.

As for the other two cutoffs, we have

\frac{d\Theta X_1}{d f X_1} = \frac{\partial \Theta X_1}{\partial w} \frac{d w}{d f X_1} + \frac{\partial \Theta X_1}{\partial f X_1}, \quad \text{and} \quad \frac{d\Theta_1}{d f X_1} = \frac{\partial \Theta_1}{\partial w} \frac{d w}{d f X_1} + \frac{\partial \Theta_1}{\partial f X_1}.

We show \frac{d\Theta X_1}{d f X_1} > 0 by contradiction. Suppose this is not true, that is, \frac{d\Theta X_1}{d f X_1} < 0 (for clearer proof, we drop the case with equality). From the profit function \pi_{X_1} and the corresponding profit line in Fig. 1, we note that an increase in \( f_{X_1} \) shifts down the intercept of the profit line \pi_{X_1}^*; if \frac{d\Theta X_1}{d f X_1} < 0, the cutoff \Theta_{X_1} shifts to the left, and the new profit line, say \pi'_{X_1}, must have a much steeper slope than \pi_{X_1}. It is clear that the two profit lines, \pi'_{X_1} (which does not move as a result of changes in \( f_{X_1} \) and \( w \)) and \pi_{X_1} must intersect at a point to the right of \Theta_1, which implies \frac{d\Theta_1}{d f X_1} > 0. Hence, the move of the two cutoff points \left( \frac{d\Theta X_1}{d f X_1} < 0 \right) implies an increase in demand for labor associated with an increase in \( f_{X_1} \). Moreover, the move of all other cutoffs \left( \frac{d\Theta D}{d f X_1} < 0, \frac{d\Theta X_2}{d f X_1} < 0, \text{ and } \frac{d\Theta_2}{d f X_1} > 0 \right) also implies an increase in demand for labor. Thus, labor demand increases and labor market is not in equilibrium. This is a contradiction. Therefore, \frac{d\Theta X_1}{d f X_1} > 0, which implies \frac{d\Theta_1}{d f X_1} < 0 following the analysis we just had.
We now turn to the change in $\tau_1$. We first modify (7) using $\tau_1$ and $\tau_2$ to substitute the respective $\tau$. We then differentiate (7) with respect to $\tau_1$ to obtain

$$0 = \int_0^1 \left\{ \int_{\Theta_D}^\infty -A\eta\alpha^\epsilon w^{-2-\alpha\eta} (\alpha\epsilon + 1) \frac{dw}{\partial \tau_1} \Theta dG(\Theta) ight. \\
- \left( A\eta\alpha^\epsilon w^{-1-\alpha\eta} \Theta_D + \frac{1}{2} f_D \right) g(\Theta_D) \frac{\partial \Theta_D}{\partial w} \frac{dw}{\partial \tau_1} \\
- \int_{\Theta_{X_1}}^{\Theta_1} A_1 \tau^\alpha \eta\alpha^\epsilon w^{-2-\alpha\eta} (\alpha\epsilon + 1) \frac{dw}{\partial \tau_1} \Theta dG(\Theta) \\
+ \int_{\Theta_{X_1}}^{\Theta_1} A_1 \tau^\alpha \eta\alpha^\epsilon w^{-1-\alpha\eta} (\alpha\epsilon + 1) \frac{dw}{\partial \tau_1} \Theta dG(\Theta) \\
+ A_1 \tau^\alpha \eta\alpha^\epsilon w^{-1-\alpha\eta} \Theta_1 g(\Theta_1) \left( \frac{\partial \Theta_1}{\partial w} \frac{dw}{\partial \tau_1} + \frac{\partial \Theta_1}{\partial \tau_1} \right) \\
- A_1 \tau^\alpha \eta\alpha^\epsilon w^{-1-\alpha\eta} \Theta_{X_1} g(\Theta_{X_1}) \left( \frac{\partial \Theta_{X_1}}{\partial w} \frac{dw}{\partial \tau_1} + \frac{\partial \Theta_{X_1}}{\partial \tau_1} \right) \\
- \int_{\Theta_{X_2}}^{\Theta_2} A_2 \tau^\alpha \eta\alpha^\epsilon w^{-2-\alpha\eta} (\alpha\epsilon + 1) \frac{dw}{\partial \tau_1} \Theta dG(\Theta) \right\} d\eta.$$

Take $\frac{dw}{d\tau_1}$ out of the integrations and reorganize it to obtain $\Psi_1 = \Psi_2$, where

$$\Psi_1 = \int_0^1 \left\{ -\eta (\alpha\epsilon + 1) \alpha^\epsilon w^{-2-\alpha\eta} \left[ \int_{\Theta_D}^\infty A \Theta dG(\Theta) ight. \\
+ \int_{\Theta_{X_1}}^{\Theta_1} A_1 \tau^\alpha \Theta dG(\Theta) + \int_{\Theta_{X_2}}^{\Theta_2} A_2 \tau^\alpha \Theta dG(\Theta) \\
- \left( A\eta\alpha^\epsilon w^{-1-\alpha\eta} \Theta_D + \frac{1}{2} f_D \right) g(\Theta_D) \frac{\partial \Theta_D}{\partial w} \frac{dw}{\partial \tau_1} \\
+ A_1 \tau^\alpha \eta\alpha^\epsilon w^{-1-\alpha\eta} \left( \Theta_1 g(\Theta_1) \frac{\partial \Theta_1}{\partial w} - \Theta_{X_1} g(\Theta_{X_1}) \frac{\partial \Theta_{X_1}}{\partial \tau_1} \right) \left( \frac{\partial \Theta_1}{\partial \tau_1} \right) \\
- \left. \frac{d\eta}{\partial \tau_1} \right\} d\eta,$$

$$\Psi_2 = \int_0^1 \left\{ A_1 \tau^\alpha \eta\alpha^\epsilon w^{-1-\alpha\eta} \left( \Theta_{X_1} g(\Theta_{X_1}) \frac{\partial \Theta_{X_1}}{\partial \tau_1} - \Theta_1 g(\Theta_1) \frac{\partial \Theta_1}{\partial \tau_1} \right) \\
- \int_{\Theta_{X_1}}^{\Theta_1} A_1 \tau^\alpha \eta\alpha^\epsilon w^{-1-\alpha\eta} \Theta dG(\Theta) \right\} d\eta.$$

As $\frac{\partial \Theta_D}{\partial w} > 0$, $\frac{\partial \Theta_{X_1}}{\partial w} > 0$, and $\frac{\partial \Theta_1}{\partial w} < 0$, and it is easy to show $\frac{\partial \Theta_{X_1}}{\partial \tau_1} < 0$ and $\frac{\partial \Theta_1}{\partial \tau_1} > 0$, we have $\Psi_1 < 0$ and $\Psi_2 < 0$, and thus $\frac{dw}{d\tau_1} > 0$. 

\[ \]
From the expressions of the cutoffs, we immediately have
\[ \frac{d\Theta_D}{d\tau_1} = \frac{d\Theta_D}{dw} \frac{dw}{d\tau_1} > 0, \quad \frac{d\Theta_{X1}}{d\tau_1} = \frac{d\Theta_{X2}}{dw} \frac{dw}{d\tau_1} > 0, \quad \text{and} \quad \frac{d\Theta_2}{d\tau_1} = \frac{d\Theta_2}{dw} \frac{dw}{d\tau_1} < 0. \]

We can also prove the following (the proof is similar to that in the case of \( f_{X1} \))
\[ \frac{d\Theta_{X1}}{d\tau_1} = \frac{\partial \Theta_{X1}}{\partial w} \frac{dw}{d\tau_1} + \frac{\partial \Theta_{X1}}{\partial \tau_1} < 0 \quad \text{and} \quad \frac{d\Theta_1}{d\tau_1} = \frac{\partial \Theta_1}{\partial w} \frac{dw}{d\tau_1} + \frac{\partial \Theta_1}{\partial \tau_1} > 0. \quad \square \]

E. Education

Differentiating (7) with respect to \( \delta_1 \) yields
\[
0 = \int_{0}^{1} \left\{ - \left( A\eta^\alpha \omega^{1-\alpha} \eta^{\alpha} \Theta_D + \frac{1}{2} f_D \right) g(\Theta_D) \frac{\partial \Theta_D}{\partial w} \frac{dw}{d\delta_1} \right. \\
+ A_1 \tau^\alpha \eta^\alpha \omega^{1-\alpha} \eta^{\alpha} \Theta_1 g(\Theta_1) \left( \frac{\partial \Theta_1}{\partial w} \frac{dw}{d\delta_1} + \frac{\partial \Theta_1}{\partial \delta_1} \right) \\
- A_1 \tau^\alpha \eta^\alpha \omega^{1-\alpha} \eta^{\alpha} \Theta_{X1} g(\Theta_{X1}) \frac{\partial \Theta_{X1}}{\partial w} \frac{dw}{d\delta_1} \\
+ A_2 \tau^\alpha \eta^\alpha \omega^{1-\alpha} \eta^{\alpha} \Theta_2 g(\Theta_2) \frac{\partial \Theta_2}{\partial w} \frac{dw}{d\delta_1} \\
- A_2 \tau^\alpha \eta^\alpha \omega^{1-\alpha} \eta^{\alpha} \Theta_{X2} g(\Theta_{X2}) \frac{\partial \Theta_{X2}}{\partial w} \frac{dw}{d\delta_1} \right\} d\eta.
\]

From that we have \( \Lambda_1 \frac{dw}{df_{X1}} = \Lambda_2 \), where
\[
\Lambda_1 = \int_{0}^{1} \left\{ - \left( A\eta^\alpha \omega^{1-\alpha} \eta^{\alpha} \Theta_D + \frac{1}{2} f_D \right) g(\Theta_D) \frac{\partial \Theta_D}{\partial w} + A_1 \tau^\alpha \eta^\alpha \omega^{1-\alpha} \eta^{\alpha} \Theta_1 g(\Theta_1) \left( \frac{\partial \Theta_1}{\partial w} - \Theta_{X1} g(\Theta_{X1}) \frac{\partial \Theta_{X1}}{\partial w} \right) \\
+ A_2 \tau^\alpha \eta^\alpha \omega^{1-\alpha} \eta^{\alpha} \Theta_2 g(\Theta_2) \left( \frac{\partial \Theta_2}{\partial w} - \Theta_{X2} g(\Theta_{X2}) \frac{\partial \Theta_{X2}}{\partial w} \right) \right\} d\eta, \quad \text{and}
\]
\[
\Lambda_2 = - \int_{0}^{1} \left( A_1 \tau^\alpha \eta^\alpha \omega^{1-\alpha} \eta^{\alpha} \Theta_1 g(\Theta_1) \frac{\partial \Theta_1}{\partial \delta_1} \right) d\eta.
\]

As \( \frac{\partial \Theta_D}{\partial w} > 0, \frac{\partial \Theta_{X1}}{\partial w} > 0, \frac{\partial \Theta_1}{\partial w} < 0 \) and \( \frac{\partial \Theta_1}{\partial \delta_1} < 0 \), we have \( \Lambda_1 < 0, \Lambda_2 > 0 \) and thus \( \frac{dw}{d\delta_1} < 0 \). Moreover, the effects of \( \delta_1 \) on the following cutoff productivities are
easily obtained:

\[
\frac{d\Theta_D}{d\delta_1} = \frac{d\Theta_D}{dw} \frac{dw}{d\delta_1} < 0, \quad \frac{d\Theta_{X1}}{d\delta_1} = \frac{d\Theta_{X1}}{dw} \frac{dw}{d\delta_1} < 0,
\]

\[
\frac{d\Theta_{X2}}{d\delta_1} = \frac{d\Theta_{X2}}{dw} \frac{dw}{d\delta_1} < 0, \quad \frac{d\Theta_2}{d\delta_1} = \frac{d\Theta_2}{dw} \frac{dw}{d\delta_1} > 0.
\]

We further prove \( \frac{d\Theta_1}{d\delta_1} < 0 \) by contradiction. Suppose \( \frac{d\Theta_1}{d\delta_1} \geq 0 \). From the above effects of \( \delta_1 \) on \( \Theta_D, \Theta_{Xi}, \Theta_2, \) and \( w \), we know that an increase in \( \delta_1 \) will always increase the domestic labor demand for skilled workers [i.e., the RHS of (7)]. Thus, the labor market equilibrium condition (7) can never hold after the education level changes. This is the contradiction. □