A Demographic Model of Economic Reform$^1$

Y. Stephen Chiu$^2$

University of Hong Kong

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$^2$School of Economics and Finance, Faculty of Business and Economics, University of Hong Kong, Pokfulam Road, Hong Kong. email: stephen.chiu@hku.hk
Abstract

This paper studies a political economy model of reform in which population structure and dynamic play a central role. It is motivated by a difference in the reform approaches that China (gradualism) and Russia (big bang) adopted. We show that in a country like China, its young population structure, together with the one-child policy (this being more subtle), allows the public to be more patient to reform setbacks and hence gradualism involves less risks and is chosen; in a country like Russia, the older population is not as patient and big bang is chosen to ensure irreversibility.
"The great events of history are often due to secular changes in the growth of population and other fundamental economic causes, which, escaping by their gradual character the notice of contemporary observers, are attributed to the follies of statesmen or the fanaticism of atheists." (J. M. Keynes, 1922, p.12)

“[O]nly a crisis—actual or perceived—produces real change. When that crisis occurs, the actions that are taken depend on the ideas that are lying around.” (M. Friedman, in Friedman and Friedman, 1982, preface p. ix) 

1 Introduction

This paper presents a political economy model of reform in which a country’s population structure and dynamic play a central role. It is motivated by a difference in the reform strategies chosen by China and Russia at the beginning of their reforms. China started its reform in 1978 with a gradual approach that allows steps to be reversed and experimentation. Russia, on the other hand, started its reform with a big bang approach in early 1990’s. While the gradual reform approach is generally viewed as one main feature of the Chinese reform, the lesson is of limited value if we are uncertain about the circumstances under which the approach is applicable.

There are two reasons why we think the role of demography deserves close examination. First, there were indeed stark differences between the two countries in terms of their demography. China’s population was much younger than its Russian counterpart at the beginning of their reforms. Despite a small 1.1 years difference in 1950 (23.9 for China versus 25.0 for Russia), the difference in their median ages was widened to 9.2 years in 1980 (22.1 for China versus 31.2 for Russia). If we take 1980 and 1990 as the comparison years for China and Russia, respectively, then the difference was further widened to 11.2 years (Russia’s median age was 33.2 in 1990). A difference in age structure
has implications in the political economy of reform, when reform has different impacts on different generations of the population.

The second reason is that gradualism in China would be difficult to explain without taking its demography into account, and the same gradualism does not seem to have been possible in Russia because of its demography. It is useful to point to two important early works on China’s reform. Qian and Xu (1993) argue that the distinguishing characteristic of Chinese gradual reform is its "sustained entry and expansion of the non-state sector," going on analyzing institutional details unique to China that enabled such phenomenon to occur. At the same time, Naughton (1995, 2007) examined the "growing-out-of-plan" reform strategy in the state sector: "[T]hroughout the 15 years of economic reform, between 1978 and about 1993, although the state sector had shrunk in relative importance, it had continued to grow in absolute terms, both in output and in employment (my emphasis)" (Naughton, 2007). This strategy allowed state enterprises to adopt gradually to market competition and operate as for-profit firms, not to mention to provide political stability.

Prescient as each analysis is, the concurrence of both observed features of the Chinese reform hinges on a peculiar pre-condition of its demography. China’s population is not expected to peak until around 2030—five decades after the beginning of reform and despite its stringent birth control policy—and by that time its size will be more than 40 percent larger than its 1980 population number. Thus, it is possible to have gradual expansion of the private sector without a contraction of its public sector, at least in the initial period, say the first one or two decades, of its reform. For a country like Russia, however, giving its shrinking population (its population peaked in 1993), the

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1 "What makes China’s reforms differ from those of Eastern Europe and the Soviet Union is the sustained entry and expansion of the non-state sector......Our analysis have demonstrated that the success of China’s particular gradual strategies depends on its initial institutional conditions (as well as other micro-and macro-economic environment which are not discussed here)..."—Qian and Xu (1993; p.1 and p.44)

2 The population size in 1980 was 977.8M (World Population Prospects: The 2015 Revision. United Nation, Department of Economic and Social Affairs, Population Division (2015)). According to the United Nations, the population size of China will peak in 2030 with the number of 1,416M (Population Estimates and Projections, World Bank Group, 01-July-2015).
same would not be feasible. If the employment in the state sector were to be maintained, there would not be spare labor supply for the non-state sector to expand; to ensure sustained expansion of the latter sector, drastic privatization and downsizing of the state sector would be inevitable.

In this paper, we develop a theory to show how certain initial population characteristics — young population structure and reduced fertility rate — bestow a government with a large leeway in maneuvering so that the generic economic reform is less likely to be reversed under popular resistance. The argument is twofold. The first point is related to population structure. Despite the long run benefits, even Pareto-improving reform may still be painful in the short run. While young people will live long enough to benefit from reform and are more likely to support it, it may not be the case for older people.

The second, more subtle point is the role played by the number of children to be had in a family. Notice that, despite the aforementioned relative tolerance, even the young do not necessarily support reform when the hardship is too extreme. We argue that, in this case, the fewer children that young adults have or are expected to have may sway them into reform supporters. We provide two specifications of the child rearing costs that give such a result. In the first specification, time cost is involved, and the higher productivity under reform suggests that it is more costly to have children. The second specification assumes that consumption given to children is increasing in their parent’s consumption. During a person’s adulthood, the child-raising period occurs in his or her early stage and hence the burden of child raising is front loaded; the person’s income profile (after netting reform costs), on the contrary, is more likely to be back loaded and is more so under reform than under no-reform due to the pain-first-gain-later nature of reform.\footnote{Under the reform scenario, the young people will undergo hardship in the short run but gain in the long run; under the no-reform scenario, they will not undergo hardship in the short run and will not gain the long run either. This suggests, in the case of indifference between the two options, the income profile must be more back loaded under reform than under no-reform.} Hence, inter-temporal consumption smoothing is more difficult to achieve under reform, and the extra hardship due to an increase in the number of children is felt more severely
under reform than under no-reform when intertemporal substitution is inelastic. It follows that the smaller the number of children, the more tolerable the person is to reform hardship.

These two points suggest that, China’s one-child policy, which hastened the slowing down of the country’s fertility rate, might have played a secret role in providing public support to its reform. More generally, they suggest that a country like China, which began with a young population structure and a low birth rate—both being favorable demographic factors—expects to face great tolerance from the public towards reform hardship. For a country like Russia, although its low birth rate is favorable towards reform, its old population structure is not. As a whole, its demographic factors are not as favorable, and the government will face less tolerance from the public towards the reform.

Based on our results in the baseline model with a generic reform, we next embed the insight into a model with two reforms where implementation of reform one may provide useful information for the implementation of reform two; this is to capture the gains from experimentation and learning emphasized in Qian and Xu (1993) and Qian, Roland, and Xu (2006). There is a trade-off here: gradualism is efficient but with lower reversal costs, while big bang is less efficient but with larger reversal costs. It is possible that, while unfavorable demographic factors will lead to a reversal under gradualism, they will still allow big bang to go through due to its higher reversal costs; in this case, big bang serves as an outside option when gradualism is not viable. On the other hand, gradualism will be chosen when it is viable under more favorable demographic factors. We thus are able to reconcile both choices can be optimal strategies.

Some remarks are in order here. First, the view that big bang reform is more difficult to reverse than gradual reform is not new to the literature.

Our emphasis of big bang reform as a commitment device is similar to the elite’s extension of the franchise to the poor so as to commit to future re-distribution policy in Acemoglu and Robinson (2000), while the importance of commitment over discretion in policy making is first famously made by Kyland and Prescott (1976). We are not the first to make such assumption.
the policy circle. As pointed out in a closely related paper by Dewatripont and Roland (1995, p.1208), "[a] big-bang strategy involves high reversal costs, which are often considered to be an advantage ex post since it reduces the reversibility of enacted reforms, which is a constant concern for reformers." Second, despite our assumption, we do not make a general claim that gradualism is more efficient than big bang in all contexts. We entertain the possibility that each of the two reforms in our model is indeed a bundle of several complementary components that ought to be implemented simultaneously for the sake of efficiency. Our point is only that, given the difficulty of moving from planned economy to market economy, the ideal reform package must have some gradualist features, and we call such an ideal reform package gradualism in our model.

Second, the emphasis of higher reversal cost under big bang also answers a question that the reader may ask: if big bang is harsher than gradual reform—and it appears to be the case—why our theory suggests that the public would prefer a greater reform hardship (under big bang) to a smaller reform hardship (under gradualism). There is no contradiction at all. Despite a greater reform hardship under big bang, the reversal cost is also greater. If the increase in the reversal cost far exceeds the increase in the reform hardship when moving from gradualism to big bang, the public may well accept the big bang but not the gradual reform.

Third, the literature has it that gradualism—in the form of divide-and-conquer strategy—has the merit of breaking or softening political resistance which thwarts big bang reform (Dewatripont and Roland 1992, 1993; Wei 1997). Our emphasis, on the contrast, is that political constraints dictate that the reform that should be ideally implemented sequentially may be feasible only when it is implemented in one go. Rather than being conflicting, what is common between these two views is that political constraints may lead to a compromise in the pacing and sequencing of reform packages. It is interesting to develop in future research a single model that allows for both hurrying up (my emphasized here) and slowing down (emphasized elsewhere) of reforms as a response to binding political constraints.
Fourth, a premise of our theory is that the young are more forward looking, comparing different policy outcomes when deciding their positions. Hence, when the government shows no interest at all in implementing reform, a young population structure is more impatient and more susceptible for revolts than an old population structure is. As a young population structure is compatible to both being too patient and being too impatient, the flexibility of the theory also poses challenges in its empirical testing.

The rest of the paper is organized as follows. Section 2 presents the baseline model with one reform. Section 3 solves the model, focusing the young workers’ support of reform and how the support varies with the number of children each young worker has or is expected to have. We examine the issue under two different public decision rules (simple majority and unanimity with transfers) and two different child rearing costs assumptions (material costs and time costs). Section 4 studies a more elaborate model with two reforms, which can either be implemented at the same time as a big bang or implemented sequentially in a gradual reform. We study the choice between big bang and gradualism, and the role that demographic factors play. Section 5 discusses some key assumptions and reviews the related literature. Section 6 concludes.

2 Baseline Model

We first provide a description of the model.

2.1 Description

We consider a discrete-time infinite horizon model. Each agent lives for four stages (periods) as stage-0 child, stage-1 young worker, stage-2 middle-aged worker, and stage-3 retiree. A stage-0 child depends on her parents for consumption and makes no decisions (we always use female pronouns to refer to agents in this paper). A young worker works and bears children and raises them until they become young workers in the next period. A middle-aged worker works, but does not need to bear or raise children, because her chil-
dren have already grown up. Parents no longer support children once the latter have grown up, nor do grown-up children support their parents. A retiree does not work and lives on her previous savings. Agents from an earlier stage move on to the next stage with certainty except that retirees will die at the end of their retiree stage. Finally, while agents can always save their income, at a fixed interest rate of \( r \), we assume they are not allowed to borrow. (Allowing them to borrow at a fixed interest rate exceeding \( r \) will not alter the qualitative nature of our results.)

The lifetime utility of a young worker at time \( t \) is

\[
U_t = A(n_t) + u(c_{1,t}) + \delta u(c_{2,t+1}) + \delta^2 u(c_{3,t+2}),
\]

(1)

where \( c_{i,t} \) is her stage-\( i \) consumption at time \( t \) and \( n_t \) is the number of children the young worker has; for simplicity, we assume that \( n_t \) is exogenously given (even though it may not be the same from period to period). The stage utility function in each stage \( i \) takes the form of constant relative risk aversion (CRRA), i.e.,

\[
u(c) = c^{1-\rho} / (1 - \rho),\]

where \( \rho > 0 \) and is not equal to unity (when \( \rho = 1 \), the stage utility function is replaced by \( u(c) = \ln(c) \)). Notice that \( 1/\rho \) is the constant elasticity of intertemporal substitution.

We assume the following child rearing cost function in most part of the paper (an alternative child rearing cost function is introduced in subsection 3.4 to check robustness).

**C1** A young-worker with \( n \) children incurs the following two child rearing costs: a fixed consumption cost of \( T(n_t) \) and a fraction of the parent’s time \( \kappa(n_t) \in (0,1) \) for child rearing that could have been spent on working, where both \( T(n_t) \) and \( \kappa(n_t) \) are differentiable and increasing in \( n \).

In this formulation, although the young parent does not derive utility from her children being fed more (as assumed in a strand of literature on population...
and economic growth, see, e.g., Galor and Weil (1996)), she has to give them a subsistent level of consumption that depends on the number of children but not the parent’s income. Moreover, because of time spent on children, the fraction of the young parent’s time available for working is only $1 - \kappa(n_t)$ now.\footnote{See, for instance, Becker and Barro (1988) and Galor and Weil (1996) on modeling cost of having children as time cost.}

### 2.2 Reform

Only labor is required for production. At the status quo, labor productivity is inefficient and low. A reform, which takes one period to complete, increases the output per worker to $y$; during the reform period, however, each worker incurs a reform cost $k$ and hence the payoff per worker is $y - k$ only. The cost may be the effort required to learn a new skill, hardship coming from adoption to a new environment, etc. Here we highlight the investment nature of reform (in the spirit of Krusell and Rios-Rull 1996); while benefiting in the long run, it is costly in the short run. While we model reform as a single act here, we will consider more complicated reform strategies in a later section. We assume that neither children nor retirees are directly affected in the reform period.\footnote{If retirees are indeed affected, they are likely to be affected negatively. Thus a population with a larger fraction of retirees makes the reform more likely to be reversed and our argument will hold more easily.}

The reform needs a certain amount of popular support to complete: in the absence of such support, it will be reversed with an output per worker of $x < y$ and will stay the same thereafter unless alternative decisions are made. We follow the interpretation in Dewatripont and Roland (1995) that a reversal means a return to a more conservative path of development, not necessarily a return to the status quo. We think that public support to reform is important even for an autocracy like China. Although an autocracy may have more muscle than a democracy, there are still limitations to its power — it cannot force the public to make wise, risky business decisions or to be innovative, etc.\footnote{This is supported by the emphasis of notions such as Pareto improvement reform or}"
reversal decision is made (the rules are used to make operational the idea of necessity of public support and should not be interpreted literally).

SM Reform reversal is decided by young and middle-aged workers through simple majority voting without transfers.

UT Reform reversal is decided by young and middle-aged workers through unanimity rule with transfers.

For either case, we assume that, since unaffected by reform, the retirees do not participate in the reversal decision making.

Figure 1 is a summary of the sequence of events at period $t$ when a reform proposal is contemplated. At time $t:1$, the children in the last period turn young workers, the young workers in the last period turn middle-aged workers, and the middle-aged workers in the last period turn retirees. At time $t:2$, the young workers each give birth to $n_t$ children, where $n_t$ is exogenously given, perfectly foreseen. At time $t:3$, reform starts. At time $t:4$, agents "vote" on continuing or reversing the reform. At time $t:5$, given the reversal decision, workers receive their income, consume, feed their children (if they are young workers), and save; retirees get back their savings, consume, and finally die. Then the surviving agents move on to the next period $t + 1$. (We defer the discussion to the next section on what will happen in the next period if reform is reversed now.)

3 Reform reversal decision

In this section, we examine the public's attitude to reform at time $t:4$. We first start with a representative young worker.

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reform-without-losers motivated by the Chinese experience (see Lau, Qian, and Roland 2000 and Qian, Roland, and Xu 2006).

*Given that the fertility rate is exogenous given, the order of voting and giving births can be switched without affecting the result.
Children and workers in the last period turn one stage older. Young workers give birth to children. Reform starts. Reform reversal decision. Workers work, receive income, all agents consume, then retirees die.

Figure 1: Event schedule in period $t$, when a reform proposal is attempted.
3.1 Young workers’ support of reform

3.1.1 Utility maximization

For regime $j = R$ (reform), $D$ (reversal, or defeat of reform), denote by $z_1^j$ and $z_2^j$ the stage-1 and stage-2 outputs (gross incomes) of a representative young worker in time $t$ when working full time; denote by $c_1^j, c_2^j, c_3^j$ her consumptions in the three stages, and denote by $s_1^j$ and $s_2^j$ her savings in stage-1 and stage-2, respectively. In order to incorporate both $SM$ and $UT$, we introduce a notion of reform hardship $h$, which equals $k$, the reform cost, under the case of $SM$ and equals $k + \Delta$ under $UT$ where $\Delta$ is the young worker’s transfer out.\(^9\)

The young worker’s intertemporal budget subject is given as follows (i) $c_1^j + s_1^j = (1 - \kappa (n)) z_1^j - T (n) - I_j h$, where $I_R = 1$ and $I_D = 0$; (ii) $c_2^j + s_2^j = z_2^j + (1 + r) s_1^j$, and (iii) $c_3^j = (1 + r) s_2^j$. Her utility maximization problem is

$$\max u (c_1^j) + \delta u (c_2^j) + \delta^2 u (c_3^j)$$

by choosing $c_1^j, c_2^j,$ and $c_3^j$ subject to the aforementioned intertemporal budget constraints, as well as the non-negativities of $c_1^j, c_2^j, c_3^j, s_1^j,$ and $s_2^j$.

It is easy to verify that consumption smoothing is feasible if $z_1$ is sufficiently large relative to $z_2$. In this case, the optimal solution to the problem satisfies the following two FOCs

$$u' (c_1^j) = \delta (1 + r) u' (c_2^j), \quad (2)$$

and

$$u' (c_2^j) = \delta (1 + r) u' (c_3^j). \quad (3)$$

And

$$c_1^j = \frac{1}{F} \left[ ((1 - \kappa (n)) z_1^j - T (n) - I_j h) + \frac{z_2^j}{1 + r} \right], \quad (4)$$

\(^9\)The reader may simply construe $h$ to be $k$ if he or she finds it easier to focus exclusively on the case of $SM$, which is also the focus of the paper.
where
\[ F \equiv 1 + \frac{(\delta (1 + r))^{\frac{1}{\beta}}}{1 + r} + \frac{(\delta^2 (1 + r)^2)^{\frac{1}{\beta}}}{(1 + r)^2}. \] (5)

When consumption smoothing between stage 1 and later stages is infeasible, the optimal solution is modified as follows: (i) the equality sign in (2) is replaced by a strictly-greater-than sign; (ii) consumption smoothing between stage 2 and stage 3 is still feasible, i.e., (3) continues to hold true; and (iii) (4) is replaced by
\[ c_1 = (1 - \kappa (n)) z_1^T - I_j h, \]
which is the disposable income in that period is wholly spent in the period.

### 3.1.2 Greatest endurable hardship

Now that we have solved the young worker’s utility maximization problem, we are ready to study her attitude towards reform reversal. If reform goes through, her current-period and next-period gross incomes are \( z_1 = y \) and \( z_2 = y \); if the reform is reversed, her current-period income is \( x \) and her next-period income, denoted by \( B(h) \), also known as the post-reversal income, depends on what will happen in the next period (in particular whether or not a reform will be attempted) and should be endogenized in a fully dynamic game, which we will discussed further at the end of subsection 3.2, as well as in Appendix B. Here, we simply posit it by a function with the following properties.

**B** (i) \( 0 \leq B(h) \leq y \); and (ii) \(-1 \leq B'(h) \leq 0 \).

Point (i) states that \( B(h) \) cannot exceed what a worker will earn subsequent to the completion of the reform. Point (ii) states that \( B \) does not decrease in \( h \) as quickly as does the current income under reform. This formulation accommodates three interesting cases:

- \( B(h) = y - h \) : the reform with the same hardship is attempted and implemented successfully next period;
- \( B(h) = x \) : the reversal outcome remains unchanged in the next period;
• \( B(h) = y - L \), where \( L \) is fixed and larger than any conceivable \( h \); it captures a more disruptive change

We use \( U_R(h, n) \) to denote the young worker’s resulting indirect utility under reform and \( U_D(h, n) \) her indirect utility function under reversal of the reform, where \( n \) is the number of children she has. We first show there exists a unique greatest endureable hardship (GEH) \( h^* \) such that, at time \( t \), the young worker prefers having reform now to having it reversed if and only if \( h \leq h^* \). (Proofs to Lemmas and Propositions are relegated to the appendix unless otherwise stated.)

**Lemma 1** Consider time \( t \) when reform is attempted. Given assumption B, there exists \( h^* > 0 \) such that for all \( h < h^* \), \( U_R(h, n) > U_D(h, n) \); for all \( h > h^* \), \( U_R < U_D \).

The result is easy to understand. We first notice that both \( U_R \) and \( U_D \) are decreasing in \( h \). However, \( U_R \) decreases at a greater rate than \( U_D \) does because \( h \) enters into the agent’s budget constraints in the first stage under reform and in the second stage under reversal and in the latter case it affects the second stage income to a lessened extent (part ii of assumption B).\(^{10} \) Hence, if \( U_R \) and \( U_D \) are ever equal at some \( h \), this \( h \) must be unique (denoted by \( h^* \)) and \( U_R \geq U_D \) if and only if \( h \leq h^* \).

We now study how \( h^* \) varies with the exogenous \( n \). Totally differentiating \( U_D(h, n) = U_R(h, n) \) and rearranging terms, we obtain

\[
\frac{dh^*}{dn} = \left( \frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} \right) \bigg/ \left( \frac{\partial U_R}{\partial h} - \frac{\partial U_D}{\partial h} \right).
\]

Since the denominator of the RHS term of (6) is negative, the sign of \( \frac{dh^*}{dn} \) is opposite to the sign of the numerator of the RHS of the equation. To determine the latter, we make use of the following Lemma.

\(^{10} \)h appears in the second period income under reversal while in the first period income under reform. Due to discounting and the opportunity of savings, an equal change of \( h \) has smaller impacts on the indirect utility under reversal.

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Lemma 2 Under assumption C1, for regime \( j = R \) (reform), \( D \) (reversal),
\[
\frac{\partial U_j}{\partial n} = A'(n) - u'(c_1^R) (\kappa'(n) z_1^R + T'(n))
\]
where \( z_1^R = y \) and \( z_1^D = x \), whether or not consumption smoothing is feasible between stage 1 and later stages.

(7) is interpreted as follows. Consider the case where consumption smoothing is infeasible under regime \( j \). Whereas the first term in the RHS of (7) is the direct effect of having one more child, the second term is its indirect effect due to a lowering of consumption. (7) can be easily shown when consumption smoothing between stage 1 and later stages is infeasible. However, Lemma 2 states that (7) holds true even if consumption smoothing is feasible. The intuition is that in the latter case, equalization of discounted marginal utility across periods will ensure that the change in the life-time utility can be represented in terms of the change in the first period’s utility and hence (7) remains true. Notice that

\[
\frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} = \begin{aligned}
u' (c_1^R) (\kappa'(n) y + T'(n)) - u'(c_1^D) (\kappa'(n) x + T'(n)) \\
\kappa'(n) (u'(c_1^R) y - u'(c_1^D) x) + T'(n) (u'(c_1^R) - u'(c_1^D))
\end{aligned} > 0
\]

\[
\begin{aligned}
&= \kappa'(n) u'(c_1^R) (y - x) + \kappa'(n) \left( (u'(c_1^R) - u'(c_1^D)) x \right) \\
&\geq 0 + T'(n) \left( u'(c_1^R) - u'(c_1^D) \right) \geq 0
\end{aligned}
\]

The first term in the RHS of (8) is strictly positive because \( y > x \), while the other two terms are non-negative because \( c_1^R \leq c_1^D \), which is due the fact that, given the equality of \( U_R \) and \( U_D \), the net income is more backloaded under reform that under reversal. Hence, although both \( \frac{\partial U_D}{\partial n} \) and \( \frac{\partial U_R}{\partial n} \) are negative, the latter is larger in magnitude. There are two forces working in the same direction. First, due to the time cost, the foregone income of having an additional child is higher under reform than under reversal. We have assumed
that the reform hardship \( h \) applies to every young worker, regardless of the fraction of time spent on working. One interpretation is that it arises from learning a new skill, and a certain degree of proficiency is needed whether the worker wants to work full time or part time. Second, the fixed consumption cost \( T(n) \) also works in the same direction as the time cost. The reason is that a fixed consumption brings to a greater marginal disutility under reform than under reversal since the young worker’s consumption is at least weakly lower under the reform than under reversal.

**Proposition 1** Suppose assumptions C1 and B holds. A young worker’s greatest endurable hardship \( h^* \) is decreasing in her children number \( n_t \), regardless of \( \rho \).

Since in the whole argument we have not made up of the parameter value of \( \rho \), our result does not depend on its exact value.

**Proof.** Omitted. ■

### 3.2 Majority voting without transfers

Now that we have discussed the young workers’ attitude towards reversal of reform given hardship \( h \), we next study the aggregate reversal decision, assuming the use of simple majority rule without transfers (assumption SM). Given no transfers, \( h \) is indeed equal to \( k \). Since the retirees are not affected by reform, we assume they do not participate at all. We use \( \alpha \geq 1 \) to denote the weight given to a vote by the middle-aged relative to a vote by the young. For each middle-aged worker, because her period income is higher under reversal (equal to \( x \)) than under reform (equal \( y - k < x \)), she will vote for reversal. For each middle-aged worker, there are \( n_{t-1} \) young workers. If \( n_{t-1} < \alpha \), the middle-aged workers are numerous enough to reverse reform. Thus, one necessary demographic condition for reform to go through unblocked is that \( n_{t-1} \geq \alpha \), i.e., the young-workers-to-middle-aged-workers ratio must exceed a critical number. Provided that this condition is satisfied, the reform will gain enough support if and only if \( k \leq h^* \), which was studied in Proposition 1.
The bottom line is that the reform will gain enough support if the population structure is young and the current fertility rate \((n_t)\) is low enough. Note that the former factor is of primary importance and only when it is satisfied that the latter factor can play a pivotal role; i.e., the importance of the two factors is lexicographic. The following proposition summarizes our discussion.

**Proposition 2** Suppose assumptions C1, B, and SM hold. Then (i) hardship \(h = k\); (ii) the reform is not reversed only if the young-workers-to-middle-aged-workers ratio is high enough \((n_{t-1} > \alpha)\), and (iii) provided that this necessary condition is satisfied, then for each fertility rate \(n_t\), there exists a cut value \(h^* (n_t)\) such that:

1. the reform is not reversed if \(k < h^* (n_t)\) and is reversed if \(k > h^* (n_t)\);
2. \(h^* (n_t)\) is strictly decreasing in \(n_t\).

**Proof.** Omitted. ■

Note that we characterized the GEH assuming a post-delay payoff function \(B(.)\). If we allow a reversed reform to be attempted again, we also need to endogenize this \(B\) function. In the fully dynamic game, there are usually multiple equilibria, each associated with a greatest endurable hardship that is consistent with the equilibrium. Among all these equilibrium greatest endurable hardships \((EGEH)\), there is a maximum one, which we call the maximum equilibrium greatest hardship \((MEGEH)\).\(^{11}\) This feature of self-fulfilling prophecy is common to dynamic games, not unique to our own model. The interested reader is referred to Appendix B for an illustration and a discussion on why the main message of Proposition 2 still holds true despite the multiple equilibria.

### 3.3 Unanimity with transfers

Here we assume the aggregate reversal decision is made by rule of unanimity with transfers \((UT)\). A representative middle-aged worker suffers an income

\(^{11}\)There is a corresponding notion of minimum \(EGEH\), which we will not explore.
loss of $x - (y - k)$ under reform relative to under reversal, this amount must be compensated in order for them to support reform. To this end, the transfer by each young worker is

$$\Delta \equiv \frac{(x - y + k)}{n_{t-1}}. \tag{9}$$

The total hardship each young worker endures under reform is thus equal to $h = k + \Delta$ (her net income in the period being equal to $y - h$). As $\Delta$ is decreasing in $n_{t-1}$, so is $h$. Although the greatest endurable hardship $h^*$, as studied in subsection 3.1, is independent of $n_{t-1}$, the actual hardship is not. This leads to a cut value of reform cost $k^*$() that depends on both $n_t$ and $n_{t-1}$ so that reform is reversed if and only if $k > k^* (n_t, n_{t-1})$.

**Proposition 3** Suppose assumptions C1, B, and D2 hold. Then (i) hardship $h = k + \Delta$ where $\Delta$ is given by (9); (ii) there exists a cut value $k^* (n_t, n_{t-1})$ such that:

1. reform is not reversed if $k < k^* (n_t, n_{t-1})$ and is reversed if $k > k^* (n_t, n_{t-1})$;
2. $k^* (n_t, n_{t-1})$ is strictly decreasing in $n_{t-1}$; and
3. $k^* (n_t, n_{t-1})$ is strictly decreasing in $n_t$.

To determine whether the reform will be reversed, we only need to check the actual reform cost against the cut value $k^* (n_t, n_{t-1})$, without any consideration of the role of the middle-aged workers. This is not the same as under SM, where the role of the middle-aged workers is stark: if $n_{t-1}$ is less than $\alpha$, reform must be reversed even though the young workers find the reform cost to be below their greatest endurable hardship.

Despite these differences, the results here are consistent with our early findings under SM that, namely: (i) the higher the young-workers-to-middle-aged-workers ratio, the more favorable it is to reform and (ii) the lower the current fertility rate, the higher is young workers’ greatest endurable hardship.
3.4 An alternative formulation of child rearing cost

To check the robustness of Proposition 1, we provide an alternative formulation of the costs of having children.

C2 The consumption given by a young worker at time $t$ to her children is equal to $\gamma(n_t) c_{1,t}$ where $c_{1,t}$ is the young worker’s consumption in that period, $n_t$ is her children number, and $\gamma(n_t)$ is differentiable and strictly increasing in $n_t$.

In this formulation, no time cost is incurred to raise children. Just like in C1, the young parent does not derive utility from her children being fed more. However, she feels obliged to feed them an amount commensurate with her own consumption. Alternatively, social norm or good provisions are such that children’s consumption level is increasing in their parents’ consumption level. (For instance, when parents go to a better restaurant, the food available for children is more expensive.) We first obtain the following lemma which is counterpart to Lemma 2 under the assumption of C1.

**Lemma 3** Under assumption C2, For regime $j = R$ (reform), $D$ (reversal),

$$
\frac{\partial U_j}{\partial n} = A'(n) - \frac{\gamma'(n)}{1 + \gamma(n)} c_1^2 u'(c_1),
$$

whether or not consumption smoothing is feasible between stage 1 and later stages.

(10) is interpreted as follows. Consider the case where consumption smoothing is infeasible under regime $j$. Whereas the first term in the RHS of (10) is the direct effect of having one more child, the second term is its indirect effect due to a lowering of consumption. Lemma 3 states that (10) holds true even if consumption smoothing is feasible.
Making use of Lemma 3, as well as (6) which continues to hold under $C2$, we obtain

$$
\text{sign} \frac{dh^*}{dn} = -\text{sign} \left( \frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} \right) = +\text{sign} \left( c^D_1 u' (c^R_1) - c^R_1 u' (c^D_1) \right). \tag{11}
$$

The young worker is more likely to face difficulty in consumption smoothing under reform than under reversal. The equality of $U_R$ and $U_D$ is thus achieved either by (i) $c^R_1 < c^D_1, c^R_2 > c^D_2$ and $c^R_3 > c^D_3$ (consumption smoothing infeasible under reform) or by (ii) $c^R_i = c^D_i$ for $i = 1, 2, 3$ (consumption smoothing feasible under reform). In the latter case, since $c^R_1 = c^D_1$, an additional child reduces the agent’s utility under reform as much as it does under reversal, and $dh^*/dn = 0$. The former case is more intriguing as there are two opposite effects here: as $c^R_1 < c^D_1$ and $u'(c^R_1) > u'(c^D_1)$, it is unclear which effect is more prominent or which one between $c^R_1 u'(c^R_1)$ and $c^D_1 u'(c^D_1)$ is larger without further information. However, for CRRA utility function with $\rho > 1$, we can determine without ambiguity that $c^R_1 u'(c^R_1) > c^D_1 u'(c^D_1)$ and $dh^*/dn < 0$ (the sign will be reversed if $\rho < 1$).\footnote{\cite{12} $c^j_1 u'(c^j_1)$ can be re-written as $(1 - \rho) u(c^j_1)$. For $\rho > 1$, $u(c^j_1)$ is negative. When $c^j_1$ is bigger, $u(c^j_1)$ is less negative and $(1 - \rho) u(c^j_1)$ is smaller.} Recall that the CRRA utility function is also a constant intertemporal elasticity substitution utility function. The result $dh^*/dn < 0$ thus arises when the intertemporal substitution is inelastic ($\rho > 1$).

**Proposition 4** Suppose reform is attempted at time $t$. Given assumptions $C2$ and $B$, there exists a critical $n^*$ such that (i) for $n_t < n^*$, $h^*$ is independent of $n_t$; and (ii) for $n_t \geq n^*$, $h^*$ decreases with $n_t$ if $\rho > 1$ (and increases with $n_t$ if $\rho < 1$).

It is worthwhile noticing that the quantitative macro literature suggests by and large that $\rho$ is likely to exceed unity. Because of this, from now on we are to
maintain the assumption that $\rho > 1$.\footnote{The literature starts with Hall (1978) which shows little intertemporal substitution. There are noticeably recent work that argue for the opposite though (see the analysis and also the updated literature review in Gruber (2013)).} Note that $n^*$ is the number of children such that the young workers’ consumption smoothing condition under reform just becomes non-binding given that $U_R = U_D$. A natural question is how large $n^*$ is or whether it is binding. If we take the view that in practice young parents do have difficulty in smoothing consumption, then we can conclude that $n$ exceeding $n^*$ is indeed the relevant range to focus on. Given these observations, we conclude that, under our assumptions, a young worker’s GEH is negatively related to the number of children she has.

Under $C2$, the actual consumption given to children is smaller when the parent consumes less. Since the interesting case is where the parent consumes less under reform than under reversal (given the indifference condition of $U_D = U_R$), the burden of having more children on the parent is mitigated. There is, however, no such a mitigating effect under cost function $C1$. There are two reasons for it. First, the amount of good given to children $T(n_t)$ is not reduced when the parent’s disposable income is reduced under reform. Second, the forgone income due to children rearing is higher under reform (earning $y$) than under reversal (earning $x < y$). As a result, the negative effect of $n_t$ on $h^*$ is more prominent under specification $C1$.

Finally, recall that Proposition 4, just like Proposition 1, is obtained independent of the voting rule to be used, given the post-reversal function $B$. The reason is that it simply examines the representative young worker’s own attitude towards reform, but not the aggregate reform reversal decision outcome.

### 3.5 Summary

In the study of the reversal decision in this section, we attempted two specifications of child rearing costs ($C1$ and $C2$ together with $\rho > 1$) and two different voting rules ($SM$ and $UT$). The common, general insight is that reform is less likely to be reversed when the population structure is young (in the sense of young-workers–to-middle-aged-workers ratio) and when the current fertility
rate is low. Therefore, for a country like China endowed with both favorable conditions, its reform enjoys great tolerance from the public; for a country like Russia with one unfavorable condition—old population structure, reform less auspicious. And if this unfavorable condition is strong enough, it may even derail the reform. Foreseeing this, the country may re-package the reform to avoid its reversal, or to do something to make reversal more difficult. We will explore this idea in the next section.

4 Big bang versus gradualism

Based on our early results, we now present a model in which the government chooses between two reform packages: big bang ($BB$) and gradualism ($GR$). Consider the same 4-stage OLG model with the following modifications. There are two reforms, $d_1 \in \{0, 1\}$ and $d_2 \in \{0, a, A\}$. They can be implemented either at the same time as a big bang ($BB$) or with $d_1$ preceding $d_2$ as a gradual reform ($GR$); both reforms are completed in the same period when $BB$ or $GR$ is attempted. For $d_1$, the choice is either no reform ($d_1 = 0$) or reform ($d_1 = 1$); and for $d_2$, 0 means deferring the decision to a later time, while $a$ and $A$ represent two different choices of reform whose optimalities depend on the state of the world which is unknown at the time the choice between $BB$ and $GR$ is made. We assume, without additional information, $a$ is a better choice, and $BB$ means a choice of $(d_1, d_2) = (1, a)$. In this case, the output to each worker will be $y$ in that period and thereafter.

On the other hand, $GR$ means a choice of $(d_1, d_2) = (1, 0)$. Subsequent to the choice of $d_1 = 1$, a signal $s \in \{0, 1\}$ is revealed where $s = 1$ occurs with probability $\pi$. If $s = 0$, $d_2 = a$ is the optimal choice; if $s = 1$, $d_2 = A$ is the optimal choice. In the former case, the output to each worker will be $y$ in that period and thereafter. In the latter case, the output per worker will be

Qian and Xu (1993), and Qian, Roland, and Xu (2006) study how a difference in institutions that affect the ability to learn from experiments will affect the choice between big bang and gradualism. Here, we do not focus on the effects of a variation in the ability to learn (which is captured by $\pi$), instead we focus on the role of demography given the learning ability.
Figure 2: Event schedule at time $t$; big bang ($BB$) and gradualism ($GR$)

Let’s examine the time line in Figure 2 more closely. At time $t.3$, the government chooses between $BB$ and $GR$. At time $t.4$, a signal $s$ is revealed. At time $t.5$, the public decides whether to reverse the reform package, where a reversal, as in the benchmark model, may mean a return to the status quo or simply taking a more conservative approach. We assume that, in case of

$y + \Lambda$ in that period and thereafter. In other words, $GR$ will result in the same payoff as $BB$ will result if $s = 0$ is realized and will result in a higher payoff otherwise. We assume that the same reform cost $k$ is incurred in both $BB$ and $GR$, and both packages are subject to reversals, to be detailed below.$^{15}$

$^{15}$In the case of $GR$, since learning occurs only if $d_1$ precedes $d_2$, we do not study the other $GR$ in which $d_2$ precedes $d_1$. This latter type of $GR$ will be dominated by $GR$ with $d_1$ preceding $d_2$. 

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reversal, each worker produces a period output of $x$ and incurs a one-time reversal cost of $\xi_i$ during the period, where $\xi_{BB} > \xi_{GR}$ due to greater difficulty in reversing two changes under $BB$ than reversing one change under $GR$. The payoff to each worker under the reversal period is thus $x - \xi_i$. For the case of $GR$, if $d_1 = 1$ is not reversed, it will be succeeded by a choice between $d_1 = a$ and $d_1 = A$. We assume that no reversal of reforms can be made after time $t.5$ (in particular, this means the choice of $d_2$ in $t.5$ under $GR$ is irreversible). At time $t.6$, workers produce and receive income according to the decisions made earlier; in particular, both $BB$ and $GR$ are completed if no reversals have been made.

In the literature, the closest to our model is that of Dewatripont and Roland (1995), in which two reforms can be packaged either as big bang or gradualism. There are two major differences. First, in their model the payoffs from reform are of a larger range so as to entertain the possibility that a reform is undesirable under some state of nature and hence there is an option value of reversal; in fact the option value of reversal is a major analytical finding in their work. In our model, we allow a smaller scope of payoffs and any policy reversal reflects a setback of reform and is undesirable. Second and more importantly, there are identical, infinitely lived agents in their model while there are OLG agents in ours and the heterogeneity manifested as intergenerational conflict is the focus of our analysis. The OLG assumption in our model also creates a wedge between the interest of the (current generation) agents and that of the benevolent social planner.

4.1 Analysis

We first study the reversal decisions under $GR$ at time $t.5$. Under policy reversal, a young worker’s current period income is $x - \xi_{GR}$ and the next-period income is denoted by $B(h)$, which we continue to assume to satisfy

\footnote{As there is no payoff uncertainty associated with $d_2$, there are no circumstances where the majority want to implement $d_2$ at $t.5$ but the same or another majority want to reverse it after $t.5$. It is thus without loss of generality to restrict the reversal decision under $GR$ to one time, at $t.5$.}
assumption $A1$. Her income profiles under $GR$ and under its reversal are summarized as follows.

<table>
<thead>
<tr>
<th>Signal Realized</th>
<th>Current-Period Income</th>
<th>Next-Period Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>$GR$ $s = 1$ (with probability $\pi$)</td>
<td>$y + \Lambda - h$</td>
<td>$y$</td>
</tr>
<tr>
<td>$GR$ $s = 0$ (with probability $1 - \pi$)</td>
<td>$y - h$</td>
<td>$y$</td>
</tr>
<tr>
<td>Reversal</td>
<td>$x - \xi_{GR}$</td>
<td>$B(h)$</td>
</tr>
</tbody>
</table>

where, like in our benchmark model, the reform hardship $h = k$ under $SM$ and $h = k + \Delta$ under $UT$. Use $U_R(GR, h, s, n_t)$ and $U_D(GR, h, n_t, \xi_{GR})$ to denote a young worker’s life-time utility from the implementation of $GR$ and from its reversal, respectively, where the utility is calculated at time $t.5$ after $s$ has been revealed. For each $s$, define $GEH h_{GR}^{*}(n_t, s, \xi_{GR})$ such that the agent strictly prefers reversal to reform if and only if $h > h_{GR}^{*}(n_t, s, \xi_{GR})$.

Next, consider time $t.5$, where the reversal decision on $BB$ is made. A young worker’s income profile under the implementation of $BB$ and under its reversal, respectively, is as follows:

<table>
<thead>
<tr>
<th>Current-Period Income (regardless of signal $s$)</th>
<th>Next-Period Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BB$ $y - h$</td>
<td>$y$</td>
</tr>
<tr>
<td>$y - h$</td>
<td>$y$</td>
</tr>
<tr>
<td>Reversal</td>
<td>$x - \xi_{BB}$</td>
</tr>
</tbody>
</table>

Use $U_R(BB, h, s, n_t)$ and $U_D(BB, h, n_t, \xi_{BB})$ to denote her life-time utility

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17 We maintain $A1$ accounts for quite a large range of reasonable possible contingencies (in any case, it is only a sufficient condition). Here are a few possibilities regarding what will happen in the next period that are consistent with $A1$. (i) The reversal is a permanent change so each worker continues to produce $x$, hence, we have $B(h) = x$. (ii) Suppose reform is attempted next period and $SM$ is used. If a big bang reform is implemented next period, then it corresponds to the case where $B(h) = y - h$. (iii) The case of gradualism taking place next period is a bit more complicated. Suppose the signal $s$ revealed this period indeed tells us exactly the state of the world which is invariant over time. Then the realization of $s = 0$ in this period implies that gradualism implemented next period corresponds to the case where $B(h) = y - h$. And $s = 1$ implies $B(h) = y + \Delta - h$, which is less than $y$ when $h$ is equal to the maximum endurable hardship (because $U_R$ and $U_D$ and the income profile under reversal is less backloaded.)
from BB’s implementation and from its reversal, respectively. For each $s$, define a GEH $h^*_BB(n_t, s, \xi_{BB})$ such that the agent strictly prefers reversal to reform if and only if $h > h^*_BB(n_t, s, \xi_{BB})$. We have the following results regarding on those GEHs.

**Lemma 4**

1. $h^*_GR(n, s = 1, \xi_{GR}) > h^*_GR(n, s = 0, \xi_{GR})$;
2. $h^*_BB(n_t, s = 1, \xi) = h^*_BB(n_t, s = 0, \xi)$;
3. $h^*_GR(n, s = 0, \xi) = h^*_BB(n_t, s, \xi)$;
4. $h^*_BB(n_t, s, \xi_{BB})$ is increasing in $\xi_{BB}$ and $h^*_GR(n_t, s, \xi_{GR})$ is increasing in $\xi_{GR}$.

Result 1 states that, under $GR$, the GEH is higher upon $s = 1$ is revealed than upon $s = 0$ is revealed; this is due to the fact that the young workers’ payoffs upon signal of $s = 1$ is higher than upon signal $s = 0$. Result 2 states that the GEH under $BB$ is independent of the revealed signal; this is due to the fact that, by assumption, the signal $s$ has no effects on the workers’ payoffs from $BB$. Result 3 states that the GEH under $GR$ and $s = 0$ is the same as its counterpart under $BB$. The reason is that in both cases $d_2 = a$ is chosen and young workers have the same payoffs. Result 4 states that the higher the reversal costs the higher the GEH; this is so because the higher the reversal costs the less attractive the reversal option is.

**Proposition 5** Assume $B$ and $C1$. We have the following results.

1. $h^*_i(n_t, s, \xi)$ is decreasing in $n_t$, where $i = BB, GR$; and
2. Propositions 2 and 3 continue to hold.

Result 1 states that a young worker’ GEH is decreasing in the number of children she has, provided that $\rho > 1$ and $n_t$ is sufficiently large. This is the same result as Proposition 1 where a generic reform is studied, despite the presence of two different reforms ($d_1$ and $d_2$) here. The point is that there is a single reversal decision and the second reform $d_2$ does not introduce any payoff.
uncertainty under $BB$ once it is chosen, nor under $GR$ upon the realization of $s$, after which the reversal decision is made. Given that proposition 1 continues to hold here, so do propositions 2 and 3, as stated in result 2.

Let’s elaborate this later result in more detailed. Recall that there are two voting rules. Under $SM$, the hardship that a young worker endures is just the reform cost, i.e., $h = k$; the reform package ($BB$ or $GR$) is not reversed only if the population structure is young (in the sense that $n_{t-1} > \alpha$), and provided that this condition is satisfied, the reform package $i$ is not reversed if and only if $k < h_i^* (n_t, s, \xi_i)$, $i = BB, GR$. Relative to the benchmark model, what are new is that the role of state of world reflected by $s$, which affects the efficacy of different choices of $d_2$, and the reversal costs $\xi_i$, as well as the implications, is explicitly stated here. And the roles of $s$ and $\xi_i$ have been given in Lemma 3 and Proposition 5.

Under $UT$, the hardship that a young worker endures is $h = k + \Delta$ where $\Delta$ is her compensation made to middle aged workers. From this relationship, together with the properties of the $GEH$s found out right above, we can derive, for each reform package $i$, a cut value $k_i^* (n_t, n_{t-1}, s, \xi_i)$ such that reform is not reversed if and only if $k < k_i^* (n_t, n_{t-1}, s, \xi_i)$. This cut value $k_i^* (n_t, n_{t-1}, s, \xi_i)$ is increasing in $n_t$ and decreasing in $n_{t-1}$. What is new here relative to the benchmark model is the role of $s$, which affects the efficacy of different choices of $d_2$, and the reversal costs $\xi_i$, as well as the corresponding implications, is explicitly stated here. And the roles of $s$ and $\xi_i$ have been given in Lemma 3 and Proposition 5. The next result is immediate.

**Proposition 6** There are parameter values over which, while $GR$ will be reversed (at least with positive probability), $BB$ will not be reversed.

Proposition 6 states that it is possible that the two reforms are feasible under $BB$ but not under $GR$. The case of $SM$ is easy to understand. If $\xi_{BB}$ happened to equal $\xi_{GR}$, then the $GEH$ under $BB$ will equal the $GEH$ under $GR$ with $s = 0$ (Result 3 of Lemma 3). However, in reality, $\xi_{BB}$ is greater than $\xi_{GR}$, then there is a range of hardship $h \in (h_{GR}^* (n_t, s = 0, \xi_{GR}), h_{BB}^* (n_t, s = 1, \xi_{BB}))$ over which young workers will oppose $GR$ but not $BB$. Provided that young
workers are pivotal in the voting \((n_{t-1} > \alpha)\), then it is indeed the case that, for this range of \(h\), \(GR\) will be reversed while \(BB\) will not be reversed.

The case of \(UT\) is similar. Notice that as \(\xi_{BB}\) is greater than \(\xi_{GR}\), there are two effects on the cut value \(k^*\) as the reform switches from \(GR\) to \(BB\). First, the option of reversal is less attractive for the young workers because of the higher reversal cost. Second, the amount needed to compensate to the middle aged workers is smaller because the option of reversing reform is also less attractive for these middle-aged workers for the same reason. These two effects work in the same direction to make the cut value \(k^*\) to be larger under \(BB\) than under \(GR\) (recall that \(k = h - \Delta\) and now \(h\) is increased and \(\Delta\) is decreased).

There is a trade-off between the two approaches of reform. On one hand, gradualism is more efficient and can be reversed with low costs. On the other hand, big bang reform is less efficient and can be reversed only with high costs. Thus there exist parameter values over which, while gradualism will be reversed (due to unfavorable demographic conditions), big bang reform will not. In real world reform decisions, the threat of reversal is noted and fast changes were proposed precisely having this in mind.

One question the reader may have is that it is hard to comprehend that the public will accept the greater hardship under \(BB\) given that it is the lesser hardship under \(GR\) that they do not like that makes \(GR\) infeasible. The reason is indeed quite simple because the public’s attitude toward reform is, roughly speaking, a comparison between the hardship and the reversal costs. When the reversal costs increases more than the hardship does when moving from \(GR\) to \(BB\), it is perfectly possibly that the public will accept \(BB\) but not \(GR\).

Here we have focused on the case where \(GR\) is a readily outside option when gradual reform is infeasible. Of course, when \(\xi_{BB}\) is not as large, it is possible that both \(BB\) and \(GR\) will be reversed when attempted. In this case, perhaps there is a third reform strategy which is to prolong the gradual reform so that it becomes politically feasible. This is consistent with a major finding in the literature that, by carefully sequencing and pacing, two reforms that should be conducted and finished in one go may become viable politically. Our
paper complements this line of research by pointing out that, to get around the political constraints, hurrying up reforms may also be an option. It is interesting to construct a model to incorporate both types of adaptations to get around the political constraints.

4.2 Ex ante approvals of reforms

Whereas we assumed reforms are subject to reversals, we have not subjected the reforms to ex ante approvals. One reason is that there may exist a "window of opportunity" (during which either the government is unconstrained, or the public is more lenient than they should be in hindsight) exists that may allow a big bang to be pushed through. The "window of opportunity" argument is first offered by A. Krueger (1993) when drawing on the experiences from reforming countries (mainly Latin American countries) in the 70’s.

Conceivably, such a “window of opportunity” was present in Russia in early 1990s’ when the government was relatively unconstrained in adopting drastic reforms. President Yeltsin was charismatic and popular. Russian reformers and their economic advisors were optimistic about market efficiency. As famously acknowledged by Milton Friedman, “only a crisis—actual or perceived—produces real change. When that crisis occurs, the actions that are taken depend on the ideas that are lying around.” (Friedman and Friedman, 1982 preface, p. ix) These two factors combined led to a rare opportunity for the Russian government to adopt drastic reform.

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18To do so will make a big bang more difficult to start and the big bang is less likely to be an outside option. We think that a model that incorporates the idea of individual uncertainty of Fernandez and Rodrik (1991) may help. Suppose although agents are ex ante identical (other than their ages in our context), they differ in their payoffs from reform. Thus it is possible that they would prefer to start a big bang reform foreseeing that if gradualism is attempted enough "losers" from reform will succeed in reversing it. We think that such a model may replicate some of the insights in Proposition 5 when the main decision becomes to start a reform rather than to reverse a started reform. But a more complete analysis is beyond the scope of this paper.

19This is reminiscent of the following famous saying, "You can fool all the people some of the time, and some of the people all the time, but you cannot fool all the people all the time."
5 Discussions

5.1 Exogenous fertility rate and support of birth control

In the analysis, we have assumed exogenous fertility rates. In the case of Russia, the favorable role of the decreasing number of children may be dominated by the unfavorable old population structure, and assuming exogenous fertility role is a benign simplification. In the case of China, the one child policy can be viewed as a binding constraint, and hence the number of children is an exogenous variable in young couples’ decision making.

Despite our stress on the importance of public support to reform, we have not modeled the public support to birth control policy. Such support may not be as important. Relative to the noncompliance of economic reforms that is difficult to detect (non-observable efforts, uncertain outputs, team production, etc.), the noncompliance of birth control is easier to identify, to punish ex post, or even to deter ex ante.

Moreover, the policy may not be as disagreeable as it appears to be, for several reasons. Given that demographic transition has been a global phenomenon, it is just a matter of time when Chinese would significantly reduce their fertility rates. Fertility choices depend on the prevalent social norm (Munshi and Myaux 2005) and the one-child policy has simply hastened the shift of norm. The public might indeed agree that, in the absence of birth control, there are more births than socially optimal because fertility exerts negative externality (see Johnson 1974). In light of Chinese’s son preferences, the availability of gender selection technology since the mid 1980’s has allowed Chinese households to experience a reduction in the number of sons that is less restrictive than literally implied by the one-child policy.

5.2 Toward a behavioral model of the one-child policy?

That birth control is helpful to the economy in a political economy sense can also be found from government officials’ speeches. A provincial leader, for example, announced in 2009 that due to the one-child policy in the last 30
years, "the province has cut down the number of births by 10.6 millions and hence has increased the per-capita GDP by RMB2,944 and contributed to one fourth of the economic social development". The ideas seem to be as follows. A new born baby is not ready for labor market participation until 15 or 20 years later. Therefore, in the first 15 to 20 years of the policy’s inception, while not the addition of a single worker to the labor force was prevented, a lot of burdens to families and society were avoided. This thus led to an increase in the per-capita GDP (compared with the case where one-child policy is not enforced), not to mention the additional effects of having a greater supply of female labor in the intervening time.

This simple math may have significant implications. A household finds it easier to feed its members above the subsistence level, and the poverty rate is reduced without any change to household incomes. Suppose young people support government’s initiatives as long as their expected living standard does not fall below a reference point, and that they consider their reference point to be their parents’ living standard during the former’s childhood. Then one-child policy makes such a reference point easier to reach. Such reference dependent preferences have been used to explore various phenomena that are hard to explain with standard models. Examples range from the individual investment and consumption decisions to labor markets to demographic changes. Despite the choice of standard, rational model in the current paper, we think that such reference dependent preferences is a prominent direction for investigation.

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21 This perspective was introduced to economics in Tversky and Kahneman (1991). Kiszegi and Rabin (2006) provide a game-theoretic formulation. Harber (2007) and Crawford and Meng (2011) use it understand New York cab drivers’ behavior. Andrew J. Cherlin (Feb. 22, 2016, New York Times) uses reference dependent preferences to explain Case and Deaton (2015)’s recent findings about the rising death rate of middle-aged non-Hispanic whites in the US between 1999 and 2013. argue that, people take their parents’ generation as a reference group. While other groups see improvements in their lives over the previous generations, non-Hispanic whites see little such progresses, thus feeling less satisfactorily.

22 This history-based reference point is plausible. That Chinese propaganda often reminds
5.3 Related literature

5.3.1 Reforms of transitional economies

Many pros and cons regarding the relative merits of gradualism and big bang have been pointed out.23 Here we can only discuss a few most related work. Dewatripont and Roland (1992a,b) and Wei (1997) find that, when a reform is thwarted by political constraints, gradualism—as a divide-and-conquer strategy—may soften the constraints and may go through despite some compromise in speed or efficacy. While forcing the policy to be conducted in a longer duration in their framework, political constraints force the policy to be hastened in our framework. Despite the difference, both lines of studies argue that political constraints may lead to a distortion in policy implementation.24

Murphy, Shleifer, and Vishny (1992) point out some pitfalls of partial reform in a model of supply diversion and that the key to success in the Chinese gradual/partial reform is its requirement that state firms have to fulfill its plans before supplying to the market, as well as the state’s monitoring ability. The article by Qian and Xu (1993), which we introduced earlier, argue that China’s plan economy was organized quite differently. Its M-form organizational structure, rather than the U-form in the former Soviet Union, allowed it to adopt experimentation and make changes gradually (See also Qian, Roland, and Xu (2006)). Fidrmuc (2000) and Gupta, Ham, and Svejnar (2008) examine the evidence about the dynamics of support of reforms in Eastern and Central European countries.

23For instance, in its introductory section (p.1235), Wei (1997) lists out six reasons (!) in favor of big bang and four reasons (!) in favor of gradualism.

24Che (2007) is concerned about the timing of privatization and the ex post performance of privatized firms. Government ownership is more efficient than private ownership when private property rights are insecure. As institutional protection of property rights is improving over time, there is a need to privatize. But the buyer’s financial constraints affect its timing, hence affecting the firm’s post-privatization performance.
5.3.2 Population

The literature on population and family economics is too vast to summarize here (please see Becker 1976 and Becker and Barro 1988 as the starting references). We are contented to mention some papers on Chinese population and economic reform. In a series of papers, Wei and his coauthors study the impacts of sexual imbalance in China. According to these studies, sexual imbalance (having more boys than girls) might lead to higher saving rates (Wei and Zhang, 2011) and even trade surplus (Du and Wei 2013). Using Chinese data, Li and Zhang (2007) find out that birth rate has a negative impact on economic growth, suggesting that Chinese one-child policy is conducive to economic growth. Liao (2013) is a theoretical piece that studies the effect of one child policy on labor market. However, there is no political economy in the model.

5.3.3 evolving voters and policy choice

There has been work that study how the current policy choice will affect the composition of future voters, or the identity of the next period’s medium voter, and as such may compromise the policy choice in the current period. The most often talked about example is that of immigration policy and pension policy. The political economy of demography is well studied in pension policy and immigration policy. These are where intergenerational conflicts are conspicuous (see, e.g., Sand and Razin 2007 and Storesletten 2000). Jin and Lagaloff (2011) study a very general class of dynamic games that have such a feature. There, the current leader faces a tradeoff between implementing his most desired policy which nonetheless may lead to a loss of political power; if he wants to preserve his future power, he must sacrifice his present policy objectives.

In our model, the composition of next period’s voters is determined by the current fertility rate, and this is discussed in the appendix B when we endogenize the $B$ function to close the model. However, in our main text we have not emphasized this direction of analysis. Instead what we emphasized about the current fertility rate is more direct, and is about its impacts on child
rearing burden on the currently young workers whereby their attitude towards reform is affected.

6 Conclusions

Our paper is motivated by the stark differences in the demographic structures and dynamics in Russia and China at the outset of their reforms. We have presented a simple theoretical model in which reforms are subject to reversibility due to discontent among the public. Besides the result that an old population structure lends less support to the reform, we have also found that the representative young worker’s attribute towards reform is more positive—her greatest endurable hardship is higher—when she has or is expected to have fewer children. This thus suggests some subtle political economy implications of the one-child policy. We have shown that, if the demographic conditions are unfavorable and the ideally paced gradual reform will be reversed, then a benevolent government may want to hasten its reforms to take advantage of its greater irreversibility. This theory thus provides not only a qualification to the applicability of the Chinese gradualism, but also a vindication to the big bang approach adopted in Russia.\(^{25}\) That said, we see our model as complementary to the existing literature and we do not claim that other factors (specific institutions, leadership, large rural population, etc.) or issues other than strategy choice are unimportant. In particular, our model is too simple to explain the high economic growth rate in China, a topic that is of much interest to economists and policy makers.\(^{26}\)

A premise of our theory is that the young are forward looking, comparing different policy outcomes when deciding their positions. Hence, when the government shows no interest at all in implementing any reforms, a young population structure is more impatient and more susceptible for revolts than

\(^{25}\)The dissatisfaction with the Russia reform is well implied by the defenses given in a book by Shleifer (2005), both in the context and in the title of the book: a natural state.

\(^{26}\)For a paper that explains its high growth rate, please see Song, Storesletten and Zilibotti (2011). There, growth is due to expansion of the more productive financially-unconnected firms at the expense of less productive financially-connected firms.
an old population structure is.\textsuperscript{27} As a young population structure is compatible to both being too patient and being too impatient, this posts challenges in the empirical testing of our theory. On the theory front, it is interesting to construct a unified model in which the public can choose either to endure hardship from government reform or to engage in revolutionary actions. We leave this for future research.

\textsuperscript{27}This reminds us of an ancient Chinese adage: "The water that bears the boat is the same that swallows it." Thus the higher the water, the higher the boat can be floated, and the more miserable when the boat is swallowed.
References


Appendix: (For online publication)

A. Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1

It suffices to show that $U_D$ intersects $U_R$ from below as $h$ varies, or $\frac{\partial U_D}{\partial h} - \frac{\partial U_R}{\partial h} > 0$.

From the optimization problem set out in the main text, we know that if consumption smoothing is feasible in regime $j$, the optimum is given generally by

$$u'(c^j_1) = \delta (1 + r) u'(c^j_2) = \delta^2 (1 + r)^2 u'(c^j_3),$$

and, using the specific functional form of the stage-utility function, we have $c^j_{k+1} = (\delta (1 + r))^{\frac{j}{2}} c^j_k$, where $k = 1, 2$. Differentiating $U_j$ with respect to $h$, we have

$$\frac{\partial U_j}{\partial h} = u'(c^j_1) \frac{dc^j_1}{dh} + \delta u'(c^j_2) \frac{dc^j_2}{dh} + \delta^2 u'(c^j_3) \frac{dc^j_3}{dh}$$

$$= u'(c^j_1) \frac{dc^j_1}{dh} + \frac{u'(c^j_1)}{1 + r} \frac{dc^j_2}{dh} + \frac{u'(c^j_1)}{(1 + r)^2} \frac{dc^j_3}{dh}$$

$$= u'(c^j_1) \left( \frac{dc^j_1}{dh} + \frac{1}{1 + r} \frac{dc^j_2}{dh} + \frac{1}{(1 + r)^2} \frac{dc^j_3}{dh} \right)$$

$$= Fu'(c^j_1) \frac{dc^j_1}{dh},$$

where the last line is obtained by replacing $c^j_2$ and $c^j_3$ by expressions of $c^j_1$ using (12) and $F$ was defined in (5).

Consider regime $R$. If consumption smoothing is feasible, we have $c^R_1 = 1 \frac{1}{\tilde{R}} \left( (1 - \kappa(n)) z^R_1 - T(n) - I_R h + \tilde{z}^R_2 (1 + r)^{-1} \right)$ and, making use of (13),

$$\frac{\partial U_R}{\partial h} = -u'(c^R_1);$$
otherwise, \( c_1^R = (1 - \kappa (n)) z_1^R - T(n) - I_R h \) and, also making use of (13),
\[
\frac{\partial U_R}{\partial h} = u'(c_1^R) \left( \frac{dc_1^R}{dh} + \frac{1}{1 + r} \frac{dc_2^R}{dh} + \frac{1}{(1 + r)^2} \frac{dc_3^R}{dh} \right)
\]
\[
= u'(c_1^R) \left( -1 + 0 + 0 \right)
\]
\[
= -u'(c_1^R).
\]

That is, whether or not consumption smoothing is feasible, \( \frac{\partial U_R}{\partial h} = -u'(c_1^R) \).

Under regime \( D \), if consumption smoothing is feasible, we have \( c_1^D = \frac{1}{1}(1 - \kappa (n)) z_1^D - T(n) + \frac{B(h)}{1 + r} \) and, making use of (13),
\[
\frac{\partial U_D}{\partial h} = \frac{B'(h)}{1 + r} u'(c_1^D);
\]
otherwise, we still have consumption smoothing between stage 2 and stage 3, where, with some manipulation, we have
\[
c_2^D = \left( 1 + \frac{(\delta (1 + r))^{\frac{1}{2}}}{1 + r} \right)^{-1} \times B(h)
\]
and, making use of (13) and (12),
\[
\frac{\partial U_D}{\partial h} = u'(c_1^D) \left( 0 + \frac{1}{1 + r} \frac{dc_2^D}{dh} + \frac{1}{(1 + r)^2} \frac{dc_3^D}{dh} \right)
\]
\[
= u'(c_1^D) \left( 0 + \frac{1}{1 + r} \frac{dc_2^D}{dh} + \frac{(\delta (1 + r))^{\frac{1}{2}}}{(1 + r)^2} \frac{dc_3^D}{dh} \right)
\]
\[
= \frac{1}{1 + r} u'(c_1^D) \left( 1 + \frac{(\delta (1 + r))^{\frac{1}{2}}}{1 + r} \right) \frac{dc_2^D}{dh}
\]
\[
= \frac{1}{1 + r} u'(c_1^D) B'(h).
\]

That is, whether or not consumption smoothing is feasible, \( \frac{\partial U_D}{\partial h} \) is the same and equals \(-\frac{B'(h)}{1 + r} u'(c_1^D)\).
Hence,
\[
\frac{\partial U_D}{\partial h} - \frac{\partial U_R}{\partial h} = \frac{B'(h)}{1 + r} u'(c_1^D) + u'(c_1^R) > 0
\]
because \(c_1^D \geq c_1^R, B'(b) \in (-1, 0)\). This completes the proof.

### A.2 Proof of Lemma 2

The proof is similar to that of Lemma 1. Given regime \(j\), when consumption smoothing is infeasible between stage 1 and later stages, the stage-1 consumption is equal to \(c_1^j = (1 - \kappa(n)) z_1^j - T(n) - I_j h \) and the effects of \(h\) on \(U_j\) are confined to \(A(n)\) and stage-1 utility.

\[
\frac{dU_j}{dn} = A'(n) - u'(c_1^j) (\kappa'(n) z_1^j + T'(n)) .
\]

When consumption smoothing is feasible,

\[
\frac{\partial}{\partial n} U_j = u'(c_1^j) \frac{dc_1^j}{dn} + \delta u'(c_2^j) \frac{dc_2^j}{dn} + \delta^2 u'(c_3^j) \frac{dc_3^j}{dn}
= u'(c_1^j) \frac{dc_1^j}{dn} + \frac{u'(c_1^j)}{1 + r} \frac{dc_2^j}{dn} + \frac{u'(c_1^j)}{(1 + r)^2} \frac{dc_3^j}{dn}
= u'(c_1^j) \left( \frac{dc_1^j}{dn} + \frac{1}{1 + r} \frac{dc_2^j}{dn} + \frac{1}{(1 + r)^2} \frac{dc_3^j}{dn} \right).
\]

Substituting (12) into it, we have

\[
\frac{\partial}{\partial n} U_j = u'(c_1^j) \left( \frac{dc_1^j}{dn} + \frac{\delta (1 + r) \frac{1}{r}}{1 + r} \frac{dc_1^j}{dn} + \frac{\delta^2 (1 + r)^2 \frac{1}{r^2}}{(1 + r)^2} \frac{dc_1^j}{dn} \right)
= u'(c_1^j) \left( \frac{1}{1 + r} \frac{dc_1^j}{dn} + \frac{\delta^2 (1 + r)^2 \frac{1}{r^2}}{(1 + r)^2} \frac{dc_1^j}{dn} \right).
\]

However, as \(c_1^j = \frac{1}{F} \left( (1 - \kappa(n)) z_1^j - T(n) - I_j h + z_2^j (1 + r)^{-1} \right)\), implying \(dc_1^j/dn = \frac{1}{F} \left( 1 - \kappa'(n) \right) z_1^j - T'(n) \). Substituting it into the above expression, we have the same outcome as under the case of infeasibility of consumption smoothing.
This completes the proof.

A.3 Proof of Proposition 3

Result (i). Trivial.

Result (ii). From \( h + k \) and \( \Delta \equiv (x + y + k) / n_{t-1} \), we obtain

\[
k = \left( h + \frac{y - x}{n_{t-1}} \right) \left( \frac{n_{t-1}}{1 + n_{t-1}} \right)
\]

or

\[
k^* (n_t, n_{t-1}) = \left( h^* (n_t) + \frac{y - x}{n_{t-1}} \right) \left( \frac{n_{t-1}}{1 + n_{t-1}} \right)
\]

\[
= h^* (n_t) \left( 1 + \frac{1}{n_{t-1}} \right)^{-1} + \frac{y - x}{1 + n_{t-1}}, \tag{14}
\]

where \( k^* \) is younger workers’ maximum transition cost and \( h^* \) is their greatest endurable hardship, characterized in Proposition 1. Applying Lemma 1 to (14), we obtain result ii.1. It is easy to show that \( \partial k^* (n_t, n_{t-1}) / \partial n_{t-1} > 0 \). This is result ii.2. Applying Proposition 1 to (14), we obtain result ii.3.

A.4 Proof of Lemma 3

For regime \( j = R, D \), if consumption smoothing is infeasible between stage 1 and later stages, then \( c^j_1 = w^j_1 / (1 + \gamma) \) and a change in \( n \) will have no effects on \( c^j_2 \) and \( c^j_3 \). As a result,

\[
\frac{\partial U_j}{\partial n} = A'(n) + u'(c^j_1) \frac{dc^j_1}{dn}
\]

\[
= A'(n) + u'(c^j_1) \frac{d}{dn} \left( \frac{w^j_1}{1 + \gamma (n)} \right)
\]

\[
= A'(n) - u'(c^j_1) \frac{w^j_1 \gamma (n)}{(1 + \gamma (n))^2}
\]

\[
= A'(n) - \frac{\gamma'(n)}{1 + \gamma (n)} c^j_1 u'(c^j_1).
\]

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Suppose, instead, the consumption smoothing is feasible under the regime. It is easy to show that the optimum is such that

\[ c_{k+1} = (\delta (1 + r) (1 + \gamma))^{\frac{1}{\sigma}} c_k \quad \text{where } k = 1, 2. \quad (15) \]

Then a change in \( n \) affects all of \( c_1^j, c_2^j, c_3^j \), in addition to \( A(n) \).

\[
\frac{\partial U_j}{\partial n} = A'(n) + u'(c_1^j) \frac{dc_1^j}{dn} + \delta u'(c_2^j) \frac{dc_2^j}{dn} + \delta^2 u'(c_3^j) \frac{dc_3^j}{dn} \\
= A'(n) + u'(c_1^j) \frac{dc_1^j}{dn} + \frac{u'(c_1^j)}{(1 + \gamma) (1 + r)} \frac{dc_2^j}{dn} + \frac{u'(c_1^j)}{(1 + \gamma) (1 + r)^2} \frac{dc_3^j}{dn} \quad (\because (15))
\]

\[
= A'(n) + \frac{u'(c_1^j)}{(1 + \gamma)} \left( (1 + \gamma) \frac{dc_1^j}{dn} + \frac{1}{1 + r} \frac{dc_2^j}{dn} + \frac{1}{(1 + r)^2} \frac{dc_3^j}{dn} \right) \quad (16)
\]

Note that the young worker’s intertemporal budget constraint is

\[ (1 + \gamma(n)) c_1^j + \frac{c_2^j}{1 + r} + \frac{c_3^j}{(1 + r)^2} = w_1^j + \frac{w_2^j}{1 + r}, \]

where \( w_1^j \) and \( w_2^j \) are the worker’s incomes (after netting the hardship) in the two stages and are independent of \( n \). Differentiating the budget constraint with respect to \( n \), and rearranging, we obtain

\[ (1 + \gamma(n)) \frac{dc_1^j}{dn} + \frac{1}{1 + r} \frac{dc_2^j}{dn} + \frac{1}{(1 + r)^2} \frac{dc_3^j}{dn} = -\gamma'(n) c_1^j. \]

Substituting it into the RHS of (14), we obtain

\[ \frac{\partial U_j}{\partial n} = A'(n) - \frac{\gamma'(n)}{1 + \gamma(n)} c_1^j u'(c_1^j). \]

A.5 Proof of Proposition 4

Because of Lemma 1, it suffices to determine the sign of \( \partial U_D/\partial n - \partial U_R/\partial n \). Using Lemma 3, as well as the fact that for CRRA utility function \( c_1^j u'(c_1^j) \)
can be re-written as $(1 - \rho) u(c_1^R)$, we have

$$\frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} = \frac{\gamma'(n)}{1 + \gamma(n)} \left( c_1^R u'(c_1^R) - c_1^R u'(c_1^P) \right)$$

$$= \frac{\gamma'(n)}{1 + \gamma(n)} (1 - \rho) \left( u(c_1^R) - u(c_1^P) \right).$$

There are three cases to consider: (a) consumption smoothing is infeasible in both regimes; (b) it is feasible under reversal but infeasible under reform; (c) it is feasible in both regimes. Consider case (a). For cases (a) and (b), $c_1^R < c_1^P$ implying that $u(c_1^R) - u(c_1^P) < 0$; for case (c), $c_1^R = c_1^P$ implying that $u(c_1^R) - u(c_1^P) = 0$. Hence, for $\rho > 1$,

$$\frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} > 0 \quad \text{for cases (a) and (b)}$$

$$\frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} = 0 \quad \text{for case (c)}.$$

The opposite is true for $\rho < 1$, in which case

$$\frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} < 0 \quad \text{for cases (a) and (b)}$$

$$\frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} = 0 \quad \text{for case (c)}.$$

A.6 Proof of Lemma 3

All results are straightforward. The fact that, given the reform package (as well as the signal revealed in the case of GR) a GEH exists such that young workers will oppose reform if and only if the actual hardship exists the GEH comes from the observation that the income profile under reform is more backloaded that its counterpart under reversal.

A.7 Proof of Proposition 5

The case of SM is straightforward and omitted. For the case of UT, the middle aged workers must be compensated for them to agree to reform. The amount to each such worker is $(x - \xi_{GR}) - (y + I_sA - k)$. Each young worker’
payout is
\[ \Delta = \frac{(x - \xi_{GR}) - (y + I_s \Lambda - k)}{n_{t-1}} \]

where \( I_s = 1 \) if \( s = 1 \) and \( I_s = 0 \) if \( s = 0 \).

hence,
\[ h = k + \Delta = k + \frac{(x - \xi_{GR}) - (y + I_s \Lambda - k)}{n_{t-1}}. \]

Rearranging,
\[ k = \left( h + \frac{(y + I_s \Lambda) - (x - \xi_{GR})}{n_{t-1}} \right) \frac{n_{t-1}}{1 + n_{t-1}}, \]

and hence
\[ k^* (n_t, n_{t-1}, s, \xi_{GR}) = h (n_t, s, \xi_{GR}) \frac{n_{t-1}}{1 + n_{t-1}} + \frac{(y + I_s \Lambda) - (x - \xi_{GR})}{1 + n_{t-1}}. \]

A.8 Proof of Proposition 6

Assume \( SM \) is used and \( n_{t-1} > \alpha \) so that young workers are sufficiently numerous to be pivotal. From result 3 and the first part of result 4 of the Lemma 3, we obtain \( h^*_{BB} (n_t, s, \xi_{BB}) > h^*_{GR} (n, s = 0, \xi) \). Coupled with result 1 of the lemma, we know that
\[ h^*_{GR} (n_t, s = 0, \xi_{GR}) < \min \{ h^*_{GR} (n_t, s = 1, \xi_{GR}), h^*_{BB} (n_t, s, \xi_{BB}) \}, \]

and therefore for
\[ h \in (h^*_{GR} (n_t, s = 0, \xi_{GR}), \min \{ h^*_{GR} (n_t, s = 1, \xi_{GR}), h^*_{BB} (n_t, s, \xi_{BB}) \}), \]

\( GR \) is reversed (at least with some probability) and \( BB \) is not reversed. Suppose if it is also true that \( h^*_{BB} (n_t, s, \xi_{BB}) > h^*_{GR} (n_t, s = 1, \xi_{GR}) \) (this happens when \( \xi_{BB} \) is sufficiently large or \( \Lambda \) is not large enough), then for all \( h \in (h^*_{GR} (n_t, s = 0, \xi_{GR}), h^*_{GR} (n_t, s = 1, \xi_{GR})) \), \( GR \) is reversed with certainty and \( BB \) is not reversed at all.
Continue to study $SM$ while assuming that $n_{t-1} < \alpha$. Suppose $\xi_{GR}$ is low enough so that middle age workers prefer reversal to $GR$ while $\xi_{BB}$ is large enough so that they prefer $BB$ to reversal. The proposition obviously holds.

The case where $UT$ is used is more interesting. In this case a young worker’s hardship $h = k + \Delta$. Note that an increase in the reversal cost $\xi_i$ reduces the compensation $\Lambda$ as well as the attractiveness of reversal to the young worker. It is straightforward to verify that the proposition holds.

**B. Endogenizing $B(h)$ function**

In Section 3, we characterized the greatest endurable hardship ($GEH$) assuming a post-delay payoff function $B(.)$. In this section, we clarify how the $GEH$ that can be supported in equilibrium is determined and whether our earlier insights—regarding the roles of current fertility rate and demographic structure—still hold.

We first state the following simple result for later use.

**Claim 1** Assume $h < y - x$. Let $\bar{h}$ be the $h^*$ solved assuming $B(h) = y - h$ and $\underline{h}$ be the $h^*$ solved assuming $B(h) = x$. Then $\bar{h} > \underline{h}$.

**Proof.** We use $U^B_D$ to denote the utility under reversal given post-delay income function $B$. Then it is clear that, for $h < y - x$, we have $U^B_{D=x} > U^B_{D=y-h}$. Because both $U_R$ and $U_D$ are decreasing in $h$, it must be the case that the $\underline{h}$ at which $U^B_{D=y-h}$ will intersect with $U_R$ is smaller than the $\bar{h}$ at which $U^B_{D=x}$ intersect with $U_R$. ■

The claim compares two future contingencies. In the first, the reversed reform will be implemented in the next period; in the second, it will not be approved in the next period either. The lemma states that the young agent’s $GEH$ under the first contingency is greater than under the second.

In the fully dynamic game where $B(.)$ endogenized, there are usually multiple equilibria, each being associated with an equilibrium greatest endurable hardship ($EGEH$). Among all the $EGEH$s, there is a maximum one, which
we call the maximum equilibrium greatest hardship (MEGEH). To fix ideas, we also assume the use of SM so the hardship from the reform during the implementation period is always $k$ (nonetheless, we still keep the use of $h$, and in this case $h = k$). It is easy to see that the MEGEH at period $t$, denoted by $h^m_t$, must be bound above by $\overline{h}_t$ and below by $\underline{h}_t$, which we recall are the GEH of the representative young worker based on her belief that in the next period the delayed reform will be approved and will not be approved, respectively. It is also easy to see that, if this former belief is consistent with some equilibrium, then $h^m_t$ is indeed equal to $\overline{h}_t$. Otherwise, it must strictly be less than $\overline{h}_t$, and may be equal to or strictly exceed $\underline{h}_t$. The latter point being more intrigue, we will illustrate it through a particular example.

Assume that (i) $n_{t-1} > \alpha$, (ii) all future fertility rates $n_{t+i} = n^*$, where $i = 1, 2, \ldots$, and (iii) $n^* > \alpha$. The second assumption ensures a constant young-workers-to-middle-aged-workers ratio ($n^*$) in future periods $t+2, t+3, \ldots$. The first and third assumptions ensure the young workers in period $t$, as well as those in period $t+2$ and onwards, are numerous enough to overwhelm their middle-aged worker counterpart in voting. We depict $h^m_t$ in Figure 2, where the horizontal axis is $n_t$ and the vertical axis is $h$. $\overline{h}(n_t)$ and $\underline{h}(n_t)$ are the GEHs defined and solved in Lemma 3, and they are downward slopping because of Proposition 1 and the assumption that $\rho > 1$.

There is a tripartite classification of $n_t$: (i) $n_t < \alpha$; (ii) $\alpha \leq n^t < n^*$; and (iii) $\alpha < n^* \leq n^t$.

Case i: $n_t < \alpha$. The current period’s fertility rate is so low that the current young workers are numerous enough to dominate their children in next period’s voting. In other words, if reform is not approved in period $t$, it will not be approved in period $t+1$ either. Thus $h^m_t$ is simply $\underline{h}(n_t)$.

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Figure 3 about here

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For the other two cases where \( n_t > \alpha \), the young workers will not be numerous enough to overwhelm the next generation in next period’s voting. Note that for period \( t + 2 \) and onwards, the ratios of young workers to middle-aged workers are constant, equal to \( n^* \); we can use \( h(n^*) \) and \( h(n^*) \) to denote the \( GEHs \) in periods \( t + 1, t + 2, ... \) based on the belief that the reform will be and will not be approved, respectively, in the subsequent period.

**Case ii:** \( \alpha \leq n^t \leq n^* \). Note that any reform with \( h \) strictly greater than \( h(n^*) \) will not be accepted at period \( t + 1 \) if the reform is voted on that period. Since \( h() \) is decreasing in its argument, we have \( h(n^*) < h(n_t) \) and \( h_t^{m\alpha} \) must be less than \( h(n_t) \). Hence, \( h_t^m = h(n_t) \) if \( h(n_t) \geq h(n^*) \) and equal to \( h(n^*) \) if the reverse is true. In other words, \( h_t^m \) is equal to \( \max \{ h(n^*), h(n_t) \} \).

**Case iii:** \( \alpha < n^* \leq n^t \). Because \( h(n_t) < h(n^*) \), the argument in case (ii) that "kills" \( h(n_t) \) as \( h_t^m \) no longer works. Therefore, \( h_t^m \) is simply \( h(n_t) \).

We summarize the above discussion as follows.

**Proposition 7** Suppose (i) \( n_{t-1} > \alpha \), (ii) all future fertility rates \( n_{t+i} = n^* \), where \( i = 1, 2, ... \), and (iii) \( n^* > \alpha \). Then

\[
MEGEH = \begin{cases} 
    h(n_t) & \text{if } n_t < \alpha \\
    \max \{ h(n_t), h(n^*) \} & \text{if } \alpha \leq n_t < n^* \\
    h(n_t) & \text{if } n_t \geq n^* > \alpha.
\end{cases}
\]

To show the proposition, we first we define our notations more formally. We use \( P_{\tau} \) to denote the population of stage-\( i \) agents at period \( \tau \) and \( n_\tau \) the number of children for each young worker at period \( \tau \). Hence, population evolves according to \( P_{\tau+1} = n_{\tau-1}P_{2\tau} \). In addition, we make an explicit assumption that we are using the \( SM \) decision rule so that the hardship borne by each young agent when undergoing reform is the same, equal to \( k \) (although we still keep the notation of \( h \), invariant over time). We omit the first case \( (n_\tau < \alpha) \) which is most straightforward. We use \( h_t^{m\alpha} \) to denote the \( MEGEH \) in period \( t \). For the other two cases, we first notice the following result:
Claim 2 Suppose $n^* \geq \alpha$. For any $\tau \geq t + 1$, the MEGEH in period $\tau$ is $\bar{h}(n^*)$.

Proof. We first realize that for any $\tau \geq t + 1$, $P_{1\tau} = n_{\tau-1}P_{2\tau} = n^*P_{2\tau} \geq \alpha P_{2\tau}$. Therefore, the currently young workers can overwhelm the currently middle aged workers. If the $1\tau$ agents hold the belief that the reform, if not accepted now, will be accepted in the next period $(\tau + 1)$, then their $GEH$ is simply $\bar{h}(n^*)$. By definition, it is impossible to support an even greater endurable hardship. This $GEH$ can be supported as an equilibrium outcome because, due to stationarity, the $1\tau + i$ agents having belief that "if the reform is not accepted in the current period, it will be accepted in the next period" can be supported, where $i = 1, 2, ...$ Hence, $h_i^m$ is indeed equal to $\bar{h}(n^*)$. □

Some general lessons can be drawn. First, the MEGEH is indeed decreasing in $n_t$ and our analysis is supportive of the insight from Proposition 1, despite the following qualifications for the neighborhood when $n_t$ is equal to $\alpha$ (as in panel a of Figure 3) (the potential non-monotonicity happens because of a shift of $h_i^m$ from $\max \{h(n_t), \bar{h}(n^*)\}$ to $h(n^*)$ when $n_t$ moves across $\alpha$). Second, the initial population structure may be pivotal in determining the popularity and feasibility of the reform. In the above example, if we invoked assumption (i) so that $n_{t-1} < \alpha$, then the reform cannot be approved in the current period because middle-aged workers are too numerous, and this is so even if the current young workers have low fertility rate which is presumably favorable to reform. Third, fertility rates in the far future may have impacts on the current MEGEH because of backward induction. The two panels in Figures show that a lowering of future constant fertility rate $n^*$ (switching from panel b to panel a) leads to a higher MEGEH for the current period.
Figure 3: Finding the equilibrium greatest endurable hardship