Task Interdependence and Non-Contractibility in Public Good Provision

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Abstract

In the context of public good provision, despite non-contractibility of investments, it is possible to specify who is in charge of tasks such as construction and maintenance. We show that complementarity between the investments of the two tasks favours unbundling of tasks through different contracts against bundling of them into a single contract, whether the built facility is privately or publicly owned. This result is counter-intuitive, because complementarity can be viewed as positive externality which should favor bundling (as in Bennett and Iossa 2006). We also obtain general conditions that determine the efficiency of traditional procurement vis-a-vis public-private partnership (PPP).

Keywords: Complementarity; Substitutability; Incomplete contracts; Public-private partnership.

JEL classification: D23; H11; L33

1 Introduction

According to modern firm theory, ownership structure and control right matter when the investment incentives are non-contractible. Inspired by the seminal paper of Hart, Shleifer and Vishny (1997) on prison, much recent work (e.g., Besley and Ghatak 2001) applies the property right approach to study the organization of public good provision. This body

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of work usually assumes that investment on cost reduction may reduce project quality, while investment on quality innovation may increase operational costs. Hence, the choice of privatization, leading to over-investment on cost reduction and under-investment on quality enhancing, should trade off the incentives of these two investments.

However, as Gilroy et al (2007) argue, the public good provision in practice involves complex arrangement. In additional to the transfer of property rights, the contracts may also specify the assignment of different tasks such as design, construction and maintenance, etc.\(^1\) As noted by Hart (2003), in the context of public-private partnerships (PPPs), the government usually bundles tasks such as building and operation of a facility in a single contract. Thus, given the richness of contractual arrangement, other factors may play a role on the determination of the optimal regime.

Hoppe and Schmitz (2010) analyze a richer set of contractual arrangement when the government cooperates with private sector to provide a good. They find that investment incentives are influenced not only by who has the control right but also by the initially specified quantity of the basic good. However, the main determinants of the optimal governance structure are the relative importance of cost reduction versus quality innovation, as well as the involved parties’ bargaining power. Francesconi and Muthoo (2011) study a "complex" partnership which produces "impure" public goods, and find that the optimal allocation of control right depends on the degree of impurity of public good.

In this paper, we follow the incomplete contracting approach, stressing that both task design and property rights should matter in shaping the right incentive in a public project. Though quality innovation and cost reduction are still assumed to be the main investment incentives, we find that the optimal task design depends not only on relative importance of these two investments, but also on a richer interaction between them.

Close to ours is Bennett and Iossa (2006), who show that, when an increase in the building investment reduces the subsequent operational costs, bundling construction and operation in contracting is more desirable than unbundling them.\(^2\) They thus yield an intuitive insight that positive externality across stages favors bundling. Building on them,

\(^1\)This idea is in the same spirit of Aghion and Tirole (1994): though the investment levels are non-contractible, it is possible to contractually specify who is in charge of which task and hence bears the investment costs.

\(^2\)While this result is obtained in the context where investments are non-contractible, Martimort and Pouyet (2008) demonstrate a similar result when investments are contractible.
we study a more comprehensive framework. In addition to the aforementioned externality identified in Bennett and Iossa, we take into account interdependence such as complementarity or substitutability between the two tasks. (To facilitate discussion, we call what Bennett and Iossa study as externality and the new element introduced here as task interdependence.) For example, a prison may be built in a more specified manner so that, while the subsequent operational cost is generally lower (i.e., positive externality), further enhancement of quality or alternation of usage would be more difficult (i.e., task substitutability).

By allowing both task interdependence and externality, the optimal choice of regime should vary in response to different combination of these two. The general result is that complementarity between the two investments favors unbundling rather than bundling, regardless of the ownership of the facility. The intuition is the following. In case of unbundling, the builder can share the benefits generated by the manager’s investment through bargaining, while not bearing any cost incurred by such investment. However, in the case of bundling, when investing in the building stage, the consortium will internalize not only the benefits but also the costs of subsequent investment, resulting in a dampened investment incentive on his part. Since task complementarity can be viewed as a kind of positive externality, our result is counterintuitive.

It should be noted that Chen and Chiu (2010) show a related result: complementarity favors unbundling over bundling under private ownership while task interdependence plays no role under public ownership. Their analysis, however, assumes that the operating investment becomes contractible once the building investment is complete and, therefore, will be chosen efficiently under standard bargaining procedures. Because of this different assumption on investment contractibility, the strategic interactions there are quite different from those in the current paper.

Table 1 compares the prediction of optimal contractual arrangement of our model with that of Bennett and Iossa (2006). Note that if we assume no task interdependence (the bottom row case), our results collapse into Bennett and Iossa’s. However, if we consider interplay between task interdependence and externality, clear-cut predictions are obtained when there exist positive externality and task substitutability, and when there exist negative externality and task complementarity; in other circumstances, the optimal
regime is uncertain, in the sense that it depends not only on the nature of the interactions, but also on the relative strength of the forces.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 and 4 analyze and compare the chosen investments under the private and public ownership, respectively. Section 5 derives the conditions which underlie the optimum of traditional procurement or PPP. Section 6 concludes.

2 Basic setup

We follow closely the model in Bennett and Iossa, which is a variant of Hart, Shleifer, and Vishny. A governmental agency (hereafter, the government) is contemplating a project consisting of two sequential tasks, namely, "building" and "operating" of a facility. We use \(a\) and \(e\) to denote the levels (also the costs) of the innovative activities in the building stage and in the operating stage, respectively, and these two activities can be undertaken by two separate firms or by a consortium. The operational cost, borne by the manager in the operation stage, is

\[
C(a, e) = C_0 - d(a, e),
\]

where \(C_0\) is the positive default cost, and \(d(a, e)\) is the reduction of operational cost caused by the investments, \(a\) and \(e\), satisfying the following properties:

(i) \(d(0, 0) = 0\).

(ii) \(d_2(a, e) > 0, d_2(a, 0) = \infty, d_2(a, \infty) = 0, d_{22}(a, e) < 0\).

(iii) If \(d_1(a, e) > 0\), then \(d_1(0, e) = \infty, d_1(\infty, e) = 0, d_{11}(a, e) < 0\); if \(d_1(a, e) \leq 0\), then \(d_1(0, e) = 0, d_1(\infty, e) = -\infty, d_{11}(a, e) < 0\).

If \(d_1(a, e) > 0\) \((< 0)\), \(a\) decreases (increases) the operational cost; these positive and negative externalities are studied in Bennett and Iossa. If \(d_{12}(a, e) > 0\) \((< 0)\), \(a\) and \(e\) are
complementary (substitutable) investments and an increase in \(a\) increases (decreases) the marginal benefit of \(e\). In Bennett and Iossa, \(d_{12}(a,e)\) is assumed to be zero.

**Remark 1** To facilitate exposition, we will use the following nomenclature unless otherwise stated. Task or investment externality refers to the case that \(d_1(a,e)\) is nonzero; in particular, positive (negative) externality refers where \(d_1(a,e) > 0\) (\(d_1(a,e) < 0\)). Task or investment interdependence refers to the case that \(d_{12}(a,e)\) is nonzero; in particular, complementarity (substitutability) refers to where \(d_{12}(a,e) > 0\) (\(d_{12}(a,e) < 0\)).

Like in Bennett and Iossa, the project yields the following social benefits:

\[
B(a,e) = B_0 + u(a) + v(e),
\]

where \(B_0\) is the positive default benefit; the terms \(u(a) \equiv \alpha U(a)\) and \(v(e) \equiv \beta V(e)\) measure the part of social benefits caused by the investments of \(a\) and \(e\), respectively. \(\alpha > 0\) and \(\beta > 0\) are scale parameters and \(U\) and \(V\) are normalized functions satisfying:

\[
U(0) = V(0) = 0; U'(0), U''(0), V(0) = 1; U'(a), V'(e) > 0; U''(a), V''(e) < 0; U'(0) = V'(0) = \infty; U'(\infty) = V'(\infty) = 0.
\]

Upon the project’s expiry, the owner of the facility claims its residual value, which equals

\[
R(a) = R_0 + t(a),
\]

where \(R_0\) is the positive default residual value, and \(t(a) \equiv \gamma T(a)\) measures the additional residual value generated by investment, \(a\), where \(\gamma\) is a scale parameter and \(T\) is a normalized function satisfying \(T(0) = 0; T(1) = 1; T'(a) > 0, T''(a) < 0; T'(0) = \infty; T'(\infty) = 0.\)

To simplify matter, we make the following assumption.

\textbf{A1} \(t'(a) + d_1(a,e) > 0\) for all \(a\) and \(e\).

This assumption implies that, although we allow \(a\) to increase the operational cost, this negative externality is constrained to a moderate range. Bennett and Iossa implicitly use a similar assumption when they analyze the negative externality case.

As in Bennett and Iossa, both \(a\) and \(e\) are observable, non-verifiable, and hence non-contractible. The sequence of events is as follows (see Figure 1). At time 0, given the
contractual regime, the government specifies the basic standards and contract payments when contracting with the firm(s). The various possible contractual regimes include: separation with builder ownership (SB), separation with manager ownership (SM), separation with public ownership (SP), integration with consortium ownership (IC), and integration with public ownership (IP) (where separation means unbundling and integration means bundling). At time 1, a is undertaken. At time 1.5, negotiation needs occur over the adoption of a. At time 2, e is undertaken. At time 2.5, negotiation may occur over on the adoption of e. At time 3, all the payoffs are realized. Note that investment costs of a or e are irreversible whether or not they will be subsequently adopted.

We use $g$ to denote the government’s payoff, $f$ the consortium’s payoff, and $f_b$ and $f_m$ the builder’s and manager’s payoffs, respectively, when they are two separate agents. To simplify our presentation, we omit such default values as $C_0, B_0, R_0$ in the discussions. As in Bennett and Iossa, negotiation is conducted through Nash bargaining with symmetric bargaining power.

Notice that the first-best investments $(a^*, e^*)$ satisfy:

$$u'(a^*) + t'(a^*) + d_1(a^*, e^*) = 1$$

(1)

and

$$v'(e^*) + d_2(a^*, e^*) = 1.$$  

(2)

We assume that unique, interior solutions to these two equations exist. Likewise, we also assume that the equilibrium in every regime is also unique; such an assumption is standard in the literature even though it is not usually explicitly stated.
3 Private ownership

3.1 Builder ownership

Suppose that the builder owns the facility and the manager is a separate agent. In this case, as explained by Bennett and Iossa, a will always be implemented without going through any bargaining, because the builder has control rights and receives the residual value generated by adopting a. However, in the operating stage, the builder negotiates with the manager on the approval of e, since he does not directly gain anything if he does not threaten to disallow the adoption of e. Notice that the net surplus generated by allowing the adoption of e is d(a, e) − d(a, 0).\(^3\) Given equal division of this net surplus (because of Nash bargaining), the ex post payoffs of the builder and the manager, respectively, are

\[ f_b = t(a) + \frac{1}{2} [d(a, e) − d(a, 0)] − a \quad \text{and} \quad f_m = \frac{1}{2} [d(a, e) + d(a, 0)] − e. \]

Given a, the manager maximizes \( f_m \) by choosing \( e = e_{SB}(a) \), and the FOC is

\[ \frac{1}{2} d_2(a, e_{SB}(a)) = 1. \] (3)

Totally differentiating it, we obtain \( \frac{\partial e_{SB}}{\partial a} = -\frac{d_2(a, e_{SB})}{d_2(a, e_{SB})} \), which is positive (negative) if there is task complementarity (substitutability).

Foreseeing the manager’s behavior, the builder maximizes \( f_b \) subject to (3) by choosing \( a = a_{SB} \), and the first-order condition (FOC) is

\[ t'(a_{SB}) + \frac{1}{2} [d_1(a_{SB}, e_{SB}) − d_1(a_{SB}, 0)] + \frac{\partial e_{SB}}{\partial a} = 1. \] (4)

3.2 Manager ownership

In this case, the manager could unilaterally adopt his own e, without going through any bargaining. Thus, given a, the manager chooses \( e = e_{SM}(a) \), and the FOC is

\[ d_2(a, e_{SM}(a)) = 1. \] (5)

\(^3\)To follow Bennett and Iossa, we assume that the owner cannot adopt the investment without the investor’s consent. This is in contrast to Aghion and Tirole (1994), where it is assumed that once an innovation has been made, it can be used by the owner.
After differentiation, we obtain \( \frac{\partial e_{IC}}{\partial a} = -\frac{d_{21}(\cdot\cdot)}{d_{22}(\cdot\cdot)} \), which is positive (negative) if there is task complementarity (substitutability).

The builder and the manager will bargain over the adoption of \( a \), however. If \( a \) is not adopted, the builder will gain nothing except the default contract payment and the manager will have continuing payoff \([d(0, e_{SM}(0)) - e_{SM}(0)]\). If \( a \) is adopted, the total surplus for the builder and the manager will be \([t(a) + d(a, e_{SM}(a)) - e_{SM}(a)]\). Therefore, under Nash bargaining solution, the builder’s ex post payoff is

\[
f_b = \frac{1}{2} [t(a) + d(a, e_{SM}(a)) - e_{SM}(a) - d(0, e_{SM}(0)) + e_{SM}(0)] - a,
\]

Foreseeing this, the builder maximizes \( f_b \) subject to (5) by choosing \( a = a_{SM} \), and the FOC is

\[
\frac{1}{2} [t'(a_{SM}) + d_1(a_{SM}, e_{SM})] = 1. \tag{6}
\]

### 3.3 Consortium ownership

Now that a consortium is in charge of investing both \( a \) and \( e \) and also owns the facility, he could adopt the investments of \( a \) and \( e \) without the government’s approval.\(^4\) According to A1, it is always profitable for him to adopt both two investments, and negotiation never happens. His payoff is \( f = t(a) + d(a, e) - a - e \). Thus, the FOC for choosing \( e = e_{IC}(a) \) is (5), the same as under SM, while the FOC for choosing \( a = a_{IC} \) is

\[
t'(a_{IC}) + d_1(a_{IC}, e_{IC}) = 1. \tag{7}
\]

### 3.4 Comparison

Under private ownership, there are three regimes to consider. The comparison between IC and SM is relatively easy because the FOCs for the choice of \( e \) are the same (both represented by (5)). The first implication is that, in case \( a \) and \( e \) exhibit complementarity, a regime that corresponds to a greater \( a \) also corresponds to a greater \( e \), and vice versa. We next examine which regime leads to a greater \( a \), without restricting to the case of

\(^4\)Following Bennett and Iossa, we assume that if the builder and manager form a consortium, they act as if a single person. Needless to say, this view of integration is different from the view expressed in Grossman and Hart (1986).
complementarity. Comparing their FOCs for a ((7) for IC and (6) for SM), we notice that the builder under SM has a smaller incentive than the consortium under IC in investing a. In particular, whereas the consortium under IC receives all of the marginal gain in residual value of the facility due to increase in a, the builder under SM receives only half of it. This suggests that IC always leads to a greater level of a. We thus obtain the following proposition (all proofs are relegated to the Appendix unless otherwise stated).

**Proposition 1** (i) Consortium ownership (IC) yields a greater a than manager ownership (SM). (ii) If a and e are complementary investments, IC also yields a greater e than SM.

The fact that SM is dominated by IC as far as a is concerned is supported by the prevalence of IC over SM in the real world. Because of this, we just need to compare regime IC and regime SB, while omitting the comparison between SM and SB.

**Proposition 2** Consortium ownership (IC) yields a greater a than does builder ownership (SB), if and only if, for all a ∈ [min{aIC, aSB}, max{aIC, aSB}],

\[ d_1(a, e_{IC}) > \frac{1}{2} [d_1(a, e_{SB}) - d_1(a, 0)] - \frac{d_{21}(a, e_{SB})}{d_{22}(a, e_{SB})}. \]  

(8)

According to (8), when \( d_{21}(\cdot) = 0 \), both \( (d_1(a, e_{SB}) - d_1(a, 0)) \) and \(-\frac{d_{21}(a, e_{SB})}{d_{22}(a, e_{SB})}\) are zero and IC leads to a greater a than SB if and only if \( d_1(\cdot) > 0 \). This result is indeed Proposition 1 and Lemma 1 in Bennett and Iossa, in which the choice of integration (separation) is preferred if and only if the building investment directly reduces (increases) the operational cost. Our proposition, however, is more general as it captures the role played by task interdependence measured by cross derivatives \( d_{12}(a, e) \) or \( d_{21}(a, e) \). According to (8), if \( d_{12}(a, e) > 0 \) (task complementarity), both \(-\frac{d_{21}(\cdot)}{d_{22}(\cdot)}\) and \( \frac{1}{2} (d_1(a, e_{SB}) - d_1(a, 0)) \) are positive; that means that, for integration to be the optimal regime, \( d_1(a, e_{IC}) \) has to be not only positive but also sufficiently large. On the contrary, if \( d_{12}(a, e) < 0 \) (task substitutability), both \(-\frac{d_{21}(\cdot)}{d_{22}(\cdot)}\) and \( \frac{1}{2} (d_1(a, e_{SB}) - d_1(a, 0)) \) are negative; i.e., for integration to lead to a greater a, \( d_1(a, e_{IC}) \) need not be positive, it suffices that it is not too negative.

The above analysis suggests that, as far as a is concerned, task complementarity favors some kind of unbundling (in the form of SB) against bundling. The intuition is as
follows. In case of SB, the builder could bargain with the manager. After the bargaining, the former party could share the benefits generated by the manager’s investment, while not bearing any cost incurred by such investment. Because of complementarity, a higher building investment leads to a higher operating investment, yielding a greater net surplus to be split. Anticipating more rents to be extracted from the manager’s investment, the builder has a greater incentive to invest. As a result, investment complementarity helps mitigate the builder’s under-investment problem. In the case of IC, on the contrary, when investing in the building stage, the consortium will internalize not only the benefits but also the costs of subsequent investment, resulting in a dampened investment incentive on his part. Because task complementarity can be viewed as a special kind of positive externality, this result sheds new, somewhat counter-intuitive, light to the issue.

One can make a strong observation regarding the comparison between IC and SB. Consider the case of complementarity. Suppose \( a_{IC} > a_{SB} \) (because of sufficiently large positive externality). If follows that \( e_{IC} > e_{SB} \) for two reasons. First, complementarity of \( a \) and \( e \) suggests that the optimal choice of \( e \) is increasing in \( a \); second, the incentive to invest \( e \) is stronger under IC than under SB. These two features reinforce each other, making \( e_{IC} > e_{SB} \). Notice that, however, according to Proposition 2, the positive externality must be strong enough. Absent such strong enough positive externality, we do not obtain \( a_{IC} > a_{SB} \) and in that case we cannot say too much about \( e_{IC} \) and \( e_{SB} \).

**Corollary 1** Suppose \( d_{12}(a, e) > 0 \). Relative to builder ownership (SB), consortium ownership (IC) yields greater \( a \) and \( e \) only if there exists strong enough positive externality.

**Proof.** Omitted.

The case of substitutability is more difficult. The reason is that whenever \( a \) is higher in one regime that in the other, substitutability between \( a \) and \( e \) will lead to a lower \( e \) in the former regime than in the latter, other things being equal. It is thus difficult to determine when one regime dominates the other in both \( a \) and \( e \).
4 Public ownership

4.1 Separation and public ownership

In this case, two different agents undertake building and operation, and the government owns the facility. Then the government will negotiate with the two agents separately, with respect to the adoption of \( a \) and \( e \).\(^5\) We first consider the bargaining with the manager. In case \( e \) is not adopted, the government gains nothing except the default values at this stage, and the manager ends up with \( d(a,0) \) due to the reduction in operational costs. Hence, the net surplus arising from the successful adoption of \( e \) is \([v(e) + d(a,e) - d(a,0)]\).

Using the Nash bargaining solution, we reckon the manager’s ex post payoff to be \( f_m = \frac{1}{2} [v(e) + d(a,e) + d(a,0)] - c \).

Thus, given \( a \), the manager maximizes \( f_m \) by choosing \( e = e_{SP}(a) \), and the FOC is

\[
\frac{1}{2} [v'(e_{SP}(a)) + d_2(a, e_{SP}(a))] = 1. \tag{9}
\]

After total differentiation, we obtain

\[
\frac{\partial e_{SP}}{\partial a} = -\frac{d_{21}(a, e_{SP})}{v'(e_{SP}) + d_{22}(a, e_{SP})}, \tag{10}
\]

which is positive (negative) in case of complementarity (substitutability). Next, we notice that, given adopted \( a \), the government’s gain in the operating stage (denoted as \( g_2(a) \)) is

\[
g_2(a) = \frac{1}{2} [v(e_{SP}(a)) + d(a, e_{SP}(a)) - d(a,0)].
\]

We then come back to consider the government’s bargaining with the builder. In case \( a \) is not adopted, she would end up with a continuing payoff of \( g_2(0) = \frac{1}{2} [v(e_{SP}(0)) + d(0, e_{SP}(0))] \), and the builder ends up with nothing except the default contract payment. The net surplus generated by the adoption of \( a \) is thus \([w(a) + t(a) + g_2(a) - g_2(0)]\). Through Nash

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\(^5\) We assume that the government’s payoff is equal to the social benefit minus payments made to the firm(s), plus the residual value if she is also the owner. In this regard, her objective is different from a benevolent social planner in typical welfare analysis.
bargaining, the builder’s expected payoff is

\[ f_b = \frac{1}{2} [u(a) + t(a) + g_2(a) - g_2(0)] - a \]

\[ = \frac{1}{2} [u(a) + t(a)] + \frac{1}{4} [v(e_{SP}(a)) + d(a, e_{SP}(a)) - d(a, 0) - v(e_{SP}(0)) - d(0, e_{SP}(0))] - a. \]

Foreseeing the bargaining outcome, the builder maximizes \( f_b \) subject to (9) by choosing \( a = a_{SP} \), and the FOC is

\[ \frac{1}{2} [u'(a_{SP}) + t'(a_{SP})] + \frac{1}{4} [d_1(a_{SP}, e_{SP}) - d_1(a_{SP}, 0)] + \frac{1}{2} \frac{\partial e_{SP}}{\partial a} = 1. \] (11)

### 4.2 Integration and public ownership

Suppose that a consortium takes charge of building and operation under the government’s ownership. There will be negotiation in both stages. We first consider the bargaining on the adoption of \( e \). In case \( e \) is not adopted, the government gains nothing except the default values in the operating stage, and the consortium gets \( d(a, 0) \). The net surplus arising from the adoption of \( e \) is thus \( v(e) + d(a, e) - d(a, 0) \). Through Nash bargaining solution, the consortium gets \( \frac{1}{2} [v(e) + d(a, e) + d(a, 0)] - e \), and the government gets \( \frac{1}{2} [v(e) + d(a, e) - d(a, 0)] \). Thus, given \( a \), the consortium’s optimal choice of \( e = e_{IP}(a) \) satisfies \( (9) \), as under \( SP \).

We next examine the bargaining on the adoption of \( a \). In case \( a \) is not adopted, the consortium ends up with a continuing payoff of \( \frac{1}{2} [v(e_{IP}(0)) + d(0, e_{IP}(0))] - e_{IP}(0) \), and the government ends up with a continuing payoff of \( \frac{1}{2} [v(e_{IP}(0)) + d(0, e_{IP}(0))] \). If \( a \) is successfully adopted, the total surplus generated is

\[ u(a) + t(a) + d(a, e_{IP}(a)) + v(e_{IP}(a)) - e_{IP}(a). \]

Hence, taken into account Nash bargaining, the consortium’s ex post payoff is

\[ f = \frac{1}{2} [u(a) + t(a) + d(a, e_{IP}(a)) + v(e_{IP}(a)) - e_{IP}(a) - e_{IP}(0)] - a. \]
Maximizing $f$ by choosing $a = a_{IP}$ subject to (9) gives the following FOC:

$$\frac{1}{2} [u'(a_{IP}) + t'(a_{IP}) + d_1(a_{IP}, e_{IP})] + \frac{1}{2} \frac{\partial e_{IP}}{\partial a} = 1. \quad (12)$$

4.3 Comparison

There are only two regimes to study under public ownership, $IP$ and $SP$. We first notice that the FOCs for $e$ under the two regimes are indeed identical, both represented by (9). So to compare the two regimes, we just need to focus on the choice of $a$. The following proposition summarizes the result.

**Proposition 3** Under public ownership, integration ($IP$) leads to a greater $a$ than does separation ($SP$), if and only if, for all $a \in [\min \{a_{IP}, a_{SP}\}, \max \{a_{IP}, a_{SP}\}]$,

$$d_1(a, e_{IP}) > \frac{1}{2} [d_1(a, e_{SP}) - d_1(a, 0)]. \quad (13)$$

According to (13), if $d_{12}(a, e) > 0$ (task complementarity), $\frac{1}{2} [d_1(a, e_{SP}) - d_1(a, 0)]$ are positive; it means that, for integration to be the optimal regime, $d_1(a, e_{IP})$ should be not only positive but also sufficiently large. On the contrary, if $d_{12}(a, e) < 0$ (task substitutability), the term $\frac{1}{2} [d_1(a, e_{SP}) - d_1(a, 0)]$ is negative; for integration to be the optimal regime, $d_1(a, e_{IP})$ need not be positive; it suffices that it is not too negative. Therefore, as for the impacts of task interdependence on the bundling decision, the intuition in case of public ownership is similar to that in case of private ownership.

It is useful to notice the difference with Chen and Chiu though. In their model, since investment $e$ is assumed to be contractible by following the investment $a$, $SP$ and $IP$ happen to make the same investment choices. In other words, task interdependence plays no role at all in the comparison between the two public ownership regimes. The difference between their result and ours is driven by two forces. First, the interim contractibility of operating investment deprives the bargaining power of the manager in case of unbundling. Whenever bargaining takes place, the total surplus generated by the building investment is always shared between the two parties: the government and the builder under $SP$ or the government and the consortium under $IP$. However, in this model, bargainings happen sequentially among the three parties, including the manager. Second, in Chen and Chiu,
the bargaining in the operating stage is triggered by the interim contractibility of $e$, so it happens before $e$ is invested, while in our non-contractability setting, the bargaining on $e$ happens ex post regarding its implementation.

Restricting to task complementarity, we obtain the following result.

**Corollary 2** Suppose $d_{12}(a,e) > 0$. Under public ownership, separation (SP) yields greater $a$ and $e$ than integration (IP) if there exists negative externality or there is positive but small enough externality.

**Proof.** Omitted. □

5 The optimal regime

5.1 Underinvestment of $a$ and $e$

The three regimes under private ownership are $SB, SM$, and $IC$, and their decisions on $a$ and $e$ are given by (4) and (3), (6) and (5), and (7) and (5), respectively. Notice that, whereas neither $u(.)$ nor $v(.)$ appears in these conditions, both of them appear in the first best problem (see (1) and (2)). As a result, when the scale measure of social benefit, $\alpha$ and $\beta$, are large enough, both $a$ and $e$ are under-invested under any private ownership regime.

For public ownership, the choices of $a$ and $e$ are responsive, but only partially, to $u(.)$ and $v(.)$ (comparing (11) and (9) for $SP$, (12) and (9) for $IP$ to (1) and (2) for the first-best problem). Thus, there still exist underinvestments of $a$ and $e$ when $\alpha$ and $\beta$ are large enough. The following lemma is thus obtained.

**Lemma 1** When $\alpha$ and $\beta$ are sufficiently large, there are under-investments of $a$ and $e$ in all regimes.

**Proof.** Omitted. □

This lemma justifies our exercise of comparing different regimes in terms of $a$; i.e., when one regime is found to correspond to a greater level of $a$, this regime is indeed a more efficient regime as far as the level of $a$ is concerned.
5.2 PPP versus traditional procurement

In previous real world practice, multiple stages involved in a public project were often organized through contracting out to separate firms, so this arrangement is often referred to as traditional procurement. However, recently, the Private Finance Initiative (PFI), a pioneer form of PPP launched in the UK, becomes a new trend in public service provision. According to Iossa and Martimort (2008), PPPs are being used across Europe, North American and many developing countries, covering the sectors of transport, energy, water, prisons, military training, waste management, schools and hospitals etc. So we want to understand whether the traditional procurement is still useful or should be replaced by PPPs with little qualifications. The following proposition compares PPP (i.e., the IC regime) versus traditional procurement (i.e., SP regime).

Proposition 4 PPP (IC) leads to a greater than does traditional procurement (SP), if and only if, for all \( a \in [\min \{a_{IC}, a_{SP}\}, \ max \{a_{IC}, a_{SP}\}] \)

\[
\frac{1}{2} [w'(a) - t'(a)] - d_1(a, e_{IC}(a)) \leq \frac{1}{4} [d_1(a, 0) - d_3(a, e_{SP}(a))] + \frac{1}{2} \frac{d_{21}(a, e_{SP}(a))}{v''(e_{SP}(a)) + d_{22}(a, e_{SP}(a))}.
\]

Proof. The proof is similar to those of Propositions 2 and 3 and is omitted. ■

According to the left hand side, IC is more likely to dominate SP in terms of \( a \) if the social benefit associated with the building task is smaller, the residual value of the project is greater, and the externality between \( a \) and \( e \) is more positive. This is the insight obtains under Bennett and Iossa. However, what is new here is the right hand side (RHS) which depends on task interdependence. More specifically, both terms in the RHS are negative, positive, and zero in the presence of complementarity, substitutability, no interdependence, respectively. Thus complementarity (substitutability) makes it more difficult for the condition to hold, i.e., more difficult for PPP to dominate traditional procurement. This insight which appears recurrently in this paper continues to hold here, thus suggesting stronger and more general justification for the use of traditional procurement.

\(^6\)One may ask stronger questions as to when IC dominates all other regimes (rather than just SP) and when SP dominates all other regimes (rather than just IC). We believe that the result is probably not enlightened enough given the space limitation.
The next corollary gives conditions under which either regime dominates the other in both $a$ and $e$, focusing on the case of task complementarity.

**Corollary 3** Suppose $d_{12}(a, e) > 0$.

1. PPP (IC) leads to greater $a$ and $e$ than does traditional procurement (SP) if (i) (14) holds for all $a \in [\min \{a_{IC}, a_{SP}\}, \max \{a_{IC}, a_{SP}\}]$ and (ii) $\beta$ is sufficiently small.

2. Traditional procurement (SP) leads a greater $a$ and $e$ than PPP (IC) does if (i) (14) holds with "<" for all $a \in [\min \{a_{IC}, a_{SP}\}, \max \{a_{IC}, a_{SP}\}]$ and (ii) $\beta$ is sufficiently large.

The corollary is easy to understand. Consider the first result. Condition (i) is to ensure that $a_{IC} > a_{SP}$, which is just a restatement of Proposition 4. In terms of $e$, there is a trade-off under the two regimes. The consortium under IC obtains the full marginal private benefit of $e$. While the manager under SP obtains half of such marginal private benefit, it also obtains the half of the social benefit associated with the investment. As a result, when this social benefit is small (i.e., $\beta$ is small), IC is the regime that provides stronger incentive for the investment. Condition (ii) is to ensure that $e_{IC} > e_{SP}$, which is easily shown under complementarity and under the condition that $a_{IC} > a_{SP}$. A similar result under task substitutability is more difficult to establish. The second result of the corollary is understood in the same manner.

Thus, according to Corollary 3, the usefulness of PPP is more limited as predicted by our model than by others. Ignoring task interdependence in their analysis, few existent studies are aware that task complementarity, another kind of positive externality, performs as a detriment of the desirability of PPP. Given abundant real life projects demonstrating task complementarity, practice might be too optimistic by following the fashion of adopting PPP regime and its suitability might be overstated in public good provision.

## 6 Concluding remarks

Literature suggests that traditional procurement is desirable only if there exists negative externality across the project phases (see, e.g., Bennett and Iossa, and Martimort and
Pouyet). However, as pointed out by Iossa and Martimort, evidence of negative externalities is difficult to find. Alternatively, the prediction of our model relates the desirable role of traditional procurement to task complementarity, which seems more plausible. A well-designed and well-built facility would not by itself increase the operational costs; it may instead demand more efforts from the manager to engage in careful maintenance, since breakdown of these novel facilities may cost more for repairs.

According to a report by the Audit Commission, the quality of PFI schools is undesirable; particularly, they have few windows, poor acoustic and air quality, compared to traditionally procured schools. We cannot attribute this phenomenon to negative externality, because the improvement of school quality does not directly increase the maintenance cost; instead, it heightens the importance of the school manager’s work, and any failure to control vandalism causes more valuable school assets to be damaged. Hence, investment on cost reduction will be crowded in by investment on quality enhancing. Traditional procurement, relieving the solo builder of such concern, may take the advantage of restoring the builder’s incentive on quality innovation.

On the other hand, the desirability of PPP could be rationalized by task substitutability. Arthur Andersen and LSE (2000) estimate that significant cost savings were realized in the prison sector. As we illustrated construction of a prison before, positive externality and task substitutability coexist here. According to the National Audit Office (2003), "innovative design solutions helped to reduce the level of 'staffing' needed to ensure security and this resulted in an overall cost reduction by approximately 30%.” Preferably, we translate "less staffing" into less necessity of cost-reduction effort in the operating stage, so it might be crowded out by effort on quality innovation, due to task substitutability.
References


Appendix

Proof of Proposition 1

Proof. To show the first result, we notice that when \( a = a_{SM} \) is chosen under IC, the subsequent \( e \) to be chosen will be \( e_{SM} \). Substituting \( (a_{SM}, e_{SM}) \) into the first order condition for the IC problem (7), we realize that the marginal benefit of investing \( a \) is two times the marginal cost of investing. As a result, the consortium finds it beneficial to expand \( a \), and it follows that \( a_{IC} \) is indeed greater than \( a_{SM} \). The second result comes from the facts that both IC and SM have the same FOC for \( e \) and that, in this FOC, because of complementarity, the choice of \( e \) is positively related to \( a \). ■

Proof of Proposition 2

Proof. Suppose that \( a_{IC} > a_{SB} \). Notice that all the maximization problems are concave (second-order conditions are satisfied). Then, making use of (7), we know that, for \( a \in [a_{SB}, a_{IC}] \),
\[
t'(a) + d_1(a, e_{IC}(a)) \geq 1,
\]
where the equality holds only when \( a = a_{IC} \); making use of (4), we know that, for all \( a \in [a_{SB}, a_{IC}] \),
\[
t'(a) + \frac{1}{2} [d_1(a, e_{SB}(a)) - d_1(a, 0)] + \frac{\partial e_{SB}(a)}{\partial a} \leq 1,
\]
where the equality holds only when \( a = a_{SB} \). Therefore, subtracting the second inequality from the first and rearranging, we obtain result:
\[
a_{IC} > a_{SB} \Rightarrow d_1(a, e_{IC}(a)) - \frac{1}{2} [d_1(a, e_{SB}(a)) - d_1(a, 0)] - \frac{\partial e_{SB}(a)}{\partial a} > 0 \text{ for all } a \in [a_{SB}, a_{IC}].
\]
(15)

Suppose that \( a_{IC} < a_{SB} \). The analysis is similar. Using (7) and (4), we know that, for \( a \in [a_{IC}, a_{SB}] \),
\[
t'(a) + d_1(a, e_{IC}(a)) \leq 1,
\]
where the equality holds only when \( a = a_{IC} \); and, for all \( a \in [a_{IC}, a_{SB}] \),

\[
t'(a) + \frac{1}{2} \left[ d_1(a, e_{SB}(a)) - d_1(a, 0) \right] + \frac{\partial e_{SB}(a)}{\partial a} \geq 1,
\]

where the equality holds only when \( a = a_{SB} \). Using these two inequalities, we obtain:

\[
a_{IC} < a_{SB} \Rightarrow d_1(a, e_{IC}(a)) - \frac{1}{2} \left[ d_1(a, e_{SB}(a)) - d_1(a, 0) \right] - \frac{\partial e_{SB}(a)}{\partial a} < 0 \text{ for all } a \in [a_{IC}, a_{SB}] .
\]

(16)

Combining (15) and (16) and making use of (3), we show the claimed result. \( \square \)

**Proof of Proposition 3**

**Proof.** Suppose that \( a_{IP} > a_{SP} \). Notice that all the second-order conditions are satisfied. Then making use of (12), we know that, for \( a \in [a_{SP}, a_{IP}] \),

\[
\frac{1}{2} [u'(a) + t'(a) + d_1(a, e_{IP}(a))] + \frac{1}{2} \frac{\partial e_{IP}(a)}{\partial a} \geq 1,
\]

where the equality holds only when \( a = a_{IP} \); from (11), we know that, for \( a \in [a_{SP}, a_{IP}] \),

\[
\frac{1}{2} [u'(a) + t'(a)] + \frac{1}{4} \left[ d_1(a, e_{SP}(a)) - d_1(a, 0) \right] + \frac{1}{2} \frac{\partial e_{SP}(a)}{\partial a} \leq 1,
\]

where the equality holds only when \( a = a_{SP} \). Subtracting the second inequality from the first, making use of the fact that \( \frac{\partial e_{IP}(a)}{\partial a} = \frac{\partial e_{SP}(a)}{\partial a} \) (\( e_{IP} \) and \( e_{SP} \) share the same FOC (i.e., (9)), and rearranging, we obtain:

\[
a_{IP} > a_{SP} \Rightarrow d_1(a, e_{IP}(a)) > \frac{1}{2} \left[ d_1(a, e_{SP}(a)) - d_1(a, 0) \right] \text{ for all } a \in [a_{SP}, a_{IP}] .
\]

(17)

Suppose \( a_{IP} < a_{SP} \). Using the same procedure, we know that, for \( a \in [a_{IP}, a_{SP}] \),

\[
\frac{1}{2} [u'(a) + t'(a) + d_1(a, e_{IP}(a))] + \frac{1}{2} \frac{\partial e_{IP}(a)}{\partial a} \leq 1,
\]

where the equality holds only when \( a = a_{IP} \); and, for \( a \in [a_{IP}, a_{SP}] \),

\[
a_{IP} < a_{SP} \Rightarrow \frac{1}{2} [u'(a) + t'(a)] + \frac{1}{4} \left[ d_1(a, e_{SP}(a)) - d_1(a, 0) \right] + \frac{1}{2} \frac{\partial e_{SP}(a)}{\partial a} \geq 1,
\]

(18)
where the equality holds only when \( a = a_{SP} \). Using these two inequalities, we obtain:

\[
d_1(a, e_{IP}(a)) < \frac{1}{2} [d_1(a, e_{SP}(a)) - d_1(a, 0)] \text{ for all } a \in [a_{IP}, a_{SP}].
\]

Combining (17) and (18), we show the claimed result. ■

Proof of Corollary 3

Proof. We show (1) first. Condition (i) is to ensure that \( a_{IC} > a_{SP} \) and is evidently true from Proposition 5. Condition (ii) is to ensure that \( e_{IC} > e_{SP} \). Define \( \beta^* \) which satisfies \( \beta^* V'(e_{IC}) = d_2(a_{IC}, e_{IC}) \). Since both \( a_{IC} \) and \( e_{IC} \), determined in (5) and (7), are independent of \( \beta \), the above definition of \( \beta^* \) is well-defined. Given any \( \beta < \beta^* \), consider the investment of \( e \) under \( SP \) (see equation (9)). The marginal benefit of investing \( e = e_{IC} \) equals

\[
\frac{1}{2} (\beta V'(e_{IC}) + d_2(a_{SP}, e_{IC})) < \frac{1}{2} (\beta^* V'(e_{IC}) + d_2(a_{IC}, e_{IC}))
\]

\[
= d_2(a_{IC}, e_{IC})
\]

\[
= 1,
\]

where the first line is due to \( \beta < \beta^* \), \( a_{SP} < a_{IC} \) ensured by (i), complementarity between \( a \) and \( e \); the second line due to the definition of \( \beta^* \); and the third line due to (5). As the marginal benefit is lower than the marginal cost, it follows that the \( e \) chosen under \( SP \) will be smaller than \( e_{IC} \), i.e., \( e_{SP} < e_{IC} \).

We show (2) here. Condition (i) is to ensure that \( a_{SP} > a_{IC} \) and is evidently true from Proposition 5. Condition (ii) is to ensure that \( e_{SP} > e_{IC} \). Use the same cutoff \( \beta^* \) as defined earlier. Given any \( \beta > \beta^* \), consider the investment of \( e \) under \( SP \) (see equation (9)). The marginal benefit of investing \( e \) equals

\[
\frac{1}{2} (\beta V'(e_{IC}) + d_2(a_{SP}, e_{IC})) > \frac{1}{2} (\beta^* V'(e_{IC}) + d_2(a_{IC}, e_{IC}))
\]

\[
= d_2(a_{IC}, e_{IC})
\]

\[
= 1,
\]
where the first line is due to $\beta > \beta^*$, $a_{SP} > a_{IC}$ ensured by (i), complementarity between $a$ and $e$; the second line due to the definition of $\beta^*$; and the third line due to (5). As the marginal benefit exceeds the marginal cost, it follows that the $e$ chosen under $SP$ will exceed $e_{IC}$, i.e., $e_{SP} > e_{IC}$. This completes the proof. ■
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Table 1: BI stands for the results in Bennett and Iossa (2006) while CC the results in the current paper.
Figure 1: The time line

regime is chosen investing \( a \) possible bargaining on the implementation of \( a \) investing \( e \) possible bargaining on the implementation of \( e \) payoffs are realized

0 1 1.5 2 2.5 3

25