Endogenous Preferential Treatment in Centralized Admissions

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Abstract

We study a model of centralized admissions in which schools are allowed to pre-commit to admitting qualified applicants who rank them as their top choices over more qualified applicants who do not. A less popular school may use the pre-commitment to steal applicants who otherwise would not choose it as their top choice (the stealing motive); a popular school may use the pre-commitment to prevent its own applicants from being stolen (the preemptive motive). We identify the conditions for these two motives to exist. We also clarify the relationship of this phenomenon with that of pre-arrangement of school places.
"We want the students that want us most!"

– a New York City educator


1 Introduction

A significant number of school and college places in the world are allocated through centralized admissions schemes. Instead of filing separate applications to different schools, students need to file only a single application (a rank-order list) through which they indicate their preferences for schools; schools then allocate their places in accordance with a strict protocol, taking into account applicants’ rank-order lists as well as all other available information. We are interested in a phenomenon in which a school gives preferential treatment to applicants who rank it as their top choice over more qualified applicants who do not.

Such preferential treatment may be an intrinsic feature of the admissions scheme being used. For instance, in the so-called Boston admissions scheme, first discussed by Abdulkadirğlu and Sönmez (2003), school places are first assigned according to applicants’ top choices. This means that an applicant’s failing to get into her top-choice school will also fail in getting into her second-choice school if the second-choice school has already filled up its places with applicants ranking it as their top choice, even though the applicant may be a much better student than many of the accepted applicants. (Regarding the old Boston scheme, also see Ergin and Sönmez 2006; Pathak and Sönmez 2008; and Abdulkadirğlu, Pathak, Roth, and Sönmez 2005, 2006.)

Our article, however, is concerned about preferential treatment that is a deliberate choice made by the school. One notable example is the system used in New York City prior to its reform in 2003 (see Abdulkadirğlu, Pathak and Roth 2005). The emphasis of preferential treatment by some New York City schools is succinctly summarized by the complaint quoted at the beginning of this article by an administrator that the new admissions scheme no longer contained information on how each school was ranked by each applicant.1

1We would like to thank Atila Abdulkadirğlu for pointing out this source to us.
There are two potential reasons for such phenomenon, which we call endogenous preferential treatment. First, a school may want to admit those who are genuinely more enthusiastic about it, because presumably students’ motivations and commitments are important to success in their studies. In contrast with this motivational consideration, a school may adopt the policy out of a strategic consideration. If preferential treatment helps sway applicants to rank a school as their top choice and if this effect is strong enough, the school may turn out admitting a better group of students. Thus, it is in the interest of the school to adopt the practise despite no intrinsic interest in each applicant’s ranking of schools. In this article, we explore under what circumstances this latter strategic consideration is sensible.

In Section 2, we present our baseline model with two schools. Starting with the Gale-Shapley mechanism (Gale and Shapley 1962), we allow schools to pre-commit to admitting qualified applicants who rank them as their top choices. We call this policy the immediate acceptance (IA) policy, in contrast with the deferred acceptance (DA) policy inherent in the Gale-Shapley mechanism. More specifically, we consider a three-stage game. First, schools—or school place offering units—announce publicly if they pre-commit to an IA policy. Then, applicants submit their rank-order lists, followed by the assignment of places in accordance with the Gale-Shapley mechanism subject to constraints of schools’ pre-commitments. Throughout the article, we assume that, albeit strategic, schools are honest and trustworthy.

Section 2 ends with a neutrality result, which roughly says the following. So long as the outcome in the game in which all schools have DA policies is deterministic and commonly known, no schools would gain from adopting IA. Exactly the same outcome, in terms of who enters what school, would result. Simple as it seems, this result suggests that uncertainty is essential for understanding the role of IA behind its strategic use. In light of this, we study in Sections 3 and 4 two types of uncertainty: demand uncertainty and ranking uncertainty.

In the demand uncertainty model, applicants are uncertain about others’ genuine demands, i.e., others’ preferences for places at different schools. This captures scenarios in which applicants may not be certain of the latest fads regarding whether computer science or business studies, say, is popular this year. In the ranking uncertainty model, applicants do not know how well they are ranked among themselves. This model captures the scenario
in which applicants are required to apply before learning their examination results or even before taking examinations.

Through the analysis, we identify two motives for pre-commitment. A school may use IA to steal applicants who otherwise would not have made it their top choice (the *stealing motive*). A less popular school is more likely to have a stealing motive when it is not too inferior to a more popular school in terms of the utility that enrolled students can obtain. On the other hand, rarely will a popular school have this motive. Under more restricted conditions, the popular school does have a motive to use IA. Although not helping to steal applicants, the IA is attractive because it can prevent the less popular school from using IA and thereby protecting the popular school from being hurt (the *preemptive motive*).

We show the conditions for such motives to exist and relate them to school popularity, differences in schools’ preferences, and relative school size. We also find that being able to influence applicants’ behaviors need not imply an improvement in the school’s average student quality. IA can be influential yet self defeating.

Our article falls into the literature that studies manipulation in matching and market design. Despite the very desirable property of stability, later work shows that the deferred acceptance mechanism introduced by Gale and Shapley is susceptible to manipulations prior to and during the match. In the mechanism in which students propose, schools may have an incentive to misrepresent their preferences (Dubins and Freedman 1981; Roth 1982; Roth and Sotomayor 1990), to misreport their capacities (Sönmez 1997), or to have pre-arrangements with students prior to the admissions exercise (Sönmez 1999).

The new manipulation of admissions policy that we identify differs from the pre-arrangement policy in that it takes place *during* the matching exercise, rather than prior to it. As will be clear below, pre-arrangement does replicate the effect of the IA policy that we study, provided that the school engages in unrestrained pre-arrangement, ready to fill up all its places for this purpose—a condition unlikely to occur in many real world applications. This suggests that the studies of pre-arrangement given the extent of its use that we actually observe is no substitute for the studies of manipulation that takes place during the centralized admissions exercise. Focusing on the existence of stable matching rules that are free of
manipulation of pre-arrangement, Sönmez (1999) assumes that in the centralized exercise agents report their preferences truthfully. In the present article, however, we focus on equilibrium behavior subsequent to admissions policy choices. As such we are able to obtain the neutrality result, implying that a similar neutrality result also holds under unrestrained pre-arrangement and that, for pre-arrangement to matter, some kind of uncertainty is essential. There is a counterpart literature for decentralized markets, known as early contracting (see Li and Rosen 1998, Roth and Xing 1994, and Suen 2000). One market in which there was a great deal of endogenous preferential treatment was the decentralized market for clinical psychologists (Roth and Xing 1997). And of course many American colleges have what is called "binding early decision." So it may be as big an issue in decentralized as in centralized markets.

Section 5 extends our analysis to a framework of three schools, generating additional insights. As most of our analysis is conducted in an environment of a few schools, however, how likely a school may benefit from endogenous preferential treatment is an open question. Generalizing Roth and Peranson (1999) and Immorlica and Mahdian (2005), Kojima and Pathak (forthcoming) argue that under reasonable assumptions the fraction of schools that can benefit from manipulations in the form of misreporting of preferences, misreporting of capacities, or pre-arrangement will become negligible as the matching market grows larger. A similar result may apply to the type of manipulation that we identify although it needs to be proved. Thus our contribution is to identify manipulation motives rather than to assess their likelihood in large markets. By analyzing endogenous preferential treatment that was a feature of the old New York City system, our article complements the literature that attempts to understand real-world school choice mechanisms and their reforms.2

Our study is also relevant to other places where centralized admissions systems are being used. It is common for commonwealth countries to assign university places using such centralized systems.3 China's national college admissions system allocates a few million

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2 Abdulkadiroğlu, Pathak and Roth (forthcoming) deal with the reforms of the New York City system, in which they focus on the role of the tie-breaking rule (also see Erdil and Ergin 2008). Mechanisms like those of Gale and Shapley have been put into practice in other markets (see Roth 1984; Roth and Peranson 1999 for the resident program in the US).

3 In Hong Kong, where a centralized admissions system for college places has been in place for two decades, anecdote has it that programmes entice students to rank them as their top choices by committing to giving
college places every year (Table 21-2, Chinese Statistical Yearbook, 2006). At present, more than half of the provinces are still using the traditional Hierarchical Choice Ranking Scheme (*dengji zhiguan*). In each such province, the provincial admissions office sends to each university – within or without the province – a short list of students exceeding by 20% the university’s quota allotted to the province. These 120% students are strictly chosen, in accordance with public exam scores, among those students ranking the university, or some of its programs, as their top choice. Short-listed students who are declined by the university may be subsequently declined by universities that they ranked as their second choice, third choice, etc., simply because their places have all been filled up. This is a case of intrinsic preferential treatment similar to that found in the old Boston system. Notice that, in case of insufficient first-choice students, the short list provided by the provincial admissions office will contain students who ranked the university, or some of its programs, as their second choice (and likewise third choice, fourth choice, etc.). These students are often treated disadvantageously: the university typically deduces some points from their public exam scores simply because they have not ranked the university as their top choice. Such a deliberate policy of the university is a practise of endogenous preferential treatment that our article is concerned about. All in all, we have the feeling that in the real world concerns about endogenous preferential treatment are not unwarranted.

### 2 The Model

A mass of applicants of size $N$ are competing to enter two schools. Each applicant is indexed by an ability attribute pair $(y_1, y_2)$, where $y_1$ may be her math score and $y_2$ her language score, for example. $(y_1, y_2)$’s are distributed over $[0, 1] \times [0, 1]$ with a cumulative function which is continuous and has no mass point. Each applicant’s ability pair, as well as the cumulative function, is commonly known, and as such she knows her exact ranking vis-a-vis the other applicants. Given attribute pair $(y_1, y_2)$, a fraction $\mu_i$ of applicants view

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4 Two remarks are in order here. First, each program of each university has a fixed, known quota for students for each province. Second, the university is free to choose the mechanism whereby students short listed by the provincial admissions office are assigned or not assigned to its various programs.
school 1 as the better school: the utility to these applicants of attending the two schools is, respectively, \( u_1 \) and \( u_2 \), where \( u_1 > u_2 > 0 \); a fraction, \( \mu_2 = 1 - \mu_1 \), of applicants view school 2 as better: the utility to these applicants of attending the two schools are \( v_1 \) and \( v_2 \), where \( 0 < v_1 < v_2 \). We call the former group of applicants type-1 applicants and the second group type-2 applicants.

The two schools have places, \( s_1 \) and \( s_2 \), where \( s_1 + s_2 < N \). We assume that school 1 is relatively more popular, i.e., \( \mu_1/s_1 > \mu_2/s_2 \). Occasionally, we will call school 1 the popular school and school 2 the less popular school. For \( i = 1, 2 \), school \( i \)'s preferences are represented by a parameter \( \alpha_i \geq 0 \). It strictly prefers applicant \( a \) with attribute pair \((y_1^a, y_2^a)\) over applicant \( b \) with attribute pair \((y_1^b, y_2^b)\) if and only if \( y_1^a + \alpha_i y_2^a < y_1^b + \alpha_i y_2^b \); hence, the smaller the attribute pair the better. We assume that, without loss of generality, \( \alpha_1 \leq \alpha_2 \), i.e., school 1 places more emphasis on attribute 1 than on attribute 2 as compared with school 2. When \( \alpha_1 = \alpha_2 \), the two schools have identical preferences; when \( \alpha_1 < \alpha_2 \), the two schools may differ on which of two applicants is better. School \( i \)'s welfare is measured by the average quality of student intakes, i.e.,

\[
\frac{\int_{[0,1] \times [0,1]} (y_1 + \alpha_i y_2) dH(y_1, y_2)}{\int_{[0,1] \times [0,1]} dH(y_1, y_2)},
\]

(1)

where \( H(y_1, y_2) \) is the cumulative function in which the intakes' attributes are distributed. Given two groups of intakes, characterized by cumulative functions \( H_1(y_1, y_2) \) and \( H_2(y_1, y_2) \), the school prefers the former group to the latter group if and only if the former group results in a better (lower) average quality for the school.

A centralized admissions scheme in the fashion of Gale and Shapley is in place; each applicant is required to submit a rank-order list of schools—indicating whether she regards school 1 as her first-choice and school 2 as her second choice or vice versa.\(^5\) Before the centralized admissions scheme actually operates, schools publicly announce which policy they use: a deferred acceptance (DA) policy or an immediate acceptance (IA) policy. Under

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\(^5\) Allowing applicants to receive multiple offers, the old New York City system that motivates our study is different from the Gale-Shapley mechanism. Nonetheless, we choose the latter mechanism as the baseline model because it is the most widely used one in the literature.
the former, the school ranks applicants based solely on their quality attributes and processes the applications strictly in accordance with the Gale-Shapley mechanism. Under the latter, Whereas still using the centralized admissions scheme, the school is committed to admitting applicants who specify it as their top choice up to its capacity. We assume that schools are honest so that the commitment is genuine and credible; this is so perhaps because schools are honest, or simply because reputation concerns are a strong enough enforcement device.\textsuperscript{6} The game is identical to the Gale-Shapley mechanism except that schools may pre-commit to an IA policy. We do not consider other kinds of preferential treatment because the IA policy that we study, the most extreme kind, presumably gives the sharpest insight.\textsuperscript{7}

We use a pair \((p_1, p_2)\), where \(p_1, p_2 = DA, IA\), to represent the admissions policies used by the two schools. Under \((DA, DA)\), there exists an equilibrium in which all type-1 applicants report school 1 as their top choice and all type-2 applicants report school 2 as their top choice. The two schools then admit according to the standard Gale-Shapley procedure. We call the strategies the truthful strategies, the equilibrium the \textit{truthful equilibrium}, and the outcome (in terms of who goes to what school) the \textit{truthful outcome}. Making use of (1), we use \(AQ_i(p_1, p_2)\) to denote school \(i\)'s equilibrium average quality under policy pair \((p_1, p_2)\), where \(i = 1, 2\), and \(p_1, p_2 \in \{DA, IA\}\). Payoffs and the structure of the game are commonly known.

The truthful outcome is easy to characterize. For \(i = 1, 2\), school \(i\) will set a cutoff standard, \(c_i\), so that all type-\(i\) applicants satisfying \(y_1 + \alpha_i y_2 \leq c_i\) are admitted and all type-\(j \neq i\) applicants satisfying this criterion yet not admitted by \(j\) are also admitted. Because school 2 is less popular, it always admits some type-1 applicants who fail to get into school 1. On the contrary, being more popular, school 1 may or may not admit any type-2 applicants.

\textsuperscript{6}For mechanisms that produce unstable matchings, there may be recontracting after the centralized procedure. However, in case pre-arrangement and under-reporting of capacity are successfully prohibited, re-contracting would be more difficult. Our own experience with HK’s system also suggests that reputation concerns do restrain re-contracting.

\textsuperscript{7}A weaker type of preferential treatment is as follows. Still using the original deferred acceptance rule, school \(i\) selects applicant \(a\) who ranks it as her top choice over applicant \(b\) who does not if and only if \(y_1^a + \alpha_i y_2^a < y_1^b + \alpha_i y_2^b + \beta_i\) where \(\beta_i > 0\) is a pre-announced choice variable of school \(i\).
A Neutrality Result

We first report a neutrality result (all proofs are relegated in Appendix A).

**Proposition 1** Under any policy pair \((p_1, p_2)\) where at least one school uses IA, the equilibrium outcome is unique and is the same as the truthful outcome.

Foreseeing the impact of the IA policy, every applicant who would be adversely affected will strategically rank the school that she would enter in the truthful outcome as her top choice; she knows what the school is under the complete information assumption. As such, she succeeds in getting into the school, and the effect of the IA policy is completely neutralized. It is clear that this constitutes an equilibrium. The uniqueness result is less straightforward. The following heuristic example, in which both school places and applicants are measured in discrete numbers, illustrates that uniqueness may not hold in general.

**Example 1** \(s_1 = s_2 = 2\) and there are four type-1 applicants and four type-2 applicants, denoted as \(a_1, a_2, a_3, a_4\) and \(b_1, b_2, b_3, b_4\), where the subscripts denote the quality of the student (the lower the number the better, and \(a_i\) and \(b_i\) are equally qualified for all \(i\)). In the truthful outcome, school 1 admits \(a_1\) and \(a_2\) and school 2 admits \(b_1\) and \(b_2\). This is still an equilibrium outcome under \((IA, IA)\). Provided that \(u_2/u_1 > 0.5, v_1/v_2 > 0.5\) and that an equal-choice tie-breaking rule is used, there also exists an equilibrium outcome in which school 1 admits \(a_1\) and \(b_2\) and school 2 admits \(b_1\) and \(a_2\), supported by strategies in which \(a_1\) and \(b_2\) report school 1 as their top choice and \(b_1\) and \(a_2\) report school 2 as their top choice.

In example 1, whereas there is a unique equilibrium outcome under \((DA, DA)\), there are multiple equilibrium outcomes under \((IA, IA)\): there exists another equilibrium outcome characterized by a coordination failure between \(a_2\) and \(b_2\). There are two points to notice. First, the new equilibrium outcome is sustained because of schools’ weak preferences over applicants (not satisfied in Proposition 1). Second, despite schools’ weak preferences, the outcome non-uniqueness is restricted to the two borderline applicants, but not more qualified ones. By ranking it as her top choice, \(a_1\) can enter school 1 and she surely will in any equilibrium; similarly, by ranking it as her top choice, \(b_1\) can enter school 2 and she surely...
will in any equilibrium. In general, through an iteration process – which will be frequently used in the later analysis – one can narrow down the set of equilibrium outcomes, and may obtain a unique outcome under favorable conditions.

Here we report a more general neutrality result, in which we allow more than two schools and impose no restrictions on the number of types of applicants in terms of their preferences.

Proposition 2 Consider an environment with complete information, strict preferences, discrete agents, and the number of schools greater than or equal to two. Suppose that all agents on one side are acceptable for all agents on the other side (better assigned than not) and that there are more applicants than school places. Let $p'$ be any policy profile in which at least one school uses IA, and $p$ be the one in which all schools use DA. Then all Nash equilibrium (NE) outcomes under $p$ are NE outcomes under $p'$. If NE outcomes under $p'$ are all stable, then the two sets of NE outcomes under $p$ and $p'$ coincide, equal to the set of all stable matchings.\(^8\)

That the truthful outcome, itself a stable matching, continues to be an equilibrium outcome when IA is used suggests that, without uncertainty, applicants are able to replicate the same outcome by optimally misreporting their preferences. We will next turn to two models of uncertainty that make IA relevant.

3 Demand Uncertainty

Here we assume that applicants are uncertain about the state of the world that determines applicants’ preferences. We call this a model of demand uncertainty. There are two states of the world, state 1 and state 2, with respective probabilities, $\pi_1$ and $\pi_2 \equiv 1 - \pi_1$. We assume that

$$\frac{\mu_1^1}{\mu_2^1} > \frac{\mu_1^2}{\mu_2^2} > \frac{s_1}{s_2},$$  \hspace{1cm} (2)\(^\text{\footnotesize{8}}\)

\(^8\)We conjecture that all Nash equilibrium outcomes under $p'$ are indeed stable. Ergin and Sönmez (2006) show that, in the Boston mechanism, the set of Nash equilibrium outcome is equal to the set of all stable matchings. It is interesting to show that it holds for a larger set of mechanisms.
where $\mu_i^k \in (0,1)$ is the fraction of applicants who in state $k$ prefers school $i$ to the other school. That is, school 1 is still more popular than school 2 under each state, but its popularity is less overwhelming under state 2. We study the same game described in the last section, except that, prior to the game, there is an additional stage in which the true state is realized (but not revealed to agents). We assume that schools are (von Neumann Morgenstern) expected average quality minimizers, i.e., school $i$’s objective is to minimize $AQ_i \equiv \pi_1 AQ_i^1 + \pi_2 AQ_i^2$, where $AQ_j^i$ is school $i$’s average quality in state $j$.

**Unilateral Incentive**

We now illustrate how demand uncertainty may enable IA to matter. To see this, we focus on the simplest case in which $\alpha_1 = \alpha_2 = 0$: both schools have identical preferences and do not care about the $y_2$ attribute. Denote by $F(.)$ the cumulative distribution function of $y_1$. The truthful equilibrium under (DA,DA) is characterized as follows. Type-1 applicants specify school $i$ as their top choice, $i = 1, 2$. Because of (2), school 1 will only admit type-1 applicants, setting cutoff values $c_1^1$ and $c_1^2$ in the two states, where $c_1^1 = F^{-1} (s_1/\mu_1^1 N) < c_1^2 = F^{-1} (s_1/\mu_2^1 N)$. School 2 sets the same cutoff value, $c_2 = F^{-1} ((s_1 + s_2)/N) > c_2^1$, in both states, admitting all type-2 applicants satisfying this condition as well as all type-1 applicants satisfying this condition who have been declined by school 1. Figure 1 illustrates these cutoffs, using a uniform distribution of $y_1$. We now study school 2’s incentive to use IA.

**Lemma 1** Under (DA,IA), in any equilibrium, (1) the reporting strategies of those applicants with $y_1 \leq c_1^1$ are the same as under the truthful equilibrium, and (2) all type-2 applicants have truthful reporting as their (at least weakly) dominant strategy.

Point 1 in the Lemma is shown by iterative elimination of dominated strategies, in much the same way whereby we showed the neutrality result in Proposition 1; point 2 is
straightforwardly true. Despite the Lemma, those type-1 applicants with $y_1 > c_1^1$ are not immune to school 2’s use of IA. One can show that, provided that $u_2 > \pi_2 u_1$, a non-truthful equilibrium exists in which (i) all type-1 applicants with $y_1 \leq c_1^1$, as well as all type-2 applicants, report truthfully, and (ii) all type-1 applicants with $y_1 \in (c_1^1, c_2^1]$ specify school 2 as their top choice (strategies of other applicants being inconsequential). In this equilibrium, whereas the state 1 outcome is exactly the same as in the truthful equilibrium, the state 2 outcome is not. Under state 2, those type-1 applicants with $y_1 \in (c_1^1, c_2^1]$ now are admitted by school 2 and their mass is equal to $|A| = s_1 (1 - \mu_2^2 / \mu_1^2)$ (the area in Region A in Figure 1). To accommodate it, school 2 sets a more stringent cutoff of $c_2' = F^{-1} \left( \left( s_1 + s_2 - |A| \right) / N \right) < c_2$, releasing less qualified applicants with $y_1 \in (c_2', c_2]$ for school 1’s admission (Region B in Figure 1). School 2 is strictly better off by stealing students from school 1.

One can show that this equilibrium is indeed unique — there does not exist any other equilibrium that corresponds to a different outcome. The key point is that those type-2 applicants with $y_1 > c_2$ reporting school 2 as their top choice acts as a threat, forcing those type-1 applicants with $y_1 \in (c_2 - \varepsilon, c_2]$ to rank school 2 as their top choice, where $\varepsilon$ is a small positive number. But this in turn implies that those type-1 applicants with $y_1 \in (c_2 - \varepsilon - \varepsilon', c_2 - \varepsilon]$ are forced to rank school 2 as their top choice, where $\varepsilon'$ is a small positive number. This process continues to apply to all type-1 applicants with $y_1 \in (c_1^1, c_2^1]$.\footnote{By reporting school 1 as her top choice, her expected payoff is at most $\pi_2 u_1$ (achieved when she is admitted by school 1 under state 2). By reporting school 2 as her top choice, she will enter school 2 in state 1 and enter school 1 or school 2 in state 2, obtaining an expected payoff at least as high as $u_2$.}

The stealing effect of school 2 does not always exist, however. When $u_2 < \pi_2 u_1$, school 2’s IA policy is unable to persuade type-1 applicants with $y_1 \in (c_1^1, c_2^1]$ to make school 2 their top choice. As a result, these students are neither stolen by school 2 in state 2, because of the pre-commitment, nor are they admitted by school 2 in state 1. School 2 has to admit poorer applicants to fill the vacancies. School 2 is thereby worse off as a whole, and the IA policy is self defeating.\footnote{In equilibrium, all type 1 students with $y_1 \in (c_2, c_2']$ where $c_2' = F^{-1} \left( \left( s_1 + s_2 + |A| \right) / N \right)$ now make school 2 their top choice and will be admitted by the school in state 1.}
The analysis of the effect of (IA,DA), the regime in which only school 1 uses IA, is more straightforward. Because under (DA,DA) school 1 already sets a more stringent admissions requirement, those type-2 applicants whom school 1 is interested in stealing must be good enough to enter school 2—their preferred school—and they have no incentive to change their behavior under (IA,DA). As a result, the truthful outcome prevails under (IA,DA), and school 1 never benefits from using IA when the other school does not use it. This asymmetry between school 1 and school 2 with respect to the stealing effect appears repeatedly in the article.

So far we have assumed that $\alpha_1 = \alpha_2 = 0$. But it is clear that the same argument holds true as long as $\alpha_1 = \alpha_2$, which need not exactly equal zero. We summarize our results as follows.

**Proposition 3** (unilateral incentive) Suppose that $\alpha_1 = \alpha_2$.

1. Suppose that $u_2 > \pi_2 u_1$ ($u_2 < \pi_2 u_1$). Under (DA,IA), there exists a unique equilibrium outcome, in which school 2’s admitted applicants’ average quality is improved (worsened) compared with under (DA,DA).

2. (IA,DA) and (DA,DA) are outcome equivalent.

A few comments are in order here. First, given $\mu_1$ and starting from $\mu_2 = \mu_1$, the number of applicants that are affected by school 2’s IA increases as $\mu_2$ decreases. That is, starting with no demand uncertainty, an increase in the difference between $\mu_1$ and $\mu_2$ always increases the number of applicants whose behavior is affected. Second, because the two schools have identical preferences, whenever school 2 experiences an improvement of its average quality through using IA, school 1 experiences a worsening whereby. Third, some applicants are better off under (DA,IA) than under (DA,DA). Those type-1 applicants in region B now get into school 1 in state 2 under (DA,IA), whereas they can only get into school 2 under (DA,DA). (Thus, it is not the case that school 2’s IA policy will be universally opposed by all applicants.) Relatedly, we also note that these type-1 applicants have a higher expected utility compared with their slightly more qualified counterparts (with slightly lower $y_1$), who
also adopt the same strategy. Somewhat ironically, just because the latter group is of better quality and is selected by school 2—their top choice but less-preferred school—the former group is able to get into school 1.

Our last comment concerns the relationship between IA and pre-arrangement identified in Sönmez (1999) (see also Kojima and Pathak; see also Li and Rosen, Roth and Xing 1994 and 1997, and Suen for pre-arrangement in decentralized markets).\footnote{Interestingly, Sönmez points out pre-arrangement can be replicated as manipulation in the centralized exercise as follows. "The intern ranks the hospitals as its top choice and the hospital ranks the intern as its top candidate" (p. 152).} Consider an alternative game in which the two schools are committed to (DA,DA) in the centralized admissions exercise. Suppose, prior to the exercise, school 2 is allowed to pre-arrange with applicants. If it is allowed to use up all its places $s_2$ for pre-arrangement and is committed to admitting every applicant subject only to its capacity constraint, then the same outcome prevails as under our model of (DA,IA): provided that $u_2 > \pi_2 u_1$, those type-1 applicants stolen by school 2 under (DA,IA) will aim at and succeed in pre-arranging with school 2. The intuition is exactly the same: the possibility of all of school 2’s places being taken up in the pre-arrangement stage compels the aforementioned type-1 applicants to pre-arrange. However, if the number of places available for pre-arrangement is limited, the outcome under (DA,IA) need not be fully replicated by pre-arrangement followed by a centralized admissions exercise with (DA,DA). To be more precise, consider the scenario in which $\alpha_1 = \alpha_2 = 0$. Suppose the total number of places that school 2 allots for pre-arrangement is less than $F(c_2) - F(c_1^2) < s_2$. Even though these places are completely used up, there are still enough remaining places to admit all type-2 applicants with $y_1 \leq c_1^2$ and all type-1 applicants with $y_1 \in (c_1^1, c_2^1)$. Therefore, it is the dominant strategy for the latter group to not pre-arrange but to participate in the centralized procedure under (DA,DA).

## Competition

We now study the equilibrium when both schools are allowed to pre-commit. If $u_2 < \pi_2 u_1$, then it follows from the previous analysis that there are only two, outcome-equivalent, pure strategy equilibria, (DA,DA) and (IA,DA). The more interesting case is where $u_2 > \pi_2 u_1$, \footnote{Interestingly, Sönmez points out pre-arrangement can be replicated as manipulation in the centralized exercise as follows. "The intern ranks the hospitals as its top choice and the hospital ranks the intern as its top candidate" (p. 152).}
where school 2 strictly benefits from using IA provided that school 1 does not use it. In this case, we have

$$AQ_1(IA, DA) = AQ_1(DA, DA),$$  \hspace{1cm} (3) 

and

$$AQ_2(DA, IA) - AQ_2(DA, DA) < 0.$$  \hspace{1cm} (4) 

(I,A,DA) is an equilibrium if it is also true that

$$\Delta \equiv AQ_2(IA, IA) - AQ_2(IA, DA) > 0,$$  \hspace{1cm} (5) 

which means that, by using IA, school 1 can deter school 2 from using IA. We say that (IA,DA) is a preemptive equilibrium for school 1 if (3) to (5) hold. It is indeed the unique equilibrium\(^\text{12}\) if it is also true that

$$AQ_1(IA, IA) \leq AQ_1(DA, IA);$$  \hspace{1cm} (6) 

otherwise, there will exist another pure strategy equilibrium: (DA,IA).

Noting that (IA,DA) is outcome equivalent to (DA,DA), we can rewrite \(\Delta\) as follows:

$$\Delta = (AQ_2(IA, IA) - AQ_2(DA, IA)) + (AQ_2(DA, IA) - AQ_2(DA, DA)).$$  \hspace{1cm} (7) 

This says that the change in the average quality for school 2 from (IA,DA) to (IA,IA) can be decomposed into two separate processes. First, from a change from (DA,DA) to (DA,IA) (the second term) and then from a change from (DA,IA) to (IA,IA) (the first term). As the

\(^{12}\text{When } AQ_1(IA, IA) < AQ_1(DA, IA), \text{ it is clear that (IA,DA) is the unique equilibrium. When } AQ_1(IA, IA) = AQ_1(DA, IA), \text{ both (IA,DA) and (DA,IA) are equilibria. Notice that school 1 is indifferent between choosing DA and IA given school 2's action, and it strictly prefers equilibrium (IA,DA) to equilibrium (DA,IA). The latter is thus unconvincing in the following sense. Even if school 2 chooses IA, school 1 can respond with IA. Foreseeing that school 1 uses IA, school 2 has the strict incentive to respond with DA, ending up with equilibrium (IA,DA). On the contrary, in (IA,DA), school 2 cannot force a switch of equilibrium in the same manner. (IA,DA) is thus the only equilibrium that survives this selection criterion.\)
first change has well been studied, we now turn to the second effect.

Under (DA,IA), applicants with $y_1 \leq c'_2$ are admitted by the same school under both states and they are admitted by their top choice schools. They are admitted as if both schools were using IA. Therefore, they could not do better by a unilateral deviation under (IA,IA).

In fact, one can show that—using iterative elimination of strictly dominated strategy that we used earlier—adopting the same strategies as under (DA,IA) constitutes the unique equilibrium. This also implies that to study the difference, $AQ_2(IA,IA) - AQ_2(DA,IA)$, one only needs to focus on the state 1 change.

**Lemma 2**  
1. In equilibrium, applicants with $y_1 \leq c'_2$ have the same reporting behavior under (IA,IA) as under (DA,IA).

2. In state 2, school 2’s student intakes under (IA,IA) are the same as under (DA,IA).

**Proof.** Omitted. ■

On the other hand, applicants with $y_1 \in (c'_2, c_2]$ need to choose between being admitted by school 2 in state 1 and not admitted by any school in state 2, on one hand, and being admitted by school 1 in state 2 and not admitted by any school in state 1, on the other. As the respective utilities from the two choices are $\pi_1 u_2$ and $\pi_2 u_1$ for type-1 applicants and $\pi_1 v_2$ and $\pi_2 v_1$ for type-2 applicants, to understand these applicants’ optimal strategies, we need to consider three cases: (i) $\pi_1 u_2 > \pi_2 u_1$ (implying $\pi_1 v_2 > \pi_2 v_1$), (ii) $\pi_1 v_2 < \pi_2 v_1$, and (iii) $u_1/u_2 > \pi_1/\pi_2 > v_1/v_2$.

**Case (i):** $\pi_1 u_2 > \pi_2 u_1$. It is easy to show that all applicants with $y_1 \in (c'_2, c_2]$ now specify school 2 as their top choice. School 2’s student intakes under (IA,IA) are just the same as under (DA,IA). Substituting $AQ_2(IA,IA) - AQ_2(DA,IA) = 0$ and (4) into (7), we find that (7) is negative and, as a result, there does not exist any preemptive equilibrium.

**Case (ii):** $\pi_1 v_2 < \pi_2 v_1$. It is easy to verify that all applicants with $y_1 \in (c'_2, c_2]$ will specify school 1 as their top choice, giving up their choice to enter school 2 in state 1. As a result, school 2 must admit an equal number of applicants to replace them in this state. It
is clear that applicants of both types with $y_1 \in (c_2', c_2]$ are the replacements (highlighted in the shaded region C in Figure 2). Therefore,

$$AQ_2(IA, IA) - AQ_2(DA, IA) = \pi_1 [n(C)m(C) - n(B)m(B)] > 0,$$

where $n(.)$ and $m(.)$ are the number and the average quality of the applicants in the region concerned, depicted in Figure 2. If this worsening effect dominates the enhancing effect in (4), there will be a worsening of school 2’s overall average quality. We also note that school 1 has exactly the same student intakes under (IA,IA) as under (DA,IA), i.e., $AQ_1(IA, IA) = AQ_1(DA, IA)$. Hence, if a preemptive equilibrium ever exists, it is indeed the unique equilibrium.

---

**Figure 2 about here**

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**Case iii:** $u_1/u_2 > \pi_1/\pi_2 > v_1/v_2$. We can show that all applicants with $y_1 \in (c_2', c_2]$ will report truthfully. The equilibrium student intakes for school 2 in state 1 are depicted as shaded regions in Figure 3. (As depicted, Region C consists of both type-1 and type-2 applicants. When applicants in Region B' are less numerous, however, Region C may comprise type-2 applicants only.) Note that those in region C are admitted to replace those type-1 applicants in Region B' who, with school 1 as their top choice, are not admissible. (Region B' refers to the set of all type-1 applicants with $y_1 \in (c_2', c_2]$.) Hence,

$$AQ_2(IA, IA) - AQ_2(DA, IA) = \pi_1 [n(C)m(C) - n(B')m(B')] > 0.$$

If the worsening effect dominates the enhancing effect of $AQ_2(DA, IA) - AQ_2(DA, DA)$, then school 2’s overall student quality is worsened (see (7)). We also note that because of pre-commitment, school 1 now needs to admit less-qualified applicants in state 2 relative to under (DA,IA). Hence, $AQ_1(IA, IA) > AQ_1(DA, IA)$, suggesting that (DA,IA) must be an equilibrium, whether or not any preemptive equilibrium exists.
Proposition 4 (Competition) Suppose that $\alpha_1 = \alpha_2$.

1. If $\pi_2 u_1 > u_2$, there are only two pure strategy equilibria: $(DA,DA)$ and $(IA,DA)$. They both lead to the same truthful outcome.

2. If $\pi_2 u_1 < u_2$ and $\pi_1 u_2 > \pi_2 u_1$, there is a unique equilibrium of $(DA,IA)$.

3. If $\pi_2 u_1 < u_2$ and $\pi_1 v_2 < \pi_2 v_1$, there is a unique equilibrium: either $(DA,IA)$ or $(IA,DA)$. If $\alpha_1 = \alpha_2 = 0$ and $y_1$ is uniformly distributed, then $(IA,DA)$ is an equilibrium if and only if $s_2/s_1 < 2\pi_1/\pi_2$.

4. If $\pi_2 u_1 < u_2$ and $u_1/u_2 > \pi_1/\pi_2 > v_1/v_2$, $(DA,IA)$ is an equilibrium, and there may exist another equilibrium, $(IA,DA)$. If $\alpha_1 = \alpha_2 = 0$ and $y_1$ is uniformly distributed, then $(IA,DA)$ is an equilibrium if

$$\frac{s_2}{s_1} < \frac{\mu_1^1}{1 - \mu_1^1} \frac{\pi_1}{\pi_2}. \tag{10}$$

For result 3, there are two ways to understand the condition $s_2/s_1 < 2\pi_1/\pi_2$. First, given that the other parameters are unchanged and given $s_1$, the greater $s_2$, the less likely that a preemptive equilibrium exists. Comparing $(IA,IA)$ with $(IA,DA)$, we find that school 2’s loss in state 1 is invariant to $s_2$ but its gain in state 2 is increasing in $s_2$. Hence, when $s_2$ is sufficiently large, IA is so attractive to school 2 that its use is never thwarted. Second, $\pi_1/\pi_2$ has to be sufficiently large for $\Delta > 0$. The reason is that the greater the likelihood of state 1, the greater weight we put on state 1’s effect, which is favorable for $\Delta > 0$. For result 4, the intuition behind the roles of $s_2/s_1$ and of $\pi_1/\pi_2$ is analogous to that of point 3 and is not repeated. Note that unlike in point 3, (10) is only a sufficient condition, and its violation does not necessarily mean the non-existence of a preemptive equilibrium.
Heterogeneous School Preferences

Thus far, our results are obtained under the assumption of identical school preferences. It is useful to see how the insights may be generalized under heterogeneous school preferences. Our working paper (Chiu and Weng 2007) discusses this in more detail; here we only want to highlight a few points. We argue that Proposition 3 still holds so long as \( \alpha_1 \) and \( \alpha_2 \) do not differ too much so that school 1 will not admit type-2 applicants under state 1. In particular, provided that school 1 does not use IA, school 2 benefits (worsens) from using IA if \( u_2 > \pi_2 u_1 \) (\( u_2 < \pi_2 u_1 \)). On the other hand, when \( \alpha_1 \) and \( \alpha_2 \) differ so much that in the truthful outcome school 1 will admit type-2 applicants under both states, the difference between school 1 and school 2 blurs. As some type-2 applicants enter school 2 in state 1 and enter school 1, their less preferred school, in state 2, school 1 now has the incentive to steal this group of applicants using IA. If the utility from being admitted by school 1 for certainty exceeds the utility from being admitted by school 2 in state 1 and not by any school in state 2 (i.e., \( v_1 > \pi_1 v_2 \)), these applicants will be successfully swayed and school 1 is better off. Of course, if \( v_1 < \pi_1 v_2 \), school 1’s IA is self defeating. A new question is what the equilibrium is like when school preferences differ sufficiently and both schools have a stealing motive. Through simulation using two specifications for the distributions of \((y_1, y_2)\), we find that there are ample scenarios in which both schools using IA can be an equilibrium outcome. It also generates some comparative statics, which are found in the working paper.

4 Ranking Uncertainty

Here we argue that applicants’ uncertainty about their relative rankings among all applicants might also make IA relevant. We call this a model of ranking uncertainty. To make the point, we focus on a very simple version. Like in the previous model, there are two schools with places, \( s_1 \) and \( s_2 \), for which a mass of applicants of size \( N \) are competing. The two schools’ preferences are represented by \( \alpha_1 = 0 \) and \( \alpha_2 \geq \alpha_1 \). Each applicant is characterized by an attribute pair \((y_1, y_2)\) uniformly distributed over \([0, 1] \times [0, 1] \). The distribution function is commonly known, and schools know each applicant’s attribute pair when evaluating
applicants. Despite this, each applicant does not know her own attribute pair (nor the attribute pair of any other applicants) and is ex ante identical. This is the quintessential case of ranking uncertainty. We also assume that applicants have identical preferences. For each applicant, the utility of attending school $i$ is $u_i, i = 1, 2$, so that $u_1 > u_2$; the utility of not attending any school is zero.

**Identical School Preferences**

**Unilateral Incentive**

We first study the case of identical school preferences, i.e., $\alpha_1 = \alpha_2 = 0$, and as such we can represent each applicant's attribute simply by her first attribute, $y_1$. Under (DA,DA), the equilibrium is such that all applicants make school 1 their top choice. It follows immediately that school 1 never has the stealing motive. This is not the case for school 2, however. Suppose under (DA,IA) that all other applicants report truthfully. By reporting truthfully, an applicant obtains an expected payoff of $(s_1u_1 + s_2u_2)/N$; by reporting school 2 as her top choice, she obtains a payoff of $u_2$ for certain. If $u_2/u_1$ is sufficiently high, such a unilateral deviation is beneficial and the truthful equilibrium is infeasible. This means that, in equilibrium, a non-negligible fraction of applicants will specify school 2 as their top choice, and if this fraction (denoted by $b$) is large enough, school 2 will indeed benefit from the IA policy. The equilibria under (DA,DA) and (DA,IA) are shown in Figure 4, where $a$ and $b$, respectively, are the corresponding number of applicants who specify school 1 and school 2 as their top choice.

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**Proposition 5** *(Unilateral incentive)* Suppose that $\alpha_1 = \alpha_2 = 0$ and that $y_1$ is distributed uniformly over the $[0,1]$ interval.

1. *(IA,DA) and (DA,DA) are outcome equivalent.*
2. Under \((DA, IA)\), if \(u_2/u_1 < s_1/(N - s_2)\), there exists a truthful equilibrium.

3. Under \((DA, IA)\), if \(u_2/u_1 > s_1/(N - s_2)\), then (a) there does not exist a truthful equilibrium, but (b) there exists an equilibrium in which \(b \in (s_2, N)\) and in this equilibrium, \(AQ_2(DA, IA) < AQ_2(DA, DA)\), if and only if \(u_2/u_1 > 1/2\).

According to point 3, the condition for school 2’s IA to influence the applicants’ behaviors is different from the condition for it to benefit from the policy. For instance, consider the parameter values \(s_1 = s_2 = N/4\) and \(u_2/u_1 = 0.4\). Then, school 2’s IA is able to influence the applicants’ behaviors but its average applicant quality will worsen. More generally, provided that \(2s_1 + s_2 < N\), there exists a nonvanishing range of \(u_2/u_1 \in (s_1/(N - s_2), 1/2)\) such that school 2’s IA is influential yet self defeating.\(^\text{13}\)

\section*{Competition}

Given that \(u_2/u_1 > s_1/(N - s_2)\), the equilibrium under \((DA, IA)\) has the following simple but powerful feature: relative to school 2’s, school 1’s cutoff value must be more stringent. (Or else, each applicant would strictly prefer to making school 1 her top choice, contradicting the claim that school 1’s cutoff value is less stringent.) This implies that school 1 admits only among those who specify it as their top choice. Taking into account the property that \(b > s_2\) (Part 3 of Proposition 5), we reckon that neither school will admit any applicant not specifying it as her top choice. In other words, the outcome under \((DA, IA)\) is exactly the same as that under \((IA, IA)\). Noting the outcome equivalence of \((DA, DA)\) and \((IA, DA)\), we conclude that, regardless of school 2’s policy choice, school 1 cannot worsen school 2 through using IA. School 2’s IA is nonpreemptable.

\begin{proposition} \textbf{(Competition)} Suppose that \(\alpha_1 = \alpha_2 = 0\) and that \(y_1\) is distributed uniformly over the \([0,1]\) interval.

1. If \(u_2/u_1 > \max \{s_1/(N - s_2), 1/2\}\), then there is a unique equilibrium outcome, which is the same as under \((DA, IA)\).

\end{proposition}

\(^{13}\)The result can be replicated by a stage of pre-arrangement by school 2 prior to the centralized admissions exercise. This is the case provided that school 2 is allowed to use up all its places \(s_2\) for pre-arrangement and is not necessarily the case otherwise.
2. If \( u_2/u_1 < \max\{s_1/(N - s_2), 1/2\} \), then there is a unique equilibrium outcome, which is the same as under (DA,DA).

**Proof.** Omitted. ■

According to the first result, when \( u_2/u_1 \) is high enough, school 2 will gain from using IA, and its use of IA is not deterred by school 1. In fact, school 2’s student intake is invariant to whether or not school 1 adopts IA. There are two pure strategy equilibria: (DA,IA) and (IA,IA) and they are outcome equivalent. According to the second point, when \( u_2/u_1 \) is not high enough, school 2 will not gain from using IA and will not use it. There are two pure strategy equilibria: (DA,DA) and (IA,DA) and they are outcome equivalent. To conclude, we have found not only an asymmetry between the two schools regarding the stealing effect but also the lack of preemptive ability on the part of the popular school.

**Heterogeneous School Preferences**

Despite its absence under homogeneous preferences, a preemptive equilibrium by school 1 may exist if school preferences are sufficiently heterogeneous. To see this, we assume that \( \alpha_1 = 0 \) and \( \alpha_2 \to \infty \) such that school 1 cares about the applicants’ first attribute, \( y_1 \), and school 2 cares only about their second attribute, \( y_2 \). This is the most extreme case of heterogeneous preferences. Under (DA,DA), applicants still report truthfully, and school 1 admits applicants whose \( y_1 \leq c_1 = s_1/N \) and school 2 admits applicants whose \( (y_1, y_2) \) satisfies \( y_1 > c_1 \) and \( y_2 \leq c_2 = s_2/(N - s_1) \), where \( c_1 \) and \( c_2 \) are the two cutoffs on \( y_1 \) and \( y_2 \) that the two schools, respectively, set in equilibrium. We have the following proposition regarding the unilateral incentive to use IA.

**Proposition 7** (Unilateral Incentive) Suppose \( \alpha_1 = 0, \alpha_2 \to \infty \), and \( (y_1, y_2) \) is distributed uniformly over the \([0,1]\) square.

1. (IA,DA) and (DA,DA) are outcome equivalent.

2. Under (DA,IA), if \( u_2/u_1 < s_1/(N - s_2) \), there exists a truthful equilibrium; if \( u_2/u_1 > s_1/(N - s_2) \), then (a) there does not exist a truthful equilibrium, but (b) there exists a unique equilibrium, in which \( b = N \) and \( AQ_2(DA, IA) < AQ_2(DA, DA) \).
The first result is derived from the fact that school 1 has no incentive to use IA given that school 2 does not use it. The bandwagon phenomenon in result 2.b is a joint outcome of school 1’s DA and the two schools’ orthogonal preferences. Because of school 1’s DA, those applicants who did not specify school 1 as their top choice are not discriminated against by school 1 when they become available for its selection. Because of the schools’ orthogonal preferences, these applicants are viewed by school 1 as good as those who specified school 1 as their top choice. As a result, they will be admitted with the same probability by school 1 as if they chose it as their top choice. Foreseeing this, they find it optimal to make school 2 their top choice provided that \( u_2 \) is sufficiently high. In the identical school preferences case, on the contrary, those specifying school 2 as their top choice and subsequently declined by school 2 are of inferior quality from school 1’s point of view and will not be admitted. This difference leads to a very significant difference in terms of the applicants’ behaviors.

Given that \( b = N \), school 2 has all the applicants to select from, and it is not surprising to see that the average quality of its intakes will improve. We note in passing that, under (DA,IA), the two schools will have cutoffs \( c_1 = s_1/(1 - c_2)N \) and \( c_2 = s_2/N \), with average qualities \( AQ_1(DA, IA) = s_1/2(N - s_2) \) and \( AQ_2(DA, IA) = s_2/2N \). Compared with their counterparts under (DA,DA), we know that school 1 is worse off whereas school 2 is better off. Therefore, the ability to influence applicants’ behavior is equivalent to gaining from the IA policy.

We next note that, if \( u_2/u_1 > s_1/(N - s_2) \) and both schools are allowed to pre-commit to IA, there are two asymmetric equilibria: (IA,DA) and (DA,IA). The policy pair (IA,IA) is not an equilibrium because given that the other school uses IA, each school finds it optimal not to use it, i.e., we have \( AQ_1(IA, IA) < AQ_1(DA, IA) \) and \( AQ_2(IA, DA) < AQ_2(IA, IA) \).

**Proposition 8 (Competition)** Suppose that \( \alpha_1 = 0, \alpha_2 \to \infty, (y_1, y_2) \) is distributed uniformly over the \([0,1]\) square, and \( u_2/u_1 > s_1/(N - s_2) \). There are two pure strategy equilibria: (IA,DA) and (DA,IA).

(IA,DA) is a preemptive equilibrium. Not directly benefiting from the use of IA, school 1 can deter school 2 from using IA. Thus, the asymmetry between the two schools that we
have seen under homogenous preferences is somewhat diluted.

Discussion

As in the demand uncertainty model, we have found both stealing and preemptive incentives in this ranking uncertainty model. Regarding the preemptive motive, we have new results. There is little the popular school can do to prevent the less popular school from choosing IA when the two schools have like preferences. When they are sufficiently different, however, the popular school can successfully use IA as a defensive policy. More importantly, universally seen as a less preferred school, school 2 is still able to steal students from school 1. This is a unique feature of the ranking uncertainty model.

Despite our focus on the quintessential case, we maintain that the general insights still hold when applicants have more information about the rankings among themselves. We illustrate this with a simple variant. Assume that, whereas still not knowing her $y_1$, each applicant observes a signal $x_1 \in [0, 1]$ of it. We assume that $x_1$ and $y_1$ are affiliated and distributed according to a cumulative function that is continuous and has no mass point. In this case, the lower her signal $x_1$, the more confident is an applicant that her $y_1$ is low.

Under (DA,DA), applicants report truthfully. As a result, school 1 admits all applicants with $y_1 \leq c_1$ and school 2 admits all applicants with $y_1 \in (c_1, c_2)$, where $c_1$ and $c_2$ are the equilibrium cutoffs set by the two schools.

Under (DA,IA), provided that $u_2/u_1$ is high enough, one can show that there exists a critical value, $x_1^*$, such that applicants with $x_1 < x_1^*$ ($x_1 > x_1^*$) write school 1 (school 2) as their first choice, that school 1 and school 2 are both oversubscribed, and that each school admits only from those who report it as their top choice. Denote their cutoffs by $c_1'$ and $c_2' > c_1'$. As $u_2/u_1$ approaches unity, in the limiting case, $c_1' = c_2' = c_2$, and exactly the same set of applicants admitted under (DA,DA) is admitted under (DA,IA). Because now school 2 gets better students than under (DA,DA), it clearly benefits from using IA. When $u_2/u_1$ is lower, it is conceivable that the stealing effect will become detrimental. Using this framework, we thus are able to replicate the main results in Propositions 4 and 5 (see Appendix B for details).
5 Multiple Schools

We now explore a framework of three schools, denoted by $q_1, q_2, \text{ and } q_3$, to check if the basic insights found still hold and if additional insights will emerge. We assume that $\alpha_1 = \alpha_2 = \alpha_3 = 0$ (i.e., schools have the same preferences). We first study demand uncertainty, assuming that all applicants who share the same ordinal preferences share the same cardinal preferences, and that their proportions are different under different states of the world. A full model for a three school case will consist of six types of applicants, classified by their preferences. To illustrate the point, however, it suffices to focus on a smaller number of types. We next study ranking uncertainty and conclude.

Multiple Stealing

Unlike in our earlier model, in general a school may steal more than one type of applicants from more than one school in more than one state. Suppose there are three types of applicants, 1, 2, and 3, with the following strict preferences: $q_1 \succ_1 q_2 \succ_1 q_3$, $q_2 \succ_2 q_3 \succ_2 q_1$, and $q_3 \succ_3 q_1 \succ_3 q_2$. Denote by $u_i(\theta)$ the utility a type $\theta$ applicant obtains from entering school $i$. Suppose there are two states, $\pi_1$ and $\pi_2 \equiv 1 - \pi_1$, such that the truthful outcome is depicted in Figure 5.

As depicted, under the truthful outcome, type-1 applicants with $y_1 \in (c_1^1, c_1^2]$ enter school 1 in state 2 and school 2, a less preferred school, in state 1; type-3 applicants with $y_1 \in (c_2^1, c_2^2]$ enter school 3 in state 1 and school 2, a less preferred school, in state 2. These applicants are thus concerned about the use of IA by school 2. School 2 is indeed able to steal the aforementioned type-1 applicants in state 2 and the aforementioned type-3 applicants.
in state 1, provided that:

\[ u_2(1) > \pi_2u_1(1) \quad \text{(for type-1 applicants)} \]
\[ u_2(3) > \pi_1u_3(3) \quad \text{(for type-3 applicants)} \]

and that no other schools use IA.\(^{14}\) As a numeric example, the two conditions are indeed satisfied for all \( \pi_1 \in (0.2, 0.5) \) if \( u_1(1) = u_3(3) = 10 \) (utility from the most preferred schools), \( u_2(1) = 8 \) (utility from the moderately preferred schools) and \( u_2(3) = 5 \) (utility from the least preferred schools). Given multiple stealing, it is an interesting question which school whose students are stolen will have the incentive to preempt and how the interaction and coordination between the two schools will be.

**A school’s Lack (Presence) of the Stealing Effect Hinders (Enhances)**

**Its Preemption**

There are three types of applicants, 1, 2 and 3, with the following strict preferences: \( q_1 \succ_1 q_2 \succ_1 q_3 , q_2 \succ_2 q_1 \succ_2 q_3 , \text{and } q_3 \succ_3 q_1 \succ_3 q_2 \). There are two states of the world with probabilities \( \pi_1, \pi_2 = 1 - \pi_1 \) so that the truthful outcome is depicted in Figure 6. We first observe that those type-1 and type-2 applicants with \( y_1 \in (c_1^1, c_2^2) \) will enter school 2 in state 2 and enter school 3, a less preferred school, in state 1. Hence, if only it is allowed to use IA, school 3 may want to steal these two groups of applicants from school 2 and will succeed in doing so provided that

\[ u_3(1) > \pi_2u_2(1) \quad \text{(for type-1 applicants)} \]
\[ u_3(2) > \pi_2u_2(2) \quad \text{(for type-2 applicants)} \]

Suppose that by using IA, school 2 can inflict damage on school 3 so that school 3 will not adopt IA anticipating that school 2 will. This may suggest the existence of a preemptive

\(^{14}\)Consider the following strategies. (i) Those aforementioned type 1 and type 3 applicants ranking school 2 as their top choice and truthfully reporting between the two remaining schools. (ii) All others reporting as their top choice the school that they will enter under the truthful outcome and truthfully reporting between the two remaining schools.
equilibrium when both school 2 and school 3 are allowed to choose IA. However, there is a
difficulty here because school 2’s IA has an impact on the reporting strategies of those type-1
applicants with $y_1 \in (c_1^0, c_2^0]$ (applicants in Region A) who are uncertain which school they
will enter. If parameters are such that school 2 is not able to steal applicants from school 1
(this happens when $\pi_2 u_{1}(1) > u_{2}(1)$), the aforementioned preemption becomes incredible
as school 2’s IA becomes self defeating (i.e., $AQ_2(DA, IA, DA) > AQ_2(DA, DA, DA)$). In
other words, school 2’s lack of the stealing effect on school 1 hinders its preemption against
the use of IA by school 3. Thinking along this line, one may conclude that, under different
parameters, the presence of a beneficial stealing effect on the part of school 2 may enhance
its otherwise ineffective preemption against school 3.

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**Horizontal Competition versus Vertical Competition**

We note that IA has no such strategic value for a school in its competition with another
school if under no states do any applicants consider it as the better school. Imagine a
model in which there are two types of applicants with preferences: $q_1 \succ_1 q_2 \succ_1 q_3$ and
$q_1 \succ_2 q_3 \succ_2 q_2$. There are two states of the world so that the proportions of the two
types are $\mu_i^1$ and $\mu_i^2 \equiv 1 - \mu_i^1$ in state $i$, $i = 1, 2$. Despite the uncertainty about the
proportion of each type, the cutoff standard set by school 1 under the truthful outcome is
known and invariant across states. Hence, school 1 is insulated from any stealing attempt
by school 2 or school 3. A generalization is that a school generally known to be superior to
other schools will not be affected by use of IA by the latter schools. We may say that the
school competition studied in Section 3 is horizontal competition: for each school, there are
applicants who find it the best, whereas the competition between $q_1$, on the one hand, with
$q_2$ and $q_3$, on the other, that we study here is vertical competition. Our discussion suggests
that in case of demand uncertainty, the IA policy matters only for schools that engage in
horizontal competition.

**Ranking Uncertainty**

This model with affiliated signals (introduced in the second half of Section 4) is extendable to a multiple school framework. Suppose there are three schools that give admitted students the utilities of $u_1, u_2, u_3$, respectively, where $u_1 > u_2 > u_3$. Suppose $u_3/u_2$ and $u_2/u_1$ are sufficiently high. Then it is conceivable that under the policy regime (DA,DA,IA) there exists a critical value $x_1^*$ so that those applicants with higher, hence worse signals will be swayed to rank school 3 as their top choice and as a result school 3 is better off using IA. It is interesting to note that school 3 steals students not only from school 2 but also from school 1, which is two places better. It is also conceivable that under (DA,IA,IA) there exists $x_1^*$ and $x_1''$ so that those with signals $x_1 < x_1^*$ rank school 1 as their top choice, those with $x_1 \in (x_1', x_1'')$ rank school 2 as their top choice, and those with $x_1 > x_1''$ rank school 3 as their top choice.

**Large Number of Schools**

The previous exploration suggests that, in a multiple school setting, conditions can be found under which some school has the motive to steal applicants using IA. We have not, however, addressed the question as to what fraction of schools will be expected to have such a motive or how likely a beneficial use of IA may occur.

It is worth noticing the recent work of Kojima and Pathak on the incentive to manipulation in large markets. Generalizing Roth and Peranson and Immorlica and Mahdian, they show that the fraction of schools that can benefit from manipulations in the form of misreporting of preferences, misreporting of capacities, or pre-arrangement will become negligible as the matching market grows larger. They also show that no manipulation is an $\varepsilon$-equilibrium between schools and applicants when the market is getting large and the sequence of markets has the property of being sufficiently thick. It is a question whether scenarios that do not satisfy their assumptions may still be interesting.\(^{15}\) Moreover, the

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\(^{15}\)In a circular city model, an enlargement of the circular city does not seem to reduce the fraction of
model they consider is very different from ours. For instance, in their game of equilibrium analysis, schools submit their preference lists prior to applicants knowing their preferences and submitting their rank-order lists. That said, we do have the feeling that a similar result may apply to the type of manipulation that we identify although it needs to be formally analyzed. We leave this for future research.

6 Concluding Remarks

We have been interested in a phenomenon in which a school deliberately gives preferential treatment to applicants who rank it as their top choice over more qualified applicants who do not. We have explored a particular form of preferential treatment in which a school uses the so-called immediate acceptance policy, committing to admitting applicants who rank it as their top choice as long as places are available. We have identified two motives of doing so. A less popular school may use the pre-commitment to steal applicants who otherwise would not choose it as their top choice (stealing motive); a popular school may use the pre-commitment to prevent its own applicants from being stolen (preemptive motive).

What can we do to reduce the scope and effects of endogenous preferential treatment? One natural answer is to reduce the scope of uncertainty. Demand uncertainty can be reduced by a more thorough survey of applicants’ preferences prior to application, whereas ranking uncertainty can be reduced by providing finer examination grades and scores or postponing the application period until applicants know their examination scores.

Should endogenous preferential treatment be banned? In our model, we assumed that schools adopted preferential treatment simply out of a strategic consideration, and the motivational consideration was completely neglected. When students’ motivations and commitments are important to success in their studies, schools naturally want to admit students who like them most, other things being equal.\textsuperscript{16} There is nothing wrong with it and banning preferential treatment does not seem justifiable. To answer the question satisfactorily, there-

\textsuperscript{16}The emphasis of one’s motivation in one’s performance and its impact on the design of organizations have received increasing attention lately (see, e.g., Besley and Ghatak 2006).
fore, requires studying a model that also takes into account the motivational consideration. We hope that this issue can be tackled in the future.
Appendix A:
Proofs of Lemma 1 and Propositions 1 to 5, 7 and 8 follow.

Proof of Proposition 1: Existence. Partition applicants into three groups according to where they end up under the truthful outcome: those who get admitted to school 1 (with measure $s_1$), those who get admitted to school 2 (with measure $s_2$), and the rest. Call them group 1, group 2, and group 0 applicants, respectively. Given the policy pair in which at least one school uses IA, consider the following strategies: each group 1 applicant specifies school 1 as her top choice, and each group 2 applicant specifies school 2 as her top choice. (It does not matter what the rest will do.) We first note that school $k$ considers every group $k$ applicant strictly better than every group 0 applicant, $k = 1, 2$. (Otherwise, it should have given a place to some group 0 applicant under (DA,DA).) Therefore, in the first round of the Gale-Shapley mechanism, school $k$ will keep all group $k$ applicants. Each school that uses IA will simply admit this group of applicants and will not proceed to the second round. The school, if any, that uses DA will find the applicants it retained in the first round to be strictly better than any applicant whom the school can newly consider in the second round. As a result, the mechanism terminates with school $k$ admitting all group $k$ applicants, $k = 1, 2$. Given the aforementioned prescription for applicants, it is clear that no applicant can benefit from a unilateral deviation; otherwise, she would have benefitted from a unilateral deviation under (DA,DA). Thus, the truthful outcome is still feasible as an equilibrium outcome under the said policy pair. It is clear that this same result holds for a general setting with multiple schools.

Uniqueness. For $i = 1, 2$, let $Q_i$ be the set of all type-$i$ applicants. Let $R_i$ be the set of applicants admitted by school $i$ in the truthful outcome and $S_i$ be the set of type-$i$ applicants among them. Given any policy regime $(p_1, p_2)$ where at least one school uses IA, we construct a recursive process that determines the equilibrium. Essentially, the recursive process helps us clarify the following equation. Suppose school $i$ happens to have received the applications from all type-$i$ and type-$j$ applicants, $j \neq i$. What is the cutoff, denoted by $\gamma_i(1)$, so that an applicant with $(y_1, y_2)$ satisfying $y_1 + \alpha_i y_2 \leq \gamma_i(1)$ can surely enter school $i$? Thus those type-$i$ applicants satisfying this criterion, denoted by the set $T_i(1)$,
can thus enter school $i$ despite the most fierce competition from type-$j$ applicants (this is the case whether the school uses DA or IA). Hence, there cannot exist another equilibrium in which any of them is not assigned to school $i$. Now that we know in equilibrium type-$i$ applicants in $T_i(1)$ will enter school $i$ (and type-$j$ applicants in $T_j(1)$ will enter school $j$), we proceed to the next round to deal with the remaining unassigned applicants. In this round, we ask a similar question: suppose school $i$ happens to have received, along with the applications from $T_i(1)$, the applications from all remaining unassigned applicants (of both types). What is the cutoff, denoted by $\gamma_i(2)$, so that an applicant with $(y_1, y_2)$ satisfying $y_1 + \alpha_i y_2 \leq \gamma_i(2)$ can surely enter school $i$? Those type-$i$ applicants satisfying this criterion can thus enter school $i$ despite the most fierce competition from type-$j$ applicants (this is the case whether the school uses DA or IA). Hence, there cannot exist another equilibrium in which any of them is not assigned to school $i$. We argue that when we continue this process, at the end, in any equilibrium each school is assigned the same set of applicants under the truthful equilibrium, and the equilibrium outcome is unique.

We now proceed to the detail. For $i = 1, 2$, define $T_i(0) = \phi$, $\gamma_i(0)$, and define $\gamma_i(k), T_i(k)$, and $U_i(k)$ in the following manner

$$T_i(k) = \{q \in Q_i \mid y_1 + \alpha_i y_2 \leq \gamma_i(k)\},$$

$$U_i(k) = \{q \in Q_j \setminus T_j(k-1) \mid y_1 + \alpha_i y_2 \leq \gamma_i(k), j \neq i\},$$

and $$|T_i(k) \cup U_i(k)| = s_i,$$

where $k = 1, 2, \ldots$ and $|.|$ are the measure of applicants in the inscribed set. Note that the cutoff $\gamma_i(k), T_i(k)$, and $U_i(k)$ are all determined within the recursive process. $T_i(k)$ is the set of type-$i$ applicants that have been assigned to school $i$ by round $k$, and it contains all type-$i$ applicants who satisfy the cutoff $\gamma_i(k)$. $U_i(k)$ is an auxiliary set that tells us the set of type-$j$ applicants who (i) have not obtained a place in school $j$ by the last round (not in $T_j(k-1)$) and (ii) satisfy the cutoff of school $i$ in this round; it is the presence of the members in $U_i(k)$ that prevents less qualified type-$i$ applicants from being assigned to school $i$ by this round. Also note that in general $U_i(k) \cap T_j(k) \neq \phi$. The third line states
that in each round these two groups of applicants just exhaust school $i$’s places.

It is easy to show that for all $k$, $T_i(k) \supseteq T_i(k - 1)$ and $\gamma_i(k) \geq \gamma_i(k - 1)$. That is, the set of type-$i$ applicants that get assigned to school $i$ is larger and the cutoff the school adopts is more lenient as $k$ increases. Next we observe the following two facts:

**Claim 1.** School $i$ must admit any member in $T_i(k)$, for all $k$, in any equilibrium.

**Claim 2.** As $k \to \infty$, $T_i(k) \to S_i$ and $\gamma_i(k) \to c_i$, where $c_i$ is school $i$’s cutoff in the truthful outcome.

We first show claim 1. Note that, as explained earlier, school $i$ must admit any member in $T_i(1)$ in any equilibrium. Assume that it is also true that school $i$ must admit any member in $T_i(k - 1)$ in any equilibrium. We now show the validity of the claim for $T_i(k)$. By definition, $T_i(k) \setminus T_i(k - 1)$ is the group of remaining unassigned type-$i$ applicants who can get assigned to school $i$ when competing with the remaining unassigned type-$j$ applicants. In other words, each applicant in $T_i(k) \setminus T_i(k - 1)$ can surely enter school $i$ by specifying it as her top choice. As entering school $i$ is most ideal for her, she must enter school $i$ in any equilibrium. Hence, by mathematical induction, claim 1 is shown.

We next show claim 2. As there are more type-$i$ applicants than places that school $i$ provides, there is a finite upper bound to $\gamma_i(k)$. As a result, $\gamma_i(k)$ will converge to some limit, denoted by $\gamma_i(\infty)$, and so will $T_i(k)$ converge to some limit, denoted by $T_i(\infty)$. We next observe that it cannot be the case that $\gamma_i(\infty) > c_i$. If this was true, then according to claim 1 in any equilibrium school $i$ will admit type-$i$ applicants that meet this criterion, contradicting to the fact that the truthful outcome does not satisfy this condition. Therefore, we have $\gamma_1(\infty) \leq c_1$ and $\gamma_2(\infty) \leq c_2$. We argue that these inequalities must indeed hold as equalities. Denote by $U_i(\infty)$ the limit of $U_i(k)$ as $k$ approaches infinity. We have $T_i(\infty) \cap U_j(\infty) = 0$; otherwise, the recursive process would not have reached its limit. This relationship allows us to rewrite $|T_i(\infty) \cup U_i(\infty)| = s_i$, $i = 1, 2$, as

\[ \{|q \in Q_1| y_1 + \alpha_1 y_2 \leq k_1(\infty) \} \cup \{|q \in Q_2| y_1 + \alpha_2 y_2 > k_2(\infty) \text{ and } y_1 + \alpha_1 y_2 \leq k_1(\infty) \} \] = s_1,

and \[ \{|q \in Q_2| y_1 + \alpha_2 y_2 \leq k_2(\infty) \} \cup \{|q \in Q_1| y_1 + \alpha_1 y_2 > k_1(\infty) \text{ and } y_1 + \alpha_2 y_2 \leq k_2(\infty) \} \] = s_2.
These two equations can be viewed as two equations with two unknowns \( k_1(\infty) \) and \( k_2(\infty) \). It is easy to see that \( c_1 \) and \( c_2 \) are the unique solution to this problem (otherwise the truthful outcome is not unique). So we have \( k_1(\infty) = c_1 \) and \( k_2(\infty) = c_2 \).

It follows that under policy regime \((p_1, p_2)\), in any equilibrium, members in \( S_i \) get admitted by school \( i \). Given this result, one can show that members in \( R_i \setminus S_i \)—type-\( j \) applicants that get into school \( i \) in the truthful outcome—will indeed enter school \( i \) in any equilibrium. Note that these applicants are clearly qualified to enter school \( i \) in any equilibrium (they satisfy the same cutoff, \( \lim_{k \to \infty} \gamma_i(k) = c_i \), that allows all members in \( S_i \) to enter school \( i \) and the only possible source of non-uniqueness is that some of them turn out to enter school \( j \). This latter possibility could not occur, however. The basic point is that there exist type-\( j \) applicants who are more qualified than members in \( R_i \setminus S_i \) from school \( j \)'s viewpoint and who eventually do not enter any school. A formal proof would require an iterative process similar to the one that have been used.

Note that what is essential to the unique equilibrium outcome is the assumption that schools have strict preferences. To illustrate this, we apply to Example 1 the recursive process studied here. Given \((p_1, p_2) = (IA, IA)\), in the first round, we have \( T_1(1) = \{a_1\}, U_1(1) = \{b_1\}, T_2(1) = \{b_1\}, \) and \( U_2(1) = \{a_1\} \). In the second round, however, for \( i = 1, 2 \), \( T_i(2) \) is undefined because school \( i \) is selecting for the remaining single place between \( a_2 \) and \( b_2 \), which it is indifferent between. The argument used in this proof thus fails to work. Q.E.D.

**Proof of Proposition 2:** Call the game under \( p \) as game 1 and the game under \( p' \) as game 2. For \( j = 1, 2 \), consider any Nash equilibrium in game \( j \) that yields a stable matching \( \pi \). Denote as \( \pi(i) \) the assigned school for applicant \( i \). Now consider game \( k \neq j \) and the strategy profile where applicant \( i \) ranks \( \pi(i) \) as her top choice (the strategies of applicants who are unassigned under \( j \) are inconsequential). We argue that, given this strategy profile, these applicants will be admitted by their top-choice schools, hence replicating \( \pi \). To see this, notice that in the first round, each school \( s \) has two sets of applicants for its selection: those who are assigned to \( s \) under game \( j \), and those who are unassigned any school under game \( j \). Notice that every member in the former group is more preferred to every member
in the latter group (implied by stability of \( \pi \) under game \( j \)) and the former group’s size is just equal to the number of places that \( s \) has. Therefore, the latter group of applicants are declined and made available for other schools’ selection in the second round. However, also because of the stability of \( \pi \) under game \( j \), they are less preferred to those applicants that their second-choice schools have kept (in case of DA schools) or admitted (in case of IA schools) in the first round. Therefore, they are also declined in the second round. These applicants are made available for the next round and are declined for the same reasons. The process stops until all applicants who are unassigned according to \( \pi \) are rejected by all schools. Because the set of Nash equilibrium outcomes under game 1 is the set of all stable matchings (each stable matching can be replicated as a Nash equilibrium outcome under game 1 using the aforementioned algorithm), the proposition is thus immediate. Q.E.D.

**Proof of Lemma 1:** We show point 1. For \( i = 1, 2 \) and \( j \neq i \), define \( c_{j,0} = 0 \), and define \( c_{i,k} \),

\[
\mu_i^j F(c_{i,k}) + \mu_j^i (F(c_{i,k}) - F(\min \{c_{i,k}, c_{j,k-1}\})) = s_i/N,
\]

where \( k = 1, 2, \ldots \). Hence, \( c_{i,1} = F^{-1}(s_i/N) \) in particular. By writing school \( i \) as their top choice, all type-\( i \) applicants with \( y_1 \leq c_{i,1} \) will surely enter school \( i \) even if it is the worse state for type-\( i \) (state \( i \)) and if the most qualified type-\( j \) applicants also report school \( i \) as their top choice. This ensures that under any equilibrium not only type-\( i \) applicants with \( y_1 < c_{i,1} \) will enter school \( i \) but also no type-\( j \) applicants with \( y_i < c_{j,1} \) will enter school \( i \). This latter additional information ensures us a larger cutoff, \( c_{i,2} > c_{i,1}, i = 1, 2 \), so that in any equilibrium not only type-\( i \) applicants with \( y_1 \leq c_{i,2} \) will enter school \( i \) but also no type-\( j \) applicants with \( y_1 < c_{j,2} \) will enter school \( i \). Continue this process. Because \( c_{i,k} \) is non-decreasing and bounded above by one, it must approach a limit point, denoted as \( c_i^* \).

In other words, in any equilibrium, type-\( i \) applicants with \( y_1 \leq c_i^* \) must report school \( i \) as their top choice and succeed in getting in. We finally note that \( c_i^* \geq c_i^1, i = 1, 2 \). Suppose not. Then either (i) or (ii) is correct. (i) \( c_i^* < c_i^1 \) and \( c_j^* \geq c_j^1 \) for some \( i = 1, 2 \) and \( j \neq i \) and (ii) \( c_i^* < c_i^1 \) and \( c_j^* < c_j^1 \). It is straightforward to verify that each leads to a contradiction. The proof of point 2 is omitted. Q.E.D.
Proof of Proposition 3: Part 1. For the case of \( u_2 < \pi_2 u_1 \), the equilibrium strategies are as follows: (i) type-1 applicants with \( y_1 \leq c_1^2 \) and all type-2 applicants report truthfully, and (ii) all type-1 applicants with \( y_1 \in (c_1^2, c_2^\prime] \) report school 2 as their top choice, where

\[
c_2^\prime = F^{-1} \left( \frac{s_1 + s_2 + |A|}{N} \right).
\]

The strategies of all other applicants are inconsequential. We omit the proof that this is an equilibrium and no other equilibria exist that give a different outcome.

Next we show the equilibrium uniqueness for the case of (DA,IA) and \( u_2 > \pi_2 u_1 \). The proof is essentially an iterative elimination of strictly dominated strategies. Assume the strategy prescription as described in the main text. Because in both states those applicants with \( y_1 \leq c_1^1 \) will surely get a place in equilibrium, we know that the total amount of places left for the remaining applicants to compete for is equal to \( r = N (F(c_2) - F(c_1^1)) \). For \( i = 1, 2 \), define \( c_{2,1}^i \) as follows:

\[
\mu_2^i (1 - F(c_1^1)) + \mu_1^i (F(c_{2,1}^i) - F(c_1^1)) = r / N.
\]

The intuition of the equation is as follows. If the remaining places, \( r \), are to be firstly given to all remaining type-2 applicants, followed by the most qualified type-1 applicants, then the cutoff for the type-1 applicants will just be \( c_{2,1}^1 \). In state \( i \), by reporting school 1 as the top choice, any type-1 applicant with \( y_1 \in (c_{2,1}^i, c_2^1] \) will fare unfavorably. On the other hand, by reporting school 2 as top choice, she can obtain a place under both states. Because \( c_{2,1}^1 \leq c_{2,1}^2 \), we conclude that any type-1 applicant with \( y_1 \in (c_{2,1}^1, c_2^1] \) now has a strictly dominant strategy, which is to report school 2 as their top choice. If \( c_{2,1}^1 = c_{2,1}^1 \), then we are done. If not, continue this process. In general, define

\[
\mu_2^i (1 - F(c_1^1)) + \mu_1^i (F(c_2) - F(c_{2,k-1}^i)) + \mu_1^i (F(c_{2,k}^i) - F(c_1^1)) = r / N.
\]

If the remaining places, \( r \), are first given to all remaining type-2 applicants and those type-1
Proof of Result 3 of Proposition 4: Step 1: Recall that

\[ \Delta = \pi_1 [n(C)m(C) - n(B)m(B)] + \pi_2 [n(A)m(A) - n(B)m(B)], \tag{A2} \]

where the regions A, B, and C are depicted in Figure 1 and Figure 2. We note that \( n(A) = n(B) = n(C) \). Denote this quantity as \( Q \). Under the assumption of uniform distribution, we have

\[ m(A) = \frac{1}{2} \left( \frac{s_1}{\mu_1^2 N} + \frac{s_1}{\mu_1^2 N} \right), \quad m(B) = \frac{s_1 + s_2}{N} - \frac{x}{2}, \quad \text{and} \quad m(C) = \frac{s_1 + s_2}{N} + \frac{x}{2}. \]

Substituting all these into (A2) and with some manipulation, we obtain

\[ \Delta (\mu_1^1, \mu_1^2) = \frac{\pi_2 Q}{2} \left( \frac{s_1}{\mu_1^2 N} + \frac{s_1}{\mu_1^2 N} \right) - \pi_2 Q \left( \frac{s_1 + s_2}{N} \right) + Q \frac{s_2}{\mu_1^2} \left( 1 - \frac{\mu_1^2}{\mu_1^2} \right) \left( \frac{\pi_1}{2} + \frac{\pi_2}{2} \right) \]

\[ = s_1 Q \left[ \frac{\pi_2}{2} \left( \frac{1}{\mu_1^2} + \frac{1}{\mu_1^2} \right) - \pi_2 \left( \frac{s_1 + s_2}{s_1} \right) + \left( 1 - \frac{\mu_1^2}{\mu_1^2} \right) \left( \frac{\pi_1}{2} + \frac{\pi_2}{2} \right) \right]. \]

Step 2: \( \partial \Delta (\mu_1^1, \mu_1^2) / \partial \mu_1^2 < 0 \).

Step 3: Note that the lowest possible value of \( \mu_1^2 \) is just \( s_1 / (s_1 + s_2) \). Substituting it into \( \Delta (\mu_1^1, \mu_1^2) \), and after regrouping, yields

\[ \Delta \left( \mu_1^1, \frac{s_1}{s_1 + s_2} \right) = s_1 Q \left[ \frac{\pi_2}{2} \left( \frac{1}{\mu_1^2} + \frac{s_1 + s_2}{s_1} \right) - \pi_2 + \left( 1 - \frac{s_1}{s_1 + s_2} \right) \left( \frac{\pi_1}{2} + \frac{\pi_2}{2} \right) \right] \]

\[ = s_1 Q \left[ \frac{s_2}{s_1} - \frac{2\pi_1}{\pi_2} \left( \frac{1}{\mu_1^2} - \frac{s_1 + s_2}{s_1} \right) \right]. \]

Hence, \( \Delta (\mu_1^1, s_1 / (s_1 + s_2)) > 0 \) if and only if (i) \( s_2 / s_1 > 2\pi_1 / \pi_2 \) and \( \mu_1^1 < s_1 / (s_1 + s_2) \) (rejected) or (ii) \( s_2 / s_1 < 2\pi_1 / \pi_2 \) and \( \mu_1^1 > s_1 / (s_1 + s_2) \).
Step 4: Using step 2 and step 3, as well as continuity, we establish that there exist parameters in which $\Delta \left( \mu_1^1, \mu_2^1 \right) > 0$ if $s_2/s_1 < 2\pi_1/\pi_2$ and there do not if $s_2/s_1 \geq 2\pi_1/\pi_2$.

Step 5: In the above derivation, as well as the exposition in the main text, we assumed that the right boundary of region A is on the left hand side of the left boundary of region B. This may not be the case, and there may be "overlapping" of the two regions. However, it can be easily shown that, all the calculations in this proof, as well as the proposition, still hold. The trick is that we can add the overlapping region to both the gain and loss in state 2 so that the specifications of Region A and Region B that we used in defining $\Delta$ remains the same. It is thus without loss of generality that we assume no overlapping of the two regions. Q.E.D.

Proof of Result 4 of Proposition 4: We first reckon how school 2's composition of admitted applicants is changed under (IA,IA) relative to (DA,IA). We first note that, during this exercise, in state 2, school 1 admits those type-1 applicants with $y_1 \in (c_2, c_1']$, replacing those type-2 applicants with $y_1 \in (c_2', c_2]$, where $c_1'$ satisfies

$$
\mu_1^2 N \left( F(c_1') - F(c_2) \right) = \mu_2^2 N \left( F(c_2) - F(c_2') \right)
$$

or

$$
c_1' = F^{-1} \left( \frac{s_1 + s_2}{N} + \frac{\mu_2^2}{\mu_1^2} |A| \right).
$$

In uniform distribution,

$$
c_1' = c_2 + x'
$$

$$
x' = x \left( 1/\mu_1^2 - 1 \right) \text{ and } x = c_2 - c_2' = s_1/N \left( 1 - \mu_2^2/\mu_1^1 \right).
$$

Back to state 1, school 2 needs to replace those type-1 applicants with $y_1 \in (c_2', c_2]$ by applicants with $y_1 > c_2$. Those applicants will be solely type-2 applicants if type-2 applicants with $y_1 \in (c_2, c_1']$ are sufficiently numerous for the replacement and will be of both types otherwise.
Case 1: If the replacement is solely by type-2 applicants, then the replacement actually comes from type-2 applicants with
\[ y_1 \in \left( (s_1 + s_2) / N, (s_1 + s_2) / N + z \right) \]
where
\[ z = \frac{1}{\mu_2^1} - 1. \]
This case happens when \( \mu_1^2 < \mu_2^1 \).

Case 2: If the replacement is by both types of applicants, then the replacement actually comes from type-2 applicants with
\[ y_1 \in \left( (s_1 + s_2) / N, (s_1 + s_2) / N + z \right) \]
and type-1 applicants with
\[ y_1 \in \left( (s_1 + s_2) / N + x', (s_1 + s_2) / N + z \right), \]
where
\[ z - x' = \mu_1^2 x - \mu_2^1 x'. \]
This case happens when \( \mu_1^2 > \mu_2^1 \).

Now recall that
\[ \Delta = \pi_1 \left[ n(C)m(C) - n(B')m(B') \right] + \pi_2 \left[ n(A)m(A) - n(B)m(B) \right]. \]  \hspace{1cm} (A3)

Notice that \( n(A) = n(B) \), denoted as \( Q \), and \( n(B') = n(C) = \mu_1^1 n(B) \). Notice also that
\[ m(A) = \frac{1}{2} \left( \frac{s_1}{\mu_1^1 N} + \frac{s_1}{\mu_2^1 N} \right), \quad m(B') = m(b) = \frac{s_1 + s_2}{N} - \frac{x}{2} \quad \text{and} \quad m(C) = \frac{s_1 + s_2}{N} + Rx, \]
where \( R \) is a positive term. We thus have
\[ \Delta (\mu_1^1, \mu_2^1) = Q \left[ \pi_2 m(A) - (\pi_2 + \pi_1 \mu_1^1) m(B) + \pi_1 \mu_1^1 m(C) \right] \]
\[ = Q \left[ \frac{\pi_2}{2} \left( \frac{s_1}{\mu_1^1 N} + \frac{s_1}{\mu_2^1 N} \right) - (\pi_2 + \pi_1 \mu_1^1) \left( \frac{s_1 + s_2}{N} - \frac{x}{2} \right) + \pi_1 \mu_1^1 \left( \frac{s_1 + s_2}{N} + Rx \right) \right]. \]
Note that the lowest possible value of \( \mu_1^2 \) is just \( s_1 / (s_1 + s_2) \). Substituting it into \( \Delta (\mu_1^1, \mu_2^1) \), and after regrouping, yields
\[ \Delta \left( \mu_1^1, \frac{s_1}{s_1 + s_2} \right) = \left( - \frac{s_1 + s_2}{s_1} + \frac{1}{\mu_1^1} \right) \left( \frac{s_2}{s_1} - \mu_1^1 (1 + 2R) \frac{\pi_1}{\pi_2} \right). \]
Hence, $\Delta \left( \mu_1, s_1/(s_1 + s_2) \right) > 0$ if and only if (i) $s_2/s_1 > \mu_1^1(1 + 2R)\pi_1/\pi_2$ and $\mu_1^1 < s_1/(s_1 + s_2)$ (rejected) or (ii) $s_2/s_1 < \mu_1^1(1 + 2R)\pi_1/\pi_2$ and $\mu_1^1 > s_1/(s_1 + s_2)$. As the second condition is always satisfied, we can focus just on the first one. Hence, $\Delta \left( \mu_1^1, s_1/(s_1 + s_2) \right) > 0$ if and only if

$$\frac{s_2}{s_1} < \mu_1^1(1 + 2R)\frac{\pi_1}{\pi_2}. \quad \text{(A4)}$$

If Case 1 is the case, then $R = z/2x$, (A4) becomes $s_2/s_1 < \mu_1^1/(1 - \mu_1)\pi_1/\pi_2$. If Case 2 is the case, then given $\mu_1^1$ and $\mu_2^2$, $R$ is bounded below by zero, and in particular $R > x'/2x = (1/\mu_1^2 - 1)/2 = \mu_1^1/2(1 - \mu_1)$. Hence, taking into account continuity, a sufficient condition for existence of the preemptive equilibrium is that $s_2/s_1 < \mu_1^1/(1 - \mu_1) \times \pi_1/\pi_2$. Q.E.D.

**Proof of Proposition 5:** Here, we prove point 3 (the other parts are straightforward).

We first note that, in equilibrium, school 1’s cutoff value must be more stringent than school 2’s cutoff value; or else, each applicant would strictly prefer to make school 1 her top choice, violating the fact that school 1’s cutoff value is less stringent. This implies that the applicant who makes school 2 her top choice and is subsequently declined by school 2 will not be admitted by school 1. We next note that $b$, the number of applicants making school 2 their top choice, must exceed $s_2$. (Suppose not. Then, by making school 2 the top choice, an applicant is certain to receive a utility of $u_2$—she is admitted by school 2 for certain. By ranking school 1 as her top choice, the applicant is admitted by school 1 with probability $p = s_1/(N - b)$ and by school 2 with probability $q = (1 - p)(s_2 - b)/(N - s_1 - b)$. The equilibrium dictates that $u_2 = pu_1 + qu_2$, or $u_2/u_1 = s_1/(N - s_2)$, which is contradictory.)

Given these two facts, we learn that, in equilibrium, no applicant will be admitted by a school that she did not specify as her top choice. Hence, indifference between the two strategies means: $s_2u_2/b = s_1u_1/(N - b)$, implying

$$b = \frac{s_2u_2N}{s_1u_1 + s_2u_2}.$$  

To show the last result, we first note that $AQ_2(DA, IA) = (s_1u_1 + s_2u_2)/2u_2N$. Under
(DA,DA), all applicants choose school 1 as their top choice. Then, school 1 admits those applicants with \( y_1 \leq c_1 = s_1/N \) and school 2 admits those applicants with \( y_1 \in (c_1, c_2) \) where \( c_2 = (s_1 + s_2)/N \). As a result, \( AQ_2(DA, DA) = (2s_1 + s_2)/2N \), which is strictly greater than \( AQ_2(DA, IA) \) if and only if \( u_2/u_1 > 1/2 \). Q.E.D.

Proof of Proposition 7: We first note that it must be the case that \( b > s_2 \). Suppose not. By specifying school 1 as her top choice, an applicant is admitted to school 1 with probability \( s_1/a \) and to school 2 with probability \((s_2-b)/a\), resulting in a utility of \( s_1u_1/a + (s_2-b)u_2/a \). By specifying school 2 as her top choice, an applicant is certain to be admitted to school 2 and obtain a utility of \( u_2 \). But

\[
  u_2 > \frac{s_1}{a} u_1 + \frac{s_2 - b}{a} u_2 \iff \frac{u_2}{u_1} > \frac{s_1}{N - s_2}.
\]

Thus, everybody indeed wants to make school 2 her top choice, and this is contradictory to the claim that, in equilibrium, \( b \leq s_2 \). Given that \( b > s_2 \), it is easy to reckon that school 1 will set a cutoff value of \( c_1 = s_1/(N - s_2) \). By reporting 1 as her top choice, an applicant is admitted to school 1 with probability \( c_1 \) and to school 2 with probability zero; by reporting 2 as her top choice, the applicant is admitted to school 2 with probability \( s_2/b \) and to school 1 with probability \((1 - s_2/b)c_1 \). Hence, making 2 as the top choice is strictly better if and only if

\[
  c_1 u_1 < \frac{s_2}{b} u_2 + \left(1 - \frac{s_2}{b}\right) c_1 u_1 \iff \frac{u_2}{u_1} > c_1 = \frac{s_1}{N - s_2},
\]

suggesting that, in equilibrium, \( b = N \). Q.E.D.

Proof of Proposition 8: Under (IA,IA) in equilibrium, the numbers of applicants who specify school 1 and who specify school 2 as their top choice are \( a = Ns_1u_1/(s_1u_1 + s_2u_2) > s_1 \) and \( b = Ns_2u_2/(s_1u_1 + s_2u_2) > s_2 \), respectively. Moreover, the average qualities of the admitted applicants are

\[
  AQ_1(IA, IA) = \frac{s_1u_1 + s_2u_2}{2u_1} \quad \text{and} \quad AQ_2(IA, IA) = \frac{s_1u_1 + s_2u_2}{2u_2}.
\]
By simple comparison, we reckon that $AQ_1(DA, IA) < AQ_1(IA, IA)$ and $AQ_2(IA, DA) < AQ_2(IA, IA)$. Q.E.D.
Appendix B:

Regarding the noisy signal model introduced in Section 4, we can establish the following claim.

**Claim 2** Suppose that $\alpha_1 = \alpha_2 = 0$, that each applicant’s attribute $(y_1, y_2)$ is unknown to her, and that she has a signal $x_1$ which is affiliated with her attribute $y_1$.

1. Under (DA,IA), there exists $t \in (0, 1)$ such that if $u_2/u_1 < t$, there exists a truthful equilibrium.

2. Under (DA,IA), there exists $\bar{t} \in (0, 1)$ such that if $u_2/u_1 \geq \bar{t}$, then (a) there does not exist a truthful equilibrium, and (b) there exists an equilibrium in which applicants with $x < x^*$ ($x > x^*$) rank school 1 (school 2) as their top choice and, in this equilibrium, $AQ_2(DA, IA) < AQ_2(DA, DA)$ (beneficial stealing).

3. (IA,DA) is outcome equivalent to (DA,DA).

4. Suppose that both schools are allowed to choose IA simultaneously. If $u_2/u_1 \geq \bar{t}$, the equilibrium outcome is exactly the same as (DA,IA).

**Proof of the Claim:** Point 1. A truthful equilibrium exists under (DA,IA) if nobody benefits from unilaterally deviating from truthful reporting even others report truthfully. Given affiliated signals, we need only to ensure no such beneficial deviation by the applicant with $x_1 = 1$:

$$u_2 \leq u_1 G(c_1|1) + u_2 (G(c_2|1) - G(c_1|1))$$

where the left-hand side is her expected payoff from deviation and the right-hand side is her expected payoff from truthful reporting (to abuse the notation a little bit, $G(y_1|x_1)$ is the cumulative function of $y_1$ conditional on the signal $x_1$). The inequality can be written as

$$\frac{u_2}{u_1} \leq \frac{G(c_1|1)}{1 - G(c_2|1) + G(c_1|1)} \equiv t < 1.$$  

Point 2. Under (DA,IA), assume that those with $x_1 < x_1^*$ ($x_1 > x_1^*$) rank school 1 (school 2) as their top choice and also that both schools are oversubscribed. Denote by $c'_1$ and $c'_2$
as their corresponding cutoffs. Then, \( x_1^*, c'_1, \) and \( c'_2 \) are solved by

\[
G(c'_1 | x_1^*) u_1 = G(c'_2 | x_1^*) u_2, \quad (A5)
\]

\[
\int_{y_1=0}^{y_1=c'_1} \int_{x_1=0}^{x_1=x_1^*} g(x_1, y_1) \, dx_1 \, dy_1 = \frac{s_1}{N},
\]

\[
\int_{y_1=0}^{y_1=c'_2} \int_{x_1=x_1^*}^{x_1=1} g(x_1, y_1) \, dx_1 \, dy_1 = \frac{s_2}{N}.
\]

Clearly, such \( x_1^*, c'_1, \) and \( c'_2 \) exist such that both schools are oversubscribed. Also, the affiliation between \( x_1 \) and \( y_1 \) ensures that it is indeed an equilibrium. Now note that as \( u_2/u_1 \) approaches unity, (A5) suggests that \( c'_1 = c'_2 = c_2 \). Relative to under (DA,DA), school 2 now admits some better applicants with \( y_1 \) in \( (0, c_1] \) and does not admit any student of worse quality. Hence, school 2 clearly benefits from using IA. Because of continuity, there exists \( t \in (0, 1) \) with the claimed properties.

Point 3 is straightforward.

Point 4. If \( u_2/u_1 \geq \bar{t} \) and under (DA,IA), both schools are oversubscribed and school 1 has a more stringent cutoff than school 2 has. This implies that no applicants choosing school 2 as their top choice who get turned down will ever be admitted by school 1. Thus the (DA,IA) is de facto (IA,IA) and applicants’ behaviors would be the same under the two regimes. Thus, school 2’s optimal admissions policy is independent of school 1’s policy and the equilibrium outcome is the same as (DA,IA). Q.E.D.
References


Figure 1: Under (DA,DA), the two schools set cutoffs, $c_k^1$ and $c_k^2$, in state $k, k = 1, 2$. Provided that $u_2 > u_1$, the number in each region indicates the school the applicant enters in equilibrium in (DA,IA). In state 2, school 2 succeeds in stealing type 1 applicants in Region A, in lieu of those in Region B (no change in state 1).
Figure 2: Case (ii). The number inside each region shows the placement of applicants in the region in state 1 under (IA,IA). Applicants in Region B will report school 1 as their top choice. There is no change in placement from (DA,IA) to (IA,IA) in state 2, which is not depicted.
Figure 3: Case (iii). The number inside each region shows the placements of the applicants in the region under (IA,IA).
Figure 4: The two dotted lines show the cutoffs under (DA,DA). The two shaded regions show the admitted students under (DA,IA).
Figure 5: The truthful outcome in each state is depicted. School 2’s IA policy may be able to steal some type-1 applicants (Region A) in state 2 and some type-3 applicants (Region B) in state 1.
Figure 6: School 3’s IA may want to steal type-1 and type 2 applicants from school 2 (Region B), and School 2’s IA may want to steal type-1 applicants from school 1 (Region A). The lack (presence) of a beneficial stealing effect by school 2 hinders (enhances) its preemption against school 3’s IA.