Flipping in the Housing Market – Extensions and Robustness

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Abstract

In this technical note, we study four extensions to the "Flipping in the Housing Market" model and check how the major results may or may not be robust to these extensions.

1 Competitive search equilibrium

The search market is segmented into submarkets, each of which is controlled by a market maker. A market maker charges (flow) entry fees $\phi_b(\theta, p)$ for buyers and $\phi_s(\theta, p)$ for sellers for buying and selling in his submarket in return for promising a tightness equal to θ and regulating the transaction price to be equal to the given p. There is free entry into market making and therefore in equilibrium, the market makers earn zero profit and end up charging zero entry fees. Each household buyer, household seller and flipper seller chooses which submarket, taking as given the fee schedule, to enter into to maximize the respective expected returns of buying and selling.

1.1 Value functions

The value to a flipper seller of selling in a submarket with tightness θ at price p is given by

$$r_F V_F(\theta, p) = \eta(\theta)(p - V_F(\theta, p)) - \phi_s(\theta, p). \quad (1)$$

Flipper sellers choose which submarket to sell to maximize $V_F(\theta, p)$. We write

$$V_F = \max V_F(\theta, p).$$

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The value to a household seller of selling in a submarket with tightness θ at price p is given by

$$rV_U(\theta, p) = \eta(\theta)(V_R + p - V_U(\theta, p)) - \phi_s(\theta, p). \tag{2}$$

These agents choose which submarket to sell to maximize $V_U(\theta, p)$. We write

$$V_U = \max V_U(\theta, p).$$

The value to a household buyer of buying in a market with tightness θ at price p is given by

$$rV_R(\theta, p) = -q + \mu(\theta)(V_M - p - V_R(\theta, p)) - \phi_b(\theta, p), \tag{3}$$

These agents choose which submarket to buy to maximize $V_R(\theta, p)$. We write

$$V_R = \max V_R(\theta, p).$$

Finally, the value function for matched households remains equal to

$$rV_M = \upsilon + \delta(\max\{V_R + V_F, V_U\} - V_M). \tag{4}$$

1.2 Market makers

A market maker maximizes profit per seller, given by,

$$\pi(\theta, p) = \phi_s(\theta, p) + \theta\phi_b(\theta, p).$$

subject to

$$V_R(\theta, p) = V_R,$$

and

$$V_U(\theta, p) = V_U,$$

if the sellers attracted are household sellers or

$$V_F(\theta, p) = V_F,$$

if the sellers attracted are flipper sellers. In equilibrium,

$$\pi(\theta, p) \leq 0,$$

for all (θ, p), with equality for any active submarkets.

1.3 Accounting identities

The same accounting identities,

$$n_M + n_U + n_R = 1, \tag{5}$$

$$n_M + n_U + n_F = H, \tag{6}$$

as in the original model hold.

1.4 stock-flow equations

There are potentially two kinds of submarkets. The first ones are submarkets that attract flipper sellers and the second ones are submarkets that attract household sellers. Let θ_F and θ_U be the respective tightness in the two kinds of submarkets and λ be the fraction of household buyers who buy in those submarkets with flipper sellers. By definition,

$$\theta_F = \frac{\lambda n_R}{n_F}, \tag{7}$$

$$\theta_U = \frac{(1-\lambda)\, n_R}{n_U}. \tag{8}$$

The steady-state stock-flow equations are as follows,

$$\dot{n}_M = 0 : ((1-\lambda)\,\mu(\theta_U) + \lambda\mu(\theta_F))\, n_R = \delta n_M, \tag{9}$$

$$\dot{n}_U = 0 : (1-\alpha)\,\delta n_M = \eta(\theta_U)\, n_U, \tag{10}$$

$$\dot{n}_R = 0 : \alpha\delta n_M + \eta(\theta_U)\, n_U = ((1-\lambda)\,\mu(\theta_U) + \lambda\mu(\theta_F))\, n_R, \tag{11}$$

$$\dot{n}_F = 0 : \alpha\delta n_M = \eta(\theta_F)\, n_F, \tag{12}$$

where α, as in the original model, is the fraction of mismatched households selling to flippers in the first instance.

1.5 Market makers profit maximization

By (1)-(3), the respective entry fees paid by household buyers, flipper sellers, and household sellers are as follows:

$$\phi_b(\theta, p) = -q + \mu(\theta)(V_M - p) - (r + \mu(\theta))\, V_R(\theta, p),$$

$$\phi_s(\theta, p) = \eta(\theta)\, p - (r_F + \eta(\theta))\, V_F(\theta, p),$$

$$\phi_s(\theta, p) = \eta(\theta)(V_R + p) - (r + \eta(\theta))\, V_U(\theta, p).$$

For a market maker that attracts flipper sellers to his submarket,

$$\pi(\theta, p) = \eta(\theta)(V_M - V_R - V_F) - r_F V_F - \theta r_H V_R - q\theta.$$

In maximizing profit, the agent chooses tightness θ_F to satisfy

$$\eta'(\theta_F)(V_M - V_R - V_F) - rV_R - q = 0. \tag{13}$$

Given free entry,

$$\eta(\theta_F)(V_M - V_R - V_F) - r_F V_F - \theta_F r V_R - q\theta_F \le 0. \tag{14}$$

For a market maker that attracts household sellers to his submarket,

$$\pi(\theta, p) = \eta(\theta)(V_M - V_U) - rV_U - \theta r_H V_R - q\theta.$$

In maximizing profit, the agent chooses tightness θ_U to satisfy

$$\eta'(\theta_U)(V_M - V_U) - rV_R - q = 0. \tag{15}$$

Given free entry,

$$\eta(\theta_U)(V_M - V_U) - rV_U - \theta_U rV_R - q\theta_U \leq 0. \tag{16}$$

1.6 Stock-flow equations

Substituting (10) into (11) gives (9). Substituting $\mu = \eta/\theta$, (8) and (7) into (11), the last equation becomes identical to (12). Thus, only two of (9)-(12) constitute independent restrictions. It is easiest to work with (10) and (12). Solving (5), (6), (10), and (12) for

$$n_U = \frac{(1-\alpha)\delta\eta(\theta_F)}{(\alpha\delta + \eta(\theta_F))\eta(\theta_U) + (1-\alpha)\delta\eta(\theta_F)}H,$$

$$n_M = \frac{\eta(\theta_U)\eta(\theta_F)}{(\alpha\delta + \eta(\theta_F))\eta(\theta_U) + (1-\alpha)\delta\eta(\theta_F)}H,$$

$$n_F = \frac{\alpha\delta\eta(\theta_U)}{(\alpha\delta + \eta(\theta_F))\eta(\theta_U) + (1-\alpha)\delta\eta(\theta_F)}H,$$

$$n_R = \frac{\alpha\delta\eta(\theta_U) + (\eta(\theta_U) + (1-\alpha)\delta)\eta(\theta_F)(1-H)}{(\alpha\delta + \eta(\theta_F))\eta(\theta_U) + (1-\alpha)\delta\eta(\theta_F)}.$$

Then, by (7) and (8), respectively,

$$\theta_U = (1-\lambda)\frac{\alpha\delta\eta(\theta_U) + (\eta(\theta_U) + (1-\alpha)\delta)\eta(\theta_F)(1-H)}{(1-\alpha)\delta\eta(\theta_F)H}, \tag{17}$$

$$\theta_F = \lambda\frac{\alpha\delta\eta(\theta_U) + (\eta(\theta_U) + (1-\alpha)\delta)\eta(\theta_F)(1-H)}{\alpha\delta\eta(\theta_U)H}. \tag{18}$$

1.7 Equilibrium

1.7.1 Fully-intermediated equilibrium

In a fully-intermediated equilibrium, by (4),

$$V_M = \frac{\upsilon + \delta(V_R + V_F)}{r + \delta}.$$

Then, by $(13)-(16)$, assuming zero profit holds exactly,

$$V_R = \frac{\upsilon r_F \eta'(\theta_F) - (r(\eta(\theta_F) - \eta'(\theta_F)\theta_F) + r_F(r+\delta))\,q}{r(r(\eta(\theta_F) - \eta'(\theta_F)\theta_F) + r_F(r+\delta+\eta'(\theta_F)))},$$

$$V_F = \frac{(\eta(\theta_F) - \eta'(\theta_F)\theta_F)(\upsilon+q)}{(r(\eta(\theta_F) - \eta'(\theta_F)\theta_F) + r_F(r+\delta+\eta'(\theta_F)))},$$

$$V_U = \frac{r_F\eta'(\theta_F)(\eta(\theta_U) - \eta'(\theta_U)\theta_U)(\upsilon+q)}{r\eta'(\theta_U)(r(\eta(\theta_F) - \eta'(\theta_F)\theta_F) + r_F(r+\delta+\eta'(\theta_F)))},$$

$$1 + \left(\frac{\eta(\theta_F)}{\eta'(\theta_F)} - \theta_F\right)\frac{r}{r_F} + \frac{r - z\delta}{\eta'(\theta_F)} - \frac{(1+z)(\eta(\theta_U) - \eta'(\theta_U)\theta_U + r)}{\eta'(\theta_U)} = 0. \quad (19)$$

In this case, $V_R + V_F - V_U$ can be shown to have the same sign as

$$S = \left(1 + \left(\frac{\eta(\theta_F)}{\eta'(\theta_F)} - \theta_F\right)\frac{r}{r_F}\right)\eta'(\theta_F) - \frac{\eta'(\theta_F)}{\eta'(\theta_U)}(\eta(\theta_U) - \eta'(\theta_U)\theta_U)(1+z) - (r+\delta)\,z.$$

Substituting from (19),

$$S = r\frac{(1+z)(\eta'(\theta_F) - \eta'(\theta_U))}{\eta'(\theta_U)},$$

which has the same sign as $\theta_U - \theta_F$.

1.7.2 No-Intermediation and partially-intermediated equilibrium

In a no-intermediation or a partially-intermediated equilibrium, by (4),

$$V_M = \frac{\upsilon + \delta V_U}{r+\delta}.$$

Then, by $(13)-(16)$, assuming zero profit holds exactly,

$$V_R = \frac{\eta'(\theta_U)\upsilon - (\eta(\theta_U) - \theta_U\eta'(\theta_U) + \delta + r)\,q}{r(\eta(\theta_U) - \theta_U\eta'(\theta_U) + \delta + r)},$$

$$V_F = \frac{(\eta(\theta_F) - \theta_F\eta'(\theta_F))\,\eta'(\theta_U)\,\upsilon}{\eta'(\theta_F)\,r_F(\eta(\theta_U) - \theta_U\eta'(\theta_U) + \delta + r)},$$

$$V_U = \frac{(\eta(\theta_U) - \theta_U\eta'(\theta_U))\,\upsilon}{r(\eta(\theta_U) - \theta_U\eta'(\theta_U) + \delta + r)},$$

$$1 + \left(\frac{\eta(\theta_F)}{\eta'(\theta_F)} - \theta_F\right)\frac{r}{r_F} + \frac{r}{\eta'(\theta_F)} - \frac{\delta z + (1+z)(\eta(\theta_U) - \theta_U\eta'(\theta_U) + r)}{\eta'(\theta_U)} = 0. \quad (20)$$

In this case, $V_R + V_F - V_U$ can be shown to have the same sign as

$$S = \left(1 + \left(\frac{\eta(\theta_F)}{\eta'(\theta_F)} - \theta_F\right)\frac{r}{r_F}\right)\eta'(\theta_U) - (\eta(\theta_U) - \theta_U\eta'(\theta_U))(1+z) - (\delta+r)\,z.$$

Substituting from (20),

$$S = r\frac{\eta'(\theta_F) - \eta'(\theta_U)}{\eta'(\theta_F)},$$

which has the same sign as $\theta_U - \theta_F$.

1.7.3 Nature of equilibrium

There is a full-intermediation equilibrium if $\theta_U - \theta_F > 0$, with θ_U and θ_F satisfying (19), whereas there is a no-intermediation or partially-intermediated equilibrium if $\theta_U - \theta_F \leq 0$, with θ_U and θ_F satisfying (20).

Both (19) and (20) define a positive relationship between the two θs. More importantly, for sufficiently small θ_F, both restriction imply $\theta_U - \theta_F > 0$. Then, there can only be a full-intermediation equilibrium for such θs. And of course, in a full-intermediation equilibrium, there is only one market tightness, given by θ_F. Setting $\theta_U = \theta_F = \theta$ in (19) and (20), respectively, yield the same equation,

$$\eta'(\theta) + (\eta - \theta\eta'(\theta))\left(\frac{r}{r_F} - (1+z)\right) - z(\delta + r) = 0. \tag{21}$$

In a partially-intermediated equilibrium, if one exists, $\theta_U = \theta_F$ and the common market tightness is at the θ that satisfies (21). Now, the LHS of (21) starts out equal to positive infinity at $\theta = 0$ and is decreasing thereafter, eventually falling below zero for large θ if

$$r_F \geq \frac{r}{1+z} \equiv \overline{r}_C. \tag{22}$$

In this case, a unique solution to (21) indeed exists, at which a partially-intermediated equilibrium holds. For any larger θ_U, given the continuity of (19) and (20), by either restriction, $\theta_U - \theta_F < 0$. There can only be a no-intermediation equilibrium then at a market tightness given by θ_U.

If (22) holds in reverse, the LHS of (21) is U-shaped, at first decreasing but eventually becoming increasing. There is either no solution or two solutions to the equation. In the first case, $\theta_U - \theta_F > 0$, for all θ_U and θ_F satisfying (19) and (20), respectively; there is always only a full-intermediation equilibrium. In the latter case, again given the continuity of (19) and (20), there are two cutoff values of θ, given by the two solutions to (21). At exactly the two cutoffs, there can be a partially-intermediated equilibrium. In between, there can only be a no-intermediation equilibrium. For θ below the first cutoff and above the second cutoff, there can only be a full-intermediation equilibrium.

Summarizing, there is always only one market tightness in equilibrium. This is obviously true for either the full or no-intermediation equilibrium. This is also true for a partially-intermediated equilibrium because at such an equilibrium, $\theta_U = \theta_F$. What kind of equilibrium holds at what θ is determined by the LHS of (21) – when it is positive, $\alpha = 1$; when it is negative, $\alpha = 0$; when it is equal to 0, $\alpha \in [0,1]$. Call the RHS of (21) $C(\theta)$. Then,

$$\alpha = \begin{cases} 1 & C(\theta) > 0 \\ [0,1] & C(\theta) = 0 \\ 0 & C(\theta) < 0 \end{cases} .$$

Recall the system of equations given by (17) and (18). Clearly, in equilibrium when $\alpha = 0$, $\lambda = 0$ and when $\alpha = 1$, $\lambda = 1$. Furthermore, dividing one equation by the other,

$$\frac{\theta_U}{\theta_F} = \frac{1-\lambda}{\lambda} \frac{\alpha}{1-\alpha} \frac{\eta(\theta_U)}{\eta(\theta_F)},$$

which implies that $\lambda = \alpha$ in a partially-intermediated equilibrium with $\theta_U = \theta_F$ in any such equilibria. That is, in equilibrium, the two equations (17) and (18) become one and the same

$$\delta\eta H\theta = \alpha\delta\eta + (\eta + (1-\alpha)\delta)\eta(1-H),$$

from which a function $\theta = \theta_S(\alpha)$ can be defined. But this is the same stock-flow equation that relates α to θ in the original model. In sum, in equilibrium,

$$\alpha = \begin{cases} 1 & C(\theta_S(1)) \geq 0 \\ \alpha_C & C(\theta_S(\alpha_C)) = 0 \\ 0 & C(\theta_S(0)) \leq 0 \end{cases} .$$

Given that $C(\theta)$ is U-shaped for $r < \overline{r}_C$, there can be multiple equilibria here, just as in the original model.

2 Two-house-limit-liquidity constraint

Consider a setting in which there is only owner-occupied housing but not rental housing. Households can hold at most two houses at the same time and they cannot become homeless. Thus, when a household becomes mismatched, the household must remain in the old house. As long as the household is not already holding two houses, it can try to buy a new house. Whenever the household is holding two houses, however, it must sell one first before entering the search market as a buyer.

2.1 States

There are four states to which a household can belong:

States	measure
Holding one house, matched	$n_{M,1}$
Holding one house, mismatched	$n_{U,1}$
Holding two houses, matched with one	$n_{M,2}$
Holding two houses, mismatched with both	$n_{U,2}$

2.2 Accounting identities

The measures of households in the four states sum to the population of households in the market,

$$n_{M,1} + n_{U,1} + n_{M,2} + n_{U,2} = 1, \tag{23}$$

whereas the measures of houses held by households in the four states plus the measure of houses held by flippers sum to the housing stock,

$$n_{M,1} + n_{U,1} + 2n_{M,2} + 2n_{U,2} + n_F = H. \tag{24}$$

The buyers are mismatched households holding just one houses:

$$B = n_{U,1}.$$

The houses for sale are those held by flippers plus the second houses held by households holding two houses,

$$S = n_F + n_{M,2} + n_{U,2}.$$

Thus,

$$\theta = \frac{n_{U,1}}{n_F + n_{M,2} + n_{U,2}}. \tag{25}$$

2.3 Stock-flow equations

When a one-house mismatched household manages to find a new house to move into, the household can either sell the old house to a flipper or to offer it for sale in the search market. Let α_M be the fraction of these households who choose to sell to flippers. When a matched household is hit by a moving shock again before it manages to sell the previously mismatched house, the household is holding two mismatched house. The household can sell one either to a flipper right away or offer it for sale in the search market. Let α_U be the fraction of these households choosing to sell in the investment market. Figure 1 explains the flows of households into and out of the four possible states from which the following steady-state stock-flow equations are defined.

$$\dot{n}_{M,1} = 0 : \delta n_{M,1} = \mu \alpha_M n_{U,1} + \eta n_{M,2} \tag{26}$$

$$\dot{n}_{U,1} = 0 : \mu n_{U,1} = \delta n_{M,1} + \eta n_{U,2} + \delta \alpha_U n_{M,2} \tag{27}$$

$$\dot{n}_{M,2} = 0 : (\delta + \eta) n_{M,2} = \mu (1 - \alpha_M) n_{U,1} \tag{28}$$

$$\dot{n}_{U,2} = 0 : \eta n_{U,2} = \delta (1 - \alpha_U) n_{M,2} \tag{29}$$

$$\dot{n}_F = 0 : \eta n_F = \mu \alpha_M n_{U,1} + \delta \alpha_U n_{M,2} \tag{30}$$

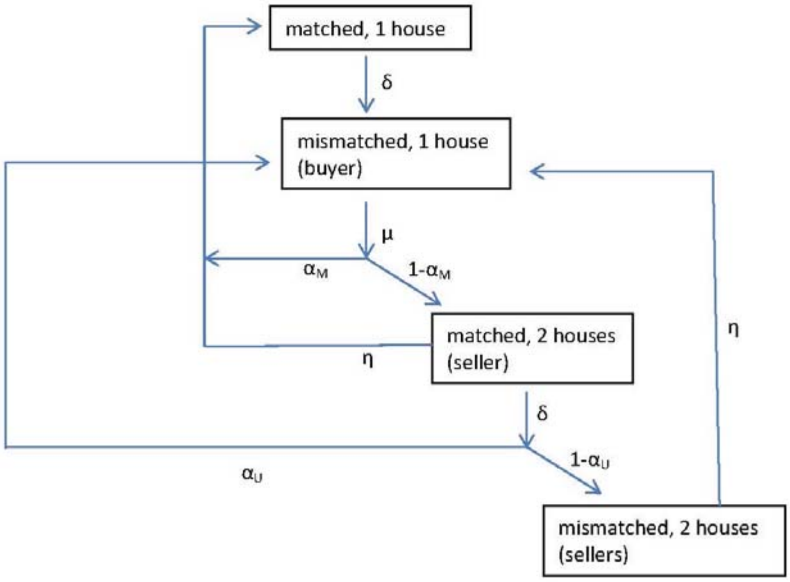

Figure 1: Flows: two-house-liquidity constraint

By $\mu = \eta/\theta$ and (25), we can show that (26), (27), and (29) imply (28) and then (26) and (30) imply (27). Thus, we only have to work with (26), (29) and (30). Together with (23) and (24), the solutions of the five states are as follows

$$n_{M,1} = \frac{\eta\left(\delta\alpha_M + \eta\right)}{\delta\left(\eta+\delta\right)}\left(H-1\right),$$

$$n_{U,1} = 1 - \frac{\eta\left(\eta+\delta\right) + \delta^2\left(1-\alpha_M\right)\left(1-\alpha_U\right)}{\delta\left(\eta+\delta\right)}\left(H-1\right),$$

$$n_{U,2} = \frac{\delta}{\eta+\delta}\left(1-\alpha_M\right)\left(1-\alpha_U\right)\left(H-1\right), \tag{31}$$

$$n_{M,2} = \frac{\eta}{\eta+\delta}\left(1-\alpha_M\right)\left(H-1\right), \tag{32}$$

$$n_F = \left(1 - \frac{\left(\eta + \delta\left(1-\alpha_U\right)\right)\left(1-\alpha_M\right)}{\eta+\delta}\right)\left(H-1\right).$$

Then (25) becomes

$$\left(\left(\eta+\delta\right)\left(\delta\theta+\eta\right) + \delta^2\left(1-\alpha_M\right)\left(1-\alpha_U\right)\right)\left(H-1\right) = \delta\left(\eta+\delta\right),$$

which implicitly defines θ as a function of α_M and α_U as long as $H \in (1,2)$. In this case, $\partial\theta/\partial\alpha_M \geq 0$ with strict inequality if $\alpha_U < 1$ and $\partial\theta/\partial\alpha_U \geq 0$ with strict inequality if $\alpha_M < 1$. For $\alpha_U = 1$, market tightness θ does not depend on α_M and similarly for $\alpha_M = 1$, θ does not depend on α_U.

There are two events for households at which a decision has to be made whether or not to sell to flippers: (1) when a one-house mismatched household finds a new house to buy and move into, (2) when a two-house matched household is hit by a moving shock.

For the first event, the old house is on the market in any case and the household is not looking for a house to buy any more. As such, it appears that whether or not the household is selling the old house to flippers should have no effects on the measures of buyers and sellers. However, if the household is not selling to flippers right away, it faces the prospect of becoming a two-house mismatched household when it is hit by a moving shock again before it is able to sell the old house in the search market. A two-house mismatched household is liquidity constrained and cannot enter the market as a buyer despite being mismatched. Thus θ can be increasing in α_M. But if any two-house matched household, whenever it is hit by a moving shock, will sell one house to flippers right away ($\alpha_U = 1$), there will never be any households prevented from entering the search market as buyers whatever households choose to do when they first move into the newly matched house. In this case and only in this case, the value of α_M will have no effects on θ.

That θ can be increasing in α_U is obvious. When more two-house mismatched households sell to flippers right away, more can enter the market as buyers immediately upon being hit by a moving shock. But if $\alpha_M = 1$, there cannot be any two-house households, matched or mismatched, in the steady state to begin with. See (31) and (32). In this case then, the value we assigned for α_U has no effect on θ.

3 Short-term flips versus long-term investment

In reality, there can be two strategies for housing market investments – short-term flip versus long-term investment in which an investor holds the house for an extended period of time, earning the rental revenue in the interim and in anticipation for a certain capital gain in the medium to long term.

In this revision, a given housing stock $H > 1$ serves as both owner-occupied housing and rental housing. A fraction of the stock is held by households, either matched or mismatched. The rest are held by flippers. Flippers either

1. offer the houses for sale in the end-user market or

2. rent them out to households in a competitive rental market.

There are the same accounting identities and steady-state stock-flow equations as in the original model, except that for $\dot{n}_F = 0$, which how reads

$$\delta \alpha n_M = \eta (\theta) \lambda n_F, \tag{33}$$

and the equation for

$$\theta = \frac{n_R}{n_U + \lambda n_F},$$

where λ denotes the fraction of the flippers' housing stock that is offered for sale. In the revised model, there is the additional rental market equilibrium condition,

$$n_R = (1 - \lambda)\, n_F. \tag{34}$$

Any equilibrium obviously must be where the rental market is active; i.e., $\lambda < 1$. There remain 4 possibilities:

1. Households sell houses to flippers ($\alpha > 0$). Flippers do not sell houses to households ($\lambda = 0$), and that all houses held by flippers are rental properties. If households sell to flippers but flippers do not, n_F must grow over time. This cannot be steady state.
2. Households sell houses to flippers ($\alpha > 0$). Flippers sell houses to households ($\lambda > 0$). For the flippers' housing stock n_F to remain stationary in the steady-state equilibrium, the two flows must be equal. See (33) above.
3. Households do not sell houses to flippers ($\alpha = 0$). Flippers do not sell houses to households ($\lambda = 0$). As in (1), all houses held by flippers are rental properties.
4. Households do not sell houses to flippers ($\alpha = 0$). Flippers sell houses to households ($\lambda > 0$). Then n_F can only fall over time. This cannot be steady state.

Specifically, solving the accounting identities and the steady-state stock-flow equations, for $\alpha > 0$,

$$\begin{aligned} n_U &= \frac{(H-1)(1-\alpha)}{\alpha}, \\ n_M &= \frac{\eta (H-1)}{\alpha\delta}, \\ n_F &= \frac{(2H-1)\alpha\delta - (H-1)(\eta+\delta)}{\alpha\delta}, \\ n_R &= \frac{H\delta\alpha - (H-1)(\eta+\delta)}{\alpha\delta}, \\ \lambda &= \frac{\alpha\delta (H-1)}{(2H-1)\alpha\delta - (H-1)(\eta+\delta)}. \end{aligned} \tag{35}$$

By (35), $\alpha > 0 \Rightarrow \lambda > 0$. Rewriting the equation,

$$\alpha = \frac{\lambda (H-1)(\eta+\delta)}{\delta (\lambda (2H-1) - (H-1))}, \tag{36}$$

which says that $\lambda > 0 \Rightarrow \alpha > 0$. Eqs (35) and (36) then indeed rule out (1) and (4) above as steady state.

For $\alpha = 0$, one can show that $\lambda = 0$ must hold. This is (3) above. Then, by (34), $n_F = n_R$. In this case, by the population and housing stock accounting identities, (5) and (6), a steady state may exist only if $H = 1$. Intuitively, when all houses held by flippers are rental properties, there can be no vacant houses. In the model housing market, there is a unit measure of households each of which always occupies one house and a measure of H houses. If no houses are vacant, the rental market can only clear if $H = 1$.

If indeed $H = 1$ and $\alpha = 0$, it is straightforward to verify that the only remaining restrictions are (5) and the equation for $\dot{n}_U = 0$ at $\alpha = 0$,

$$\eta n_U = \delta n_M.$$

There are two equations in 3 unknowns $\{n_M, n_U, n_R\}$. Thus, such a steady state is indeterminate. Apparently, when neither households nor flippers sell house to each other, the exists no restrictions to pin down the steady-state value for $n_F \, (= n_R)$. What we need to do is to go into an analysis of the dynamics to check how initial conditions will lead to a no-transactions-between-households-flippers steady state.

4 Selling to flippers right before buying a new house

In original model, if a mismatched household is not selling to a flipper immediately after being hit by a moving shock, it will try to sell the old house in the search market. All this time, the household is not searching for a new match. Given instantaneous sale in the investment market, however, nothing prevents a household staying in a mismatched house from selling the old house to a flipper right before the household buys a new house. That is, strictly speaking, a mismatched household could well remain in the old house while searching for a house to buy, a one-house-limit liquidity constraint notwithstanding. And in case a new house is found first before the old house is sold, the household can then and only then just sell the old house to a flipper. Below, we modify the model to allow for this possibility.

In the revised model, a mismatched household may

1. sell right away to flippers in the investment market, move to rental housing and search for a new house to buy right after.
2. continue to stay in the mismatched house; search for a buyer for the old house and a new house to buy in the search market at the same time.
    (a) if a buyer is found first, the household moves to rental housing to continue searching for a new house.

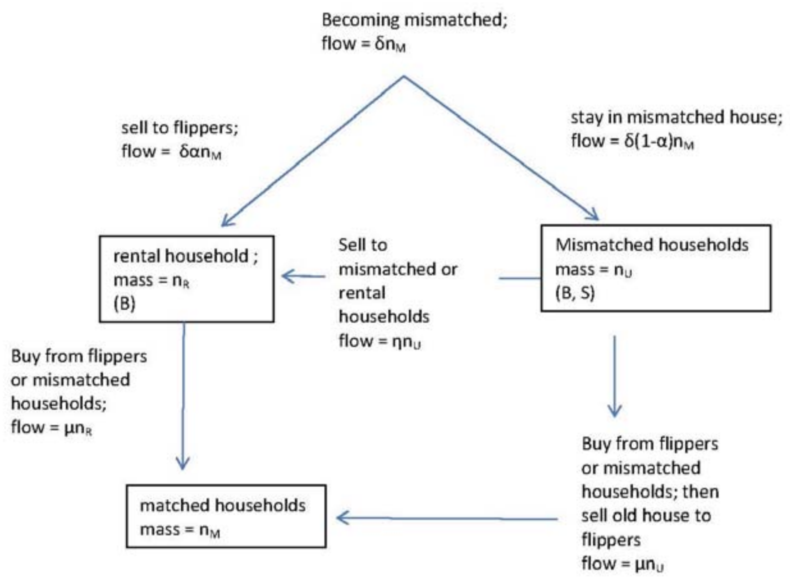

Figure 2: Flows: selling to flippers right before buying

(b) if a new house is found first, the household buys the new house and sells the old house to a flipper in the investment market at the same moment.

4.1 Accounting identities and steady-state stock-flow equations

There are the same accounting identities (5) and (6), but now given that all unmatched households are buyers in the search market,

$$\theta = \frac{n_R + n_U}{n_U + n_F}. \tag{37}$$

More specifically, Figure 2 explains the flows of households into and out of the various states, from which the following steady-state stock-flow equations can be defined.

$$\dot{n}_M = 0 \Rightarrow \mu(\theta)(n_R + n_U) = \delta n_M,$$

$$\dot{n}_U = 0 \Rightarrow \delta(1-\alpha) n_M = (\eta(\theta) + \mu(\theta)) n_U,$$

$$\dot{n}_R = 0 \Rightarrow \eta(\theta) n_U + \delta\alpha n_M = \mu(\theta) n_R,$$

$$\dot{n}_F = 0 \Rightarrow \mu(\theta) n_U + \delta\alpha n_M = \eta(\theta) n_F.$$

Notice that the equation for $\dot{n}_R = 0$ is the same equation as in the original model. But all others are now amended to reflect the fact that buyers in the search market also include households staying in the old mismatched houses.

Solving the equations for

$$n_M = \frac{\eta}{\delta + \eta} H,$$

$$n_U = \frac{(\delta + \eta - \eta H)(1 - \alpha)}{(\delta + \eta)(\delta + \eta - \eta H + H\delta)} \delta H, \tag{38}$$

$$n_F = \frac{(\delta + \eta - \eta H)\alpha + H\delta}{(\delta + \eta)(\delta + \eta - \eta H + H\delta)} \delta H, \tag{39}$$

$$n_R = \frac{(\delta + \eta)^2 + ([\eta H - 2\eta - 2\delta]\eta + [\delta + \eta - \eta H]\alpha\delta) H}{(\delta + \eta)(\delta + \eta - \eta H + H\delta)}, \tag{40}$$

Substituting (38)-(40) into (37) and simplifying yields,

$$H\delta\theta = \delta + (1 - H)\eta,$$

the solution of which gives the value for θ. Now, the important point is that in equilibrium, θ is completely independent of α. That is, in equilibrium, the tightness is the search market does not depend on how many mismatched households choosing to sell to flippers in the first instance. Come to think of it, this is not surprising. As in the original model, all "unmatched houses" are on the market, whether or not they were held by flippers or by their original owners. In the revised model, but not in the original model, however, all "unmatched households" are would-be buyers, whether or not they have sold to flippers at the outset. In this case, the buyer-seller ratio in the search market should not depend on how many households choosing to dispose their old houses right away in the investment market.

4.2 Value functions and prices

Let the fraction of flipper sellers among all sellers in the search market be

$$\phi = \frac{n_F}{n_F + n_U}, \tag{41}$$

and the fraction of rental household buyers among all buyers in the search market be

$$\gamma = \frac{n_R}{n_R + n_U}. \tag{42}$$

In the original model, $\phi = \alpha$ in the steady state. But the two are generally not equal to each other in the revised model.

There can be four types of matches in the present environment:

1. Seller - flipper

(a) Buyer - household in rental housing; price p_{FR}

(b) Buyer - household in mismatched house; price p_{FU}

2. Seller - mismatched household

(a) Buyer - household in rental housing; price p_{UR}

(b) Buyer - household in mismatched house; price p_{UU}

We then have the following value functions:

$$r_F V_F = \eta\left(\gamma p_{FR} + (1-\gamma)\, p_{FU} - V_F\right), \tag{43}$$

$$p_{FB} = V_F, \tag{44}$$

$$rV_M = \upsilon + \delta\left(\max\left\{V_R + p_{FB}, V_U\right\} - V_M\right), \tag{45}$$

$$\begin{aligned} rV_U \;=\;& \eta\left(V_R + \gamma p_{UR} + (1-\gamma)\, p_{UU} - V_U\right) + \\ & \mu\left(V_M - \phi p_{FU} - (1-\phi)\, p_{UU} + p_F - V_U\right), \end{aligned} \tag{46}$$

$$rV_R = -q + \mu\left(V_M - \left(\phi p_{FR} + (1-\phi)\, p_{UR}\right) - V_R\right). \tag{47}$$

The bargaining equations for the four types of matches are as follows:

$$(1-\beta_F)\left(p_{FR} - V_F\right) = \beta_F\left(V_M - p_{FR} - V_R\right), \tag{48}$$

$$(1-\beta_F)\left(p_{FU} - V_F\right) = \beta_F\left(V_M - p_{FU} + p_F - V_U\right), \tag{49}$$

$$(1-\beta_H)\left(V_R + p_{UR} - V_U\right) = \beta_H\left(V_M - p_{UR} - V_R\right), \tag{50}$$

$$(1-\beta_H)\left(V_R + p_{UU} - V_U\right) = \beta_H\left(V_M - p_{UU} + p_F - V_U\right). \tag{51}$$

4.3 Which market to sell?

By (38)-(40), (41) and (42) become, respectively,

$$\phi = \frac{(\delta + \eta - \eta H)\,\alpha + H\delta}{\delta + \eta - \eta H + H\delta}, \tag{52}$$

$$\gamma = \frac{(\delta+\eta)^2 + \left([\eta H - 2\eta - 2\delta]\,\eta + [\delta + \eta - \eta H]\,\alpha\delta\right) H}{(\delta+\eta)^2 + \left([\eta H - 2\eta - 2\delta]\,\eta + [\delta + \eta - \eta H]\,\delta\right) H}. \tag{53}$$

We can next solve (43)-(53) for the various asset values and prices as functions of α and substitute the resulting expressions into $D = V_R + V_F - V_U$, which is seen to have the same sign as

$$\widehat{D} = \left[r - (1+z)\, r_F\right]\eta\beta - \left((1-\beta)\,\mu + r + \delta\right) r_F z,$$

for the special case of $\beta_F = \beta_H = \beta$. For

$$r_F \geq \frac{r}{1+z}, \tag{54}$$

$\widehat{D}$ is negative for sure. That is, if flippers have no financing/bargaining advantage over ordinary households, households have no reason to sell to flippers as soon as they become mismatched. This is only be expected since households can search for a new match while staying in and trying to sell the old house. This effectively liberates them from any liquidity constraint to which they may be subjected. But if flippers do possess a sufficiently large financing advantage, whereby the condition in (54) is reversed, $\widehat{D}$ is monotone increasing in θ and can become positive for sufficiently large θ. This corresponds to where the financing advantage enables flippers to pay a high enough price to lure households to sell right away in the investment market.

In all, equilibrium is unique. Mismatched households may or may not find it optimal to sell to flippers in the first instance. No matter, equilibrium tightness in the search market is isomorphic to households's decisions on whether to use the investment market at the outset.

4.4 Selling first before entering the search market as buyers

There are a number of possibilities to defend our original assumption that households must first sell the old house before entering the search market as buyers.

1. Consider a discrete time version of the model. Say a period is divided into two subperiods where the investment market is open in the first subperiod only and the search market is open next in the second subperiod.

2. Selling in the investment market is not literally instantaneous but takes a short while. Then, buying in the search market and selling in the investment market simultaneously is not really feasible. The original model is the limit of this small-time-lag-in-investment-market model, when the time lag goes to zero.

3. There is a certain exogenously given probability that a household has no access to the investment market at each moment of time. Then, not knowing if it can sell the old house at any moment of time if necessary, buying in the search market and selling in the investment market simultaneously is not an option. The original model is the limit of this no-investment-market-access model, when the no-access-probability goes to zero.

4. The investment market is not free of friction but also a search market. In this environment and with the liquidity constraint binding, mismatched households must sell before they can buy. The original model is the limit of this model when the entry cost for flippers goes to zero.