Online Appendix to Inter-Dealer Trades in OTC Markets – Who Buys and Who Sells?

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1 Inter-dealer Market Price in-between the Marginal Values of Inventory

In the main text, we argue that in general, the price in the inter-dealer market can only be equal to one of the three marginal values of inventory in Proposition 1 in equilibrium. The following provides the formal result.

Proposition A1 Equilibrium obtains for a given $p \in (\beta (V_1^{SD} - V_0^{SD}) , \beta (V_1^{LD} - V_0^{LD}))$ only for $A = A_S$. Equilibrium obtains for a given $p \in (\beta (V_2^{LD} - V_1^{LD}) , \beta (V_1^{SD} - V_0^{SD}))$ only for $A = A_B$.

Proof. Consider an inter-dealer market price such that

$$\beta (V_1^{LD} - V_0^{LD}) > p > \beta (V_1^{SD} - V_0^{SD}) > \beta (V_2^{LD} - V_1^{LD}) .$$

The buyers are $L_0$s, whereas the sellers are $S_1$s and $L_2$s. This means that $n_1^{LD} = n^{LD}$, $n_0^{SD} = n^{SD}$, and $n_0^{LD} = n_2^{LD} = n_1^{SD} = 0$, whereby $n_S^D = n^{LD}$ and $n_B^D = n^{D}$. Substituting into (69) and (70) and manipulating yields (74) or $\Omega_S (A) = 0.1$

Consider a price such that

$$\beta (V_1^{LD} - V_0^{LD}) > \beta (V_1^{SD} - V_0^{SD}) > p > \beta (V_2^{LD} - V_1^{LD}) .$$

These equation numbers refer to those in the main text.
The buyers are \( L_0 \) and \( S_0 \), whereas the sellers are \( L_2 \)s. This means that \( n_1^{LD} = n^{LD}, n_1^{SD} = n^{SD} \), and \( n_0^{LD} = n_2^{LD} = n_0^{SD} = 0 \), whereby \( n_3^D = n^D \) and \( n_2^D = n^{LD} \). Substituting into (75) and (76) and manipulating yields (83) or \( \Omega_B(A) = 0 \). 

Together with Proposition 3, the Proposition says that at precisely the boundary between the Selling and the Balanced Equilibria, the inter-dealer market can clear at any price in between the Selling and the Balanced Eq. prices, whereas at precisely the boundary between the Balanced and Buying Eq., the inter-dealer market can clear at any price in between the Balanced and the Buying Eq. prices. At a given boundary, however, the allocations of the two equilibria concerned are identical and so exactly where \( p \) lies in between the two marginal values of inventory is immaterial.

2 Pre- and Post-crisis Equilibrium Types and \( A^D \)

2.1 Generalization of Corollary 3

In the main text, we restrict attention to \( A < \frac{1+e^\gamma}{\gamma} \) in Corollary 3. The following generalizes the Corollary to all values of \( A \).

Corollary A1 Let \( n^{LD} = \gamma n^D \) and \( n^{SD} = (1 - \gamma) n^D \). The evolution of equilibrium type is as depicted in the table below.

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition</th>
<th>( n^D ) Range</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( A &gt; \frac{1}{\gamma} )</td>
<td>( n^D &lt; n_{BE}^D )</td>
<td>Buying Eq.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{BE}^D \leq n^D &lt; \frac{2(A - \frac{1}{2})}{1+\gamma} )</td>
<td>Balanced-Buying Eq.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{2(A - \frac{1}{2})}{1+\gamma} \leq n^D \leq n_{SE}^D )</td>
<td>Balanced-Selling Eq.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n^D &gt; n_{SE}^D )</td>
<td>Selling Eq.</td>
</tr>
<tr>
<td>2.</td>
<td>( \frac{e^\gamma}{\gamma} &lt; A \leq \frac{1}{\gamma} )</td>
<td>( n^D \leq \frac{2(A - \frac{1}{2})}{1+\gamma} )</td>
<td>Buying Eq. or Balanced-Buying Eq.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{2(A - \frac{1}{2})}{1+\gamma} \leq n^D \leq n_{SE}^D )</td>
<td>Balanced-Selling Eq.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n^D &gt; n_{SE}^D )</td>
<td>Selling Eq.</td>
</tr>
<tr>
<td>3.</td>
<td>( \gamma \frac{e^\gamma}{\gamma} &lt; A \leq \frac{e^\gamma}{\gamma} )</td>
<td>( n^D \leq n_{SE}^D )</td>
<td>Balanced-Selling Eq.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n^D &gt; n_{SE}^D )</td>
<td>Selling Eq.</td>
</tr>
<tr>
<td>4.</td>
<td>( A \leq \gamma \frac{e^\gamma}{\gamma} )</td>
<td>for all ( n^D &gt; 0 )</td>
<td>Selling Eq.</td>
</tr>
</tbody>
</table>

Proof. Case 1 for \( A > \frac{1+e^\gamma}{\gamma} \) has been established in Corollary 3 in the main text. The following continues the proof of Corollary 3.
Consider Case 2 for \( \frac{e}{\delta} < A \leq \frac{1}{\delta} \). The first three intervals \((89)-(91)\) remain non-empty for \( A > \frac{e}{\delta} \). And then, \( \lim_{nD \to 0} \Omega_B \geq 0 \) and \( \lim_{nD \to A - \frac{e}{\delta}} \Omega_B = \infty \). This means that the Balanced-Buying Eq. must hold near the lower and upper bounds of the interval \( nD \in \left(0, \frac{2(A-e)}{1+\gamma}\right) \). However, with \( A \leq \frac{1}{\delta} \), it cannot be ascertained that \( \Omega_B \) is monotone increasing within the interval.

There is then the possibility that \( \Omega_B \) may fall below zero for a range of \( nD \) over which the Buying Eq. would hold. For any larger \( nD \), the analysis in the proof of Corollary 3 remains valid.

Suppose \( A \leq \frac{e}{\delta} \) as for Cases (3) and (4). Out of \((89)-(92)\), only \((92)\) can hold for any \( nD > 0 \). The only possible equilibria are the Balanced-Selling Eq. and the Selling Eq.. Notice that \( \lim_{nD \to \infty} \Omega_S < 0 \) always. For \( A \leq \gamma \frac{e}{\delta} \) as for Case 4, \( \lim_{nD \to 0} \Omega_S = -\infty \) and \( \partial \Omega_S / \partial nD > 0 \). This means that \( \Omega_S \) is negative throughout and so the Selling Eq. holds for all \( nD > 0 \). For \( A > \gamma \frac{e}{\delta} \) as for Case 3, \( \lim_{nD \to 0} \Omega_S = \infty \). Then, \( \Omega_S = 0 \) must hold for at least one \( nD \). At such \( nD \)'s, it can be shown that \( \partial \Omega_S / \partial nD < 0 \). This means that \( \Omega_S > 0 \) for \( nD < n_{SE}^D \) and \( \Omega_S < 0 \) for \( nD > n_{SE}^D \). The Balanced-Selling Eq. holds initially and then gives way to the Selling Eq. for \( nD \geq n_{SE}^D \).

2.2 Aggregate Dealers’ Inventory Capacity and \( \gamma \)

Aggregate dealers’ inventory capacity \( A^D = (1 + \gamma) nD \) may vary with either \( nD \) or \( \gamma \). In Section 3.3 of the main text, we study the evolution of equilibrium type and the comparative statics of \( A^D \) with respect to \( nD \). In the following, we repeat the same analysis with respect to \( \gamma \). This serves to illustrate how the results of Section 3.3 also hold where the change in dealers’ aggregate inventory capacity pre- and post-crisis is due to a change in \( \gamma \).

**Corollary A2** For \( A - \frac{e}{\delta} < nD \), there exists some \( \gamma_{SE} \in \left(\frac{1}{nD} (A - \frac{e}{\delta}), 1\right) \) satisfying,

\[
\frac{\delta}{1-\delta} \frac{A - \gamma_{SE} (nD + \frac{e}{\delta}) \gamma_{SE} + 1}{\gamma_{SE} nD} - \mu \left(\frac{1}{\gamma_{SE} nD} (A - \frac{e}{\delta}) \gamma_{SE} + 1\right) = 0.
\]

For \( A - \frac{e}{\delta} > nD \), there exists some \( \gamma_{BE} \in \left(0, \frac{1}{\delta (A-nD)}\right) \) satisfying,

\[
\frac{\delta}{1-\delta} \frac{\gamma_{BE} (nD - A) + \frac{e}{\delta} \gamma_{BE} + 1}{\gamma_{BE} nD} - \mu \left(\frac{nD}{1-\delta} - \frac{1-\gamma_{BE}}{A - \frac{e}{\delta} - nD} 1 + \gamma_{BE}\right) = 0.
\]
The evolution of equilibrium type is as depicted in the table below.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Balanced-Buying Eq.</th>
<th>Balanced-Selling Eq.</th>
<th>Selling Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A - \frac{\epsilon}{\delta} \leq \frac{n^D}{2}$</td>
<td>$\gamma \leq \gamma_{SE}$</td>
<td>$\gamma &gt; \gamma_{SE}$</td>
<td>Balanced-Selling Eq.</td>
</tr>
<tr>
<td>$\frac{n^D}{2} &lt; A - \frac{\epsilon}{\delta} &lt; n^D$</td>
<td>$\gamma &lt; \frac{2(A - \frac{\epsilon}{\delta}) - n^D}{n^D}$</td>
<td>$\frac{2(A - \frac{\epsilon}{\delta}) - n^D}{n^D} &lt; \gamma \leq \gamma_{SE}$</td>
<td>Balanced-Buying Eq.</td>
</tr>
<tr>
<td></td>
<td>$\gamma &gt; \gamma_{SE}$</td>
<td>$\gamma &gt; \gamma_{BE}$</td>
<td>Selling Eq.</td>
</tr>
<tr>
<td>$A - \frac{\epsilon}{\delta} = n^D$</td>
<td>for all $\gamma \in [0, 1]$</td>
<td>Balanced Eq. with no trades between small and large dealers</td>
<td></td>
</tr>
<tr>
<td>$A - \frac{\epsilon}{\delta} &gt; n^D$</td>
<td>$\gamma \leq \gamma_{BE}$</td>
<td>$\gamma &gt; \gamma_{BE}$</td>
<td>Balanced-Buying Eq.</td>
</tr>
</tbody>
</table>

**Proof.** Consider $\Omega_S$ in (93) as a function of $\gamma$, which is well-defined over $\gamma \in \left[\frac{1}{n^D} (A - \frac{\epsilon}{\delta}), 1\right]$. The interval is non-empty for $A - \frac{\epsilon}{\delta} \leq n^D$. By Proposition 3(a), the Selling Eq. holds for $A \in \left[0, n^{LD} + \frac{\delta}{\epsilon}\right]$ and $\Omega_S < 0$. The condition $A < n^{LD} + \frac{\delta}{\epsilon}$ is precisely the condition $\gamma > \frac{1}{n^D} (A - \frac{\epsilon}{\delta})$. Note that $\lim_{\gamma \to (A - \frac{\epsilon}{\delta})} \Omega_S (\gamma) = \infty$ and $\lim_{\gamma \to 1} \Omega_S (\gamma) < 0$, from which it follows that there exists some $\gamma_{SE} \in \left((A - \frac{\epsilon}{\delta}) \frac{1}{n^D}, 1\right)$ at which $\Omega_S (\gamma_{SE}) = 0$, as defined in the Corollary. In addition, it can be shown that $\Omega'_S (\gamma) < 0$. Hence, the Selling Eq. holds for $A - \frac{\epsilon}{\delta} < n^D$ and that $\gamma > \gamma_{SE}$.

By Proposition 3(b), the Balanced-Selling Eq. holds for $A \in \left[A_S, n^{LD} + \frac{n^{SD}}{2} + \frac{\delta}{\epsilon}\right]$. For $A < n^{LD} + \frac{\delta}{\epsilon}$, the condition $A \geq A_S$ is $\gamma \leq \gamma_{SE}$. The condition $A < n^{LD} + \frac{n^{SD}}{2} + \frac{\delta}{\epsilon}$ can be written as $\gamma > \frac{1}{n^D} (A - \frac{\epsilon}{\delta}) - 1$, the RHS of which is below 0 for $A - \frac{\epsilon}{\delta} < \frac{n^D}{2}$. In this case, the Balanced-Selling Eq. holds for $\gamma \in [0, \gamma_{SE}]$. Otherwise, the range is $\gamma \in \left(\frac{1}{n^D} (A - \frac{\epsilon}{\delta}) - 1, \gamma_{SE}\right]$.

Consider $\Omega_B$ in (94) as a function of $\gamma$, which is well-defined for $A - \frac{\epsilon}{\delta} \geq n^D$. By Proposition 3(d), the Buying Eq. holds for $\Omega_B < 0$, in addition to $A - \frac{\epsilon}{\delta} > n^D$. Note that $\lim_{\gamma \to 0} \Omega_B (\gamma) = \infty$ and $\lim_{\gamma \to \frac{1}{n^D} (A - \frac{\epsilon}{\delta})} \Omega_B (\gamma) < 0$, from which it follows that there exists some $\gamma_{BE} \in \left(0, \frac{\epsilon}{\delta(A - n^D)}\right)$ at which $\Omega_S (\gamma_{BE}) = 0$, as defined in the Corollary. In addition, it can be shown that $\Omega'_B (\gamma) < 0$.

In sum, for $A - \frac{\epsilon}{\delta} > n^D$, the Buying Eq. holds where $\gamma > \gamma_{BE}$.

By Proposition 3(c), the Balanced-Buying Eq. holds for $A \in \left(n^{LD} + \frac{n^{SD}}{2} + \frac{\delta}{\epsilon}, A_B\right]$. For $A - \frac{\epsilon}{\delta} > n^D$, the condition $A \leq A_B$ is $\gamma \leq \gamma_{BE}$ and the condition $A \geq n^{LD} + \frac{n^{SD}}{2} + \frac{\delta}{\epsilon}$ is not binding. The Balanced-Buying Eq. holds for $\gamma \in [0, \gamma_{BE}]$. In case $A - \frac{\epsilon}{\delta} < n^D$,
because \( A_B > n^D + \frac{\varepsilon}{\delta} \), the condition \( A \leq A_B \) is non-binding and the only binding condition is
\[
A > n^{LD} + \frac{n^{SD}}{2} + \frac{\varepsilon}{\delta},
\]
which is equivalent to \( \gamma < \frac{1}{n^{DR}} (A - \frac{\varepsilon}{\delta}) - 1 \), the RHS of which is positive where \( A - \frac{\varepsilon}{\delta} > \frac{n^D}{2} \). In this case, the Balanced-Buying Eq. holds for \( \gamma \in [0, \frac{1}{2} (A - \frac{\varepsilon}{\delta}) - 1] \).

The above completes the proofs of Cases 1, 2 and 4. To prove Case 3, note that as \( A - \frac{\varepsilon}{\delta} \to n^D \), the upper bound of the interval \( \gamma \in [0, \frac{1}{2} (A - \frac{\varepsilon}{\delta}) - 1] \) in Case 2 tends to 1. The Balanced-Selling Eq. holds for all \( \gamma \in [0, 1) \) in the limit. Moreover, as \( A - \frac{\varepsilon}{\delta} \to n^D \), \( \gamma_{BE} \to 1 \) from the definition of \( \Omega_B (\gamma) \). Case 4 then says that in the limit, the Balanced-Buying Eq. holds for all \( \gamma \in [0, 1] \). The is possible only if the sales and purchases by large dealers just clear the inter-dealer market. ■

If there were no large dealers (\( \gamma = 0 \)), any inter-dealer trades are the sales of the asset from an \( S_1 \) to an \( S_0 \). The equilibrium terms of trade must then be such that the two parties are indifferent between trading and not trading as in the Balanced Eq. For positive but small \( \gamma \), there can still only be a Balanced Eq. in which small dealers remain indifferent between trading and not trading. Consider Case 2 where the asset supply is at a relatively low but not the lowest level. With only few large dealers in the market intermediating the sales and purchases of an asset the supply of which is at a moderate level, more than one-half of small dealers would have to hold a unit inventory as in the Balanced-Buying Eq.. As dealers’ aggregate inventory capacity increases with \( \gamma \), fewer than one-half of the remaining small dealers now need to hold a unit as in the Balanced-Selling Eq. As \( \gamma \) goes up further to reach a second threshold and beyond, dealers’ aggregate inventory capacity increases to the point where no small dealers need to hold any inventory at all as in the Selling Eq. A similar reasoning explains the transition in the other cases.

Our claim in Section 3.3 of the main text is that the model can be interpreted to say that during the transition from pre- to post-crisis as dealers’ aggregate inventory capacity declines, selected small dealers expand their inventory holdings. The first two cases of the Proposition, which is for \( A - \frac{\varepsilon}{\delta} < n^D \), can indeed be interpreted to say just that. In Case 1, as \( \gamma \) falls from a large value pre-crisis to a small value post-crisis, the market may evolve from the Selling Eq. to the Balanced-Selling Eq. Pre-crisis then, no small dealers hold any inventory. Post-crisis, a fraction smaller than \( 1/2 \) hold a unit-inventory. In Case 2, as \( \gamma \) declines, the market may
evolve from the Selling, to the Balanced-Selling, and eventually to the Balanced-Buying Eq.  
– the evolution is from no small dealers holding any inventory to a fraction smaller than 1/2  
holding a unit inventory to a fraction greater than 1/2 doing so.

In these two cases, aggregate dealers’ inventory holding $A^D$ tends to decline in the interim.

**Proposition A2** While the market remains in the Selling Eq., $A^D$ does not vary with $\gamma$. Once  
the Balanced-Selling Eq. holds, $A^D$ begins to decline with $\gamma$ for a sufficiently small $\frac{\partial \eta(\theta)}{\eta(\theta)}$. In  
Case 2 where the market would eventually turn into the Balanced-Buying Eq, $A^D$ continues to  
decline with $\gamma$ thereafter.

**Proof.** In the Selling Eq., by (108), $A^D$ is independent of $\gamma$ from which it follows that $\frac{\partial A^D}{\partial \gamma} = 0$.

For the Balanced-Selling Eq., write $\frac{(1+\gamma)n^D-2A^D}{\delta + A^D - A} \frac{1}{1+\gamma}$ as $\theta$ in (104),

\[
\frac{\delta}{1 - \delta \theta} B - \mu(\theta) = 0,
\]  
where

\[
B = \frac{A - A^D - A^D A^D}{\delta + A^D - A} A^D.
\]

By the proof of Proposition 5, the LHS of (A1) is decreasing in $A^D$. We seek to show that it  
is increasing in $\gamma$. Differentiating,

\[
\frac{\delta}{1 - \delta \theta} \left( -\frac{1}{\theta^2} \frac{\partial \theta}{\partial \gamma} B + \frac{\delta}{1 - \delta \theta} \frac{1}{\partial \gamma} B - \mu'(\theta) \frac{\partial \theta}{\partial \gamma} \right)
\]

\[
= -\frac{\mu(\theta)}{\theta} \frac{\partial \theta}{\partial \gamma} + \frac{\mu(\theta)}{B} \frac{\partial B}{\partial \gamma} - \mu'(\theta) \frac{\partial \theta}{\partial \gamma} \quad \text{by substituting in from (A1)}
\]

\[
\propto -\eta'(\theta) \frac{\partial \theta}{\partial \gamma} + \eta(\theta) \frac{1}{B} \frac{\partial B}{\partial \gamma}
\]

\[
= -\eta'(\theta) \frac{\partial \theta}{\partial \gamma} + \eta(\theta) \frac{2A^D}{(1+\gamma)[(1+\gamma)n^D - 2A^D]} + \eta(\theta) \frac{A^D n^D}{A - A^D - \frac{A^D}{(1+\gamma)n^D - A^D} \delta}
\]

which is positive if and only if

\[
\eta'(\theta) \theta < \frac{\frac{n^D}{[1+\gamma](n^D - A^D)^2} (1+\gamma)[(1+\gamma)n^D - 2A^D]}{2 \left( A - A^D - \frac{A^D}{(1+\gamma)n^D - A^D} \delta \right)}.
\]

Then, for a sufficiently small $\frac{\eta'(\theta) \theta}{\eta(\theta)}$, (A2) holds.
For the Balanced-Buying Eq., write \( \frac{2AD - (\gamma + 1)nD}{A - A^D} \) as \( \theta \) in (100),
\[
\delta \frac{1}{1 - \delta \theta} \frac{nD(A^D - A + \frac{A^D}{(1+\gamma)nD - A^D})}{A^D(A - \frac{\xi}{\delta} - A^D)} - \mu(\theta) = 0. \tag{A3}
\]
By the proof of Proposition 5, the LHS of (A3) is increasing in \( A^D \). We seek to show that it is decreasing in \( \gamma \). The derivative with respect to \( \gamma \) is proportional to
\[
\frac{\eta'(\theta)\theta}{\eta(\theta)} \frac{2}{(1 + \gamma)[2A^D - (1 + \gamma)nD]} - \frac{\eta(\theta)}{A^D - A + \frac{A^D}{(1+\gamma)nD - A^D}} \frac{\frac{\xi}{\delta} \frac{nD}{(1+\gamma)nD - A^D}}{2},
\]
which is strictly negative if and only if
\[
\frac{\eta'(\theta)\theta}{\eta(\theta)} < \frac{nD(1 + \gamma)(2A^D - (1 + \gamma)nD)}{2 \left( \frac{A - A^D}{\theta} + \frac{A^D}{(1+\gamma)nD - A^D} \right) ((1 + \gamma)nD - A^D)^2}. \tag{A4}
\]
In the Balanced-Buying Eq., \( \frac{A - A^D}{\theta} > 1 \). Then, the RHS of the above exceeds
\[
\frac{1 + \gamma}{2 \left( 1 + \gamma - \frac{A^D}{nD} \right)} > \frac{1 + \gamma}{2 \left( 1 + \gamma - \frac{1 + \gamma}{2} \right)} = 1.
\]
The last inequality uses the fact that \( A^D > \frac{1 + \gamma}{2}nD \) in the Balanced-Buying Eq. Then, because \( \frac{\eta'(\theta)\theta}{\eta(\theta)} < 1 \), (A4) is guaranteed to hold.

We can rule out Case 3 as a knife-edge scenario. In Case 4, the market may evolve from the Buying Eq. to the Balanced-Buying Eq. as \( \gamma \) declines. In this case, pre-crisis, all small dealers hold a unit inventory, whereas post-crisis, only a fraction continue to hold a unit inventory. Even so, where there are absolutely more small dealers in the market at a smaller \( \gamma \), small dealers’ aggregate inventory holding post-crisis may still be greater despite only a smaller fraction of them continue to hold a unit inventory. This is guaranteed to be the case if \( \gamma \) has gone down sufficiently in the transition since the fraction of small dealers continuing to hold a unit inventory in the Balanced-Buying Eq. is bounded above 1/2.

### 3 Evaluating Conjectures 1 and 2

#### 3.1 Plotting Figure 2

For each \( A \), the conditions in Proposition 3 are invoked to determine which equilibrium type obtains. For the Selling Eq., by Table 1, \( n_1^{SD} = n_2^{LD} = 0 \), \( n_0^{SD} = n^{SD} \), while \( n_1^{LD} \) is the
solution to (72) from which we also obtain \( n^0_{LD} = n^{LD} - n^1_{1LD} \). For the Balanced-Selling and the Balanced-Buying Eq., by Table 1, \( n^1_{LD} = n^{LD} \), \( n^0_{LD} = n^2_{LD} \), while \( n^1_{1LD} \) is the solution to (78) for the Balanced-Selling and (81) for the Balanced-Buying Eq., from which we also obtain \( n^0_{SD} = n^{SD} - n^1_{1SD} \). For the Buying Eq., by Table 1, \( n^0_{SD} = n^2_{SD} = 0 \), \( n^1_{1SD} \) is the solution to (87) from which we also obtain \( n^1_{1LD} \) is the solution to (72) from which we also obtain \( n^0_{LD} = n^{LD} - n^1_{1LD} \). We next use (67) to solve for \( \theta \) and then (65) and (66), respectively, to solve for \( n^i_B \) and \( n^i_S \).

The equations for the inter-dealer market price \( p \) for the Buying and Balanced Eqs., respectively, are from (58) and (61), whereas in the Buying Eq., \( p = 0 \). Because a dealer-buyer’s gain from trade is \( p - p_I\),

\[ p_I = p - \frac{z_I}{2}, \]

Then, use (6) and (53) to obtain,

\[ p_I = \frac{1 - \beta + \frac{\eta(\theta)n_{1D}^D}{n^D}}{2(1 - \beta) + \frac{\eta(\theta)n_{1D}^D}{n^D}}p. \]

Because a dealer-seller’s gain from trade is \( p - p_I\),

\[ p_I = p + \frac{z_I}{2}. \]

Then, use (10) and (56) to obtain,

\[ p_I = \frac{(1-\delta)(1 - \beta + \frac{\eta(\theta)n_{1D}^D}{n^D}) + \left(\frac{\eta(\theta)n_{1D}^D}{n^D}\right) + \left(1 - \left(1 - \frac{\eta(\theta)n_{1D}^D}{n^D}\right)\right)}{\left(2(1 - \beta) + \frac{\eta(\theta)n_{1D}^D}{n^D}\right)}p. \]

The computations are carried out in the Gauss program “new-theory-12.e”. The output of the program consists of five columns of values. The first is the value for \( A \), the second is the equilibrium type (1 for Selling Eq., 2 for Balanced-Selling Eq, 3 for Balanced-Buying Eq., 4 for Buying Eq.), the third, forth and the fifth columns, respectively, are the values for \( \rho_{DS}, \rho_{DB}, \) and \( \rho_{RT} \). These numbers are then used to plot Figure 2.

### 3.2 The Robustness of Conjectures 1 and 2

We normalize \( n^D = 1 \) and write \( n^{LD} = \gamma \) and \( n^{SD} = 1 - \gamma \). We repeat the calculations for Figure 2 by varying one parameter at a time, for \( \gamma \in [0.01,0.98] \), \( \delta \in [0.01,0.99] \), and
$e \in [0.01, 1]$. The calculations are not extended to even smaller and larger values of the respective parameters only because the numerical procedure in Gauss breaks down then at such extreme values. Conjecture 1 remains valid throughout. Conjecture 2 also remains to hold, except for $\delta < 0.03$ and for a small part of the Selling Eq., when $A < 1.5$. A value of $\delta$ below 0.03, coupled with the benchmark value of $e = 0.15$, gives rise to $e/\delta > 5$, which can be thought of as the demand for the asset.\footnote{In each period, a measure of $e$ high-valuation investors enter the market, each of whom remains as a high-valuation investor for an average of $1/\delta$ periods. This means that in each given period, $e/\delta$ so many investors demand to hold a unit of the asset.} This means that Conjecture 2 only fails to hold where the demand for the asset is more than 3.33 times the supply.

4 Transaction Volume in the Frictional Inter-dealer Market Model – Plotting Figure 3

We first substitute (43), (44), the last two equations of the Section, $\theta = \frac{n^D}{n^B + n^S}$ and (22) to (35) and (36). This yields two equations in two unknowns: $n^I_B$ and $n^I_S$. Solving the equation and plugging in the solutions into the aforementioned equations, we obtain the solutions for $n^S_D$, $n^{SD}_1$, $n^{LD}_0$, $n^{LD}_1$, and $n^{LD}_2$, which are in turn used to calculate $SD_b$ and $SD_s$ in Figure 3a, and $SD = SD_b + SD_s + \alpha \frac{n^{LD}_0 n^{LD}_2}{n^D}$ in Figure 3b. The computations are carried out in the Gauss program “new-theory-3.e”.

5 Monotonicity of the LHSs of (78) and (81)

In this section, we show that the LHSs of (78) and (81) in the proof of Proposition 3 are monotone decreasing and increasing, respectively.

The LHS of (78) is defined over $n^{SD}_1 \in \left[ \max \left\{ A - n^{LD} - \frac{e}{\delta}, 0 \right\}, \bar{n}^{SD}_1 \right]$. First suppose $A - n^{LD} - \frac{e}{\delta} \leq 0$, so that the interval to consider is $n^{SD}_1 \in \left[ 0, \bar{n}^{SD}_1 \right]$. Differentiating with respect
to \( n_1^{SD} \) yields an expression that has the same sign as

\[
\left( -1 - \frac{n^D + n^{LD} \varepsilon}{(n^D - n_1^{SD})^2 \delta} \right) (n_1^{SD} + n^{LD}) (n^{SD} - 2n_1^{SD}) + (2n^{LD} + 4n_1^{SD} - n^{SD}) (A - n_1^{SD} - n^{LD} - \frac{n_1^{SD} + n^{LD} \varepsilon}{n^D - n_1^{SD} \delta}).
\] (A5)

The above is negative at \( n_1^{SD} = \tilde{n}_1^{SD} \) but it seems possible that it can be positive over a range of \( n_1^{SD} \) where \( 2n^{LD} + 4n_1^{SD} - n^{SD} > 0 \). In this case, it must change sign, from positive to negative at least once and be equal to zero at some \( n_1^{SD} \in (0, \tilde{n}_1^{SD}) \), for it to become negative at \( n_1^{SD} = \tilde{n}_1^{SD} \). Differentiating (A5) and factoring 2 from the resulting expression,

\[
- \frac{n^D + n^{LD}}{(n^D - n_1^{SD})^2 \delta} \varepsilon (n_1^{SD} + n^{LD}) (n^{SD} - 2n_1^{SD}) + 2 \left( A - n_1^{SD} - n^{LD} - \frac{n_1^{SD} + n^{LD} \varepsilon}{n^D - n_1^{SD} \delta} \right).
\] (A6)

Setting (A5) equal to 0,

\[
A - n_1^{SD} - n^{LD} - \frac{n_1^{SD} + n^{LD} \varepsilon}{n^D - n_1^{SD} \delta} \delta = \left( 1 + \frac{n^D + n^{LD} \varepsilon}{(n^D - n_1^{SD})^2 \delta} \right) \frac{(n_1^{SD} + n^{LD}) (n^{SD} - 2n_1^{SD})}{(2n^{LD} + 4n_1^{SD} - n^{SD})}
\]

and substituting it to (A6) and factoring out terms that are guaranteed positive,

\[
2 \left( \frac{n^D - n_1^{SD}}{2n^{LD} + 4n_1^{SD} - n^{SD}} \right)^3 + 3 \left( n^{SD} - 2n_1^{SD} \right) \left( n^D + n^{LD} \right) \frac{\varepsilon}{\delta} > 0
\]

if \( 2n^{LD} + 4n_1^{SD} - n^{SD} > 0 \). We begin the analysis presuming that (A5) is positive over a range of \( n_1^{SD} \in (0, \tilde{n}_1^{SD}) \) from which it follows that there must exist a \( n_1^{SD} \) at which (A5) is just equal to 0. At this point, we show that (A5) must be increasing. In at least one such \( n_1^{SD} \), (A5) must be turning from being positive to negative, however. Thus, we have arrived at a contradiction and this implies that (A5) is negative throughout for \( n_1^{SD} \in (0, \tilde{n}_1^{SD}) \).

Because we have not invoked any restriction related to the lower bound \( n_1^{SD} = 0 \), the same proof above is equally applicable for \( A - n^{LD} - \frac{\varepsilon}{\delta} > 0 \).

We next show that the LHS of (81) is increasing over \( n_1^{SD} \in [\tilde{n}_1^{SD}, \min\{A - \frac{\varepsilon}{\delta} - n^{LD}, n^{SD}\}] \), Write \( F(n_1^{SD}) = L_1(n_1^{SD}) \times L_2(n_1^{SD}) \), where

\[
L_1(n_1^{SD}) = \frac{-A}{n_1^{SD} + n^{LD}} + \frac{1}{n^D - n_1^{SD} \delta} \varepsilon + 1,
\]

\[
L_2(n_1^{SD}) = \frac{1}{2n_1^{SD} - n^{SD}}.
\]
Then, the LHS of (81) is equal to
\[
\frac{\delta}{1-\delta} (n^D + n^{LD}) F(n^S_D).
\]
Suppose for now
\[
A - n^{LD} - \frac{e}{\delta} < n^S_D,
\]
so that the upper bound to be considered is \(n^S_D = A - \frac{e}{\delta} - n^{LD}\). We first show \(F'(A - \frac{e}{\delta} - n^{LD}) \geq 0\). Differentiating and because \(L^0_2 = -2L^2_2\),
\[
\begin{align*}
L_1'(A - \frac{e}{\delta} - n^{LD})L_2(A - \frac{e}{\delta} - n^{LD}) - 2L_1(A - \frac{e}{\delta} - n^{LD})L_2^2(A - \frac{e}{\delta} - n^{LD}) = \\
\propto L_1'(A - \frac{e}{\delta} - n^{LD}) - 2L_1(A - \frac{e}{\delta} - n^{LD})L_2(A - \frac{e}{\delta} - n^{LD}) = \\
\propto \frac{A}{(A - \frac{e}{\delta})^2}[4(A - \frac{e}{\delta}) - (2n^{LD} + n^S_D)] - \frac{e}{\delta} \left[\frac{3(2n^{LD} + n^S_D) - 4(A - \frac{e}{\delta})}{(2n^{LD} + n^S_D) - (A - \frac{e}{\delta})^2} - 2\right].
\end{align*}
\]
 Denote \(2n^{LD} + n^S_D\) as \(x\). Differentiating (A7) with respect to \(x\) yields,
\[
-\frac{A}{(A - \frac{e}{\delta})^2} - \frac{e}{\delta} \frac{5(A - \frac{e}{\delta}) - 3x}{[x - (A - \frac{e}{\delta})]^3},
\]
which is maximized at \(x = 2(A - \frac{e}{\delta})\) and equal to
\[
-\frac{1}{A - \frac{e}{\delta}} < 0.
\]
Therefore, (A7) is decreasing in \(x\) when \(x \in [A - \frac{e}{\delta}, 2(A - \frac{e}{\delta})]\). By (82), \(x\) is indeed smaller than \(2(A - \frac{e}{\delta})\). Then,
\[
F'(A - \frac{e}{\delta} - n^{LD}) \geq \frac{A}{(A - \frac{e}{\delta})^2} 2(A - \frac{e}{\delta}) - \frac{e}{\delta} \left[\frac{1}{(A - \frac{e}{\delta})^2} 2(A - \frac{e}{\delta}) - 2\right] \propto A - \frac{e}{\delta} - A + \frac{e}{\delta} = 0.
\]
This proves that \(F'(A - \frac{e}{\delta} - n^{LD}) \geq 0\).

Next, given that \(L'_2 = -2L^2_2\) and \(L''_2 = -4L^3_2L'_2\),
\[
F'' = L''_1L_2 - 4L_1L_2^2L'_2 - 4L'_1L_2^2 = \\
\propto L''_1 - 4L_1L'_2 - 4L'_1L_2 = \\
= L''_1 - 4F'.
\]
where
\[
L''_1 = -\frac{A}{(n_{1 SD} + n_{1 LD})^3} + \frac{1}{(n_D - n_{1 SD})^3} \cdot \frac{e}{\delta},
\]
which is strictly increasing in \(n_{1 SD}\). Therefore, there exists a unique \(\tilde{n}_{1 SD}^*\) such that \(L''_1(\tilde{n}_{1 SD}^*) = 0\).

If \(F\) is not monotone increasing in \(n_{1 SD} \in (\tilde{n}_{1 SD}, A - \frac{e}{\delta} - n_{1 LD})\) and given \(F'(A - \frac{e}{\delta} - n_{1 LD}) \geq 0\), there must exist some \(n_{1h}^*\) and \(n_{1l}^*\) such that \(n_{1h}^* > n_{1l}^* \in (\tilde{n}_{1 SD}, A - n_{1 LD} - \frac{e}{\delta})\), \(F'(n_{1l}^*) = 0\), \(F''(n_{1l}^*) < 0\), \(F''(n_{1h}^*) > 0\) and \(F'(n_{1 SD}^*) < 0\) for any \(n_{1 SD}^* \in (n_{1l}^*, n_{1h}^*)\). If \(\tilde{n}_{1 SD}^* \leq \tilde{n}_{1 SD}\), then \(L''_1 > 0\) for any relevant \(n_{1 SD}^*\). In this case, \(F' = 0\) implies \(F'' > 0\). This is a contradiction. If \(\tilde{n}_{1 SD}^* > \tilde{n}_{1 SD}\), then \(\tilde{n}_{1 SD}^*\) must lie in the interval of \((n_{1l}^*, n_{1h}^*)\). This means that \(F'(\tilde{n}_{1 SD}^*) < 0\). On the other hand, from \(F''(\tilde{n}_{1 SD}^*) = 0\) we can calculate
\[
\tilde{n}_{1 SD}^* = \frac{n_D - (\frac{e}{A\delta})^{1/3} \cdot n_{1 LD}}{1 + (\frac{e}{A\delta})^{1/3}}, \tag{A8}
\]
and we know that
\[
-\frac{A}{(\tilde{n}_{1 SD}^* + n_{1 LD})^3} = \frac{e}{\delta} \cdot \frac{1}{(n_D - n_{1 SD}^*)^3}. \tag{A9}
\]
From (A8), \(\tilde{n}_{1 SD}^* < A - n_{1 LD} - \frac{e}{\delta}\) if and only if
\[
2n_{1 LD} + n_{1 SD} < (A - \frac{e}{\delta})(1 + (\frac{e}{A\delta})^{1/3}). \tag{A10}
\]
Then,

\[
F'(\tilde{n}^{SD}_1) \propto L'_1(\tilde{n}^{SD}_1) - 2L_1(\tilde{n}^{SD}_1)L_2(\tilde{n}^{SD}_1)
\]

\[
= -\frac{A}{(\tilde{n}^{SD}_1 + n^{LD})^3}[\tilde{n}^{SD}_1 + n^{LD} + 2(\tilde{n}^{SD}_1 + n^{LD})^2]
\]

\[\quad - \frac{e}{\delta (n^D - \tilde{n}^{SD}_1)^3}[2(\tilde{n}^{SD}_1 + n^{LD})^2 - n^D + \tilde{n}^{SD}_1] - \frac{2}{2n^{SD}_1 - n^{SD}}\]

\[= \frac{3A}{(\tilde{n}^{SD}_1 + n^{LD})^3}(2n^{LD} + n^{SD}) - \frac{2}{2n^{SD}_1 - n^{SD}} \text{ (after plugging in (A9))}
\]

\[= 3A \left( 1 + \left( \frac{e}{A\delta} \right)^{1/3} \right)^3 - 2 \frac{1 + \left( \frac{e}{A\delta} \right)^{1/3}}{1 - \left( \frac{e}{A\delta} \right)^{1/3}} \text{ (after plugging in (A8))}
\]

\[\propto 3A \left( 1 + \left( \frac{e}{A\delta} \right)^{1/3} \right)^2 \left( \frac{2}{1 - \left( \frac{e}{A\delta} \right)^{1/3}} \right)
\]

\[> 3 \left( 1 + \left( \frac{e}{A\delta} \right)^{1/3} \right)^{1/3} - \frac{2}{1 - \left( \frac{e}{A\delta} \right)^{1/3}} \text{ (after plugging in (A10))}
\]

\[\propto 3 \left( 1 + \left( \frac{e}{A\delta} \right)^{1/3} \right)^{1/3} + \frac{2}{1 - \left( \frac{e}{A\delta} \right)^{1/3}} - 2
\]

\[> 0 \text{ (because } \frac{e}{A\delta} < 1).\]

This is a contradiction and so \(n^{SD}_1\) is monotone increasing over \((\tilde{n}^{SD}_1, A - n^{LD} - \epsilon)\).

We can construct a proof similar to the above for the case of \(A - n^{LD} - \frac{\epsilon}{2} \geq n^{SD}\).