

# Quality competition in sequential trade equilibrium

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September 1998

## Abstract

This paper incorporates quality competition in the sequential trade models of Prescott (1975), Butters (1977) and Eden (1990). When quality changes are allowed for, firms adjust their probabilities of trade mostly in terms of qualities instead of by prices. In equilibrium, prices and qualities may actually be negatively correlated. Nevertheless, the equilibrium is optimum given the information structure.

Keywords : quality competition, sequential trading, uncertainty and pricing.

JEL classifications : L15, D41, D80.

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## 1. INTRODUCTION

In the sequential trading models of Prescott (1975), Butters (1977) and Eden (1990), homogenous products do not sell at the same price in a competitive market, violating the law of one price. With demand uncertainty and if the uncertainty is only resolved through the actual trading process, there will exist a tradeoff between prices charged and probabilities of trade. Firms charging different prices will be indifferent in equilibrium with those charging lower prices expecting a higher probability of trade. In practice, however, an alternative and perhaps more prominent channel where firms may raise their probabilities of trade is through producing a better product.

In this note, I study the implications of incorporating quality competition in the sequential trading model. In the model considered, firms may raise the probabilities of trade through producing a better product and/or by offering to sell at a lower price. Between the two channels of adjustment, quality appears to be the primary one. This is true in two ways. First, there will always be quality dispersion in equilibrium whereas there may not be any equilibrium price dispersion. Second, the probability of trade and product quality are always positively correlated but there exists no definite correlation between the former and the price charged. That is to say in equilibrium, better quality products would always be sold first while they may or may not be the lowered priced ones.

More interesting is the correlation between prices and qualities. First, efficiency unit pricing is the exception rather than the rule. Prices in general do not rise in proportion to product quality. Furthermore, prices and qualities may not even be positively correlated in equilibrium. Better quality products may in fact sell at lower prices! At first sight, this seems only possible under some sort of information asymmetry or market failure in general. The analysis that follows demonstrates that this need not be so. In fact, the equilibrium is optimum given the information structure.

## 2. ANALYSIS

The model is a straightforward extension of Prescott (1975). Let  $x$  be the quality of hotel rooms and  $p$  the price paid. The net utility a buyer of hotel room derives is

$$v = x - p.$$

The number of buyers  $n$  is stochastic with continuously differentiable distribution function  $F(n)$  and density  $f(n)$ . The cost of producing a room of quality  $x$  is  $c(x)$  where  $c$  is increasing, twice continuously differentiable and strictly convex. I assume that  $c'(0) > 0$  so that there exists some  $\bar{x}$  such that

$$c'(x) \begin{matrix} < \\ > \end{matrix} \frac{c(x)}{x} \quad \text{for } x \begin{matrix} < \\ > \end{matrix} \bar{x}. \quad (\text{A1})$$

There is free entry in the production of hotel rooms. Each seller may build one room. Assume that sellers would advertise the prices they choose to charge before  $n$  is known and do not change prices afterward. Eden (1990) explains that the rigid price assumption can be motivated by a more fundamental assumption that  $n$  is only revealed to sellers as actual purchases take place. If true, sellers receive no additional information about  $n$  in the interim and therefore have no incentives to change the prices advertised. On the other hand, buyers are assumed to possess perfect information about the price and quality distributions. As a result, they would rent from sellers offering higher net utility first before renting units offering lower net utility.

Let  $h(v)$  be the number (mass) of sellers offering net utility equal to  $v$  and define

$$H(v) = \int_v^\infty h(v) dv.$$

Given that units offering higher net utility would be rented first, a seller offering net utility  $v$  will trade if  $n \geq H(v)$  the probability of which is  $1 - F(H(v))$ . Therefore the expected profit for a seller offering quality  $x$  at price  $p$  is

$$\pi(x, p) = p[1 - F(H(\alpha x - p))] - c(x).$$

The first order condition for  $p$  gives

$$1 - F(H(v)) - pf(H(v))h(v) = 0, \quad (1)$$

and that for  $x$  is

$$pf(H(v))h(v) - c'(x) = 0. \quad (2)$$

Combining (1) and (2) yields

$$1 - F(H(v)) = c'(x). \quad (3)$$

The above defines  $x$  as an implicit function of  $v$ . Since  $H$  is decreasing in  $v$  and  $c'' > 0$ , we have  $x'(v) > 0$ .

**Proposition 1** *In equilibrium, sellers that offer a higher quality room also offer higher net utility and therefore enjoy a higher probability of trade.*

This result is reassuring in that it rules out the perverse situation where prices would be set in such a way that the lower quality rooms will offer higher net utility so that they would be rented first. Such an equilibrium would violate optimality in an extreme manner.

The mechanics of the relationship described in Proposition 1 is as follows. Consider a seller contemplating a small increase in  $v$ . This can be accomplished through either increasing  $x$  or reducing  $p$  by  $dv$ . The cost of increasing  $x$  by 1 unit is just  $c'(x)$ . Since expected revenue is equal to the price charged times the probability of trade, the cost of reducing  $p$  by 1 is simply equal to the probability of trade. If the costs of the two options are not equal, profit is not maximized. This gives rise to (3). From the convexity of  $c'(\cdot)$ , sellers choosing a large  $x$  incurs a large marginal cost of increasing  $v$ . But the marginal cost of increasing  $v$  must also equal the probability of trade. Hence,  $x$  and the probability of trade are positively correlated.

There is also a free entry condition that  $\pi(x, p) = 0$  for all sellers

$$p[1 - F(H(v))] - c(x) = 0. \quad (4)$$

Eqs. (3) and (4) imply

$$p(x) = \frac{c(x)}{c'(x)}. \quad (5)$$

What is interesting about the pricing rule in (5) is that it yields no definite prediction as to the sign of the correlation between  $p$  and  $x$  in equilibrium. Higher quality rooms may or may not rent for a higher price. For example, the parameterization  $c(x) = \phi e^{\varphi x} - d$  where  $\phi > d$  yields

$$p(x) = \frac{1}{\varphi} - \frac{d}{\phi \varphi e^{\varphi x}}$$

which is constant for all  $x$  if  $d = 0$ , and is increasing (decreasing) if  $d < (>) 0$ .

To understand why the price function may slope downward, consider again a seller contemplating raising  $v$  to earn a higher probability of trade. By definition  $1 = \frac{dx}{dv} - \frac{dp}{dv}$ . From proposition 1,  $\frac{dx}{dv} > 0$ . But what should be the magnitude of  $\frac{dx}{dv}$ ? It depends on the cost of raising  $x$ , i.e. the curvature of the cost function. If the convexity in  $c(\cdot)$  is large, the increase in  $x$  should be small, and can certainly be smaller than 1. If that is the case,  $\frac{dp}{dv}$  will have to be negative to attain the given increase in  $v$ . Then the better rooms will sell at lower prices.

In the equilibrium where  $n$  is known to all sellers once its value is realized, it can be shown that an efficiency unit pricing rule, where prices rise in proportion to qualities, will emerge. In contrast, (5) implies that in the sequential trade equilibrium,  $p(x)$  is linear in equilibrium only for cost functions of the form:  $c(x) = ax^b$ .<sup>1</sup>

Rosen (1974) also shows that efficiency unit pricing will not in general apply if it is not possible to break up a unit of quality  $x$  into two units of quality  $\frac{1}{2}x$ . In particular, the equilibrium pricing function tends to be strictly convex in quality. Rosen's result comes from the fact that only a strictly convex pricing rule can support an equilibrium in which high demand consumers would buy the better quality products. With efficiency unit

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<sup>1</sup>The observant would have noticed that this cost function does not satisfy (A1). However, the analysis can proceed without (A1) with no change in substance.

pricing, second order conditions for utility maximization will be violated and consumers will be at the corner.

The present result is different. I do not assume imperfect quality-quantity substitution as in Rosen. Preference is linear in  $x$  and demanders are identical. Indeed, in the equilibrium where sellers may change prices ex-post, efficiency unit pricing obtains. Here, sellers compete by offering high net utility  $v$ . This is accomplished by increasing  $x$  and not adjusting  $p$  upward by more than  $dx$ . The adjustment in  $p$  relative to  $x$  depends on the curvature of the cost function. There is no reason to presume the adjustment should be proportional.

Each seller must obviously offer non-negative net utility.

$$p(x) \leq x.$$

From (5) and (A1), the smallest  $x$  that satisfies the condition is  $\bar{x}$ . Proposition 1 implies that the firm selling  $\bar{x}$  is also the firm offering the smallest  $v$ . By virtue of (3), total entry  $H$  solves

$$1 - F(H) = c'(\bar{x}). \tag{6}$$

Since  $F(0) = 0$ , the sellers offering the highest  $v$  will sell its room with probability 1. Remember that this is also the highest quality room offered in equilibrium. From (3), this  $x$  solves

$$1 = c'(x).$$

In the absence of demand uncertainty, all rooms offered will sell with probability 1 and hence all rooms will be of the quality defined above. This shows demand uncertainty has an adverse effect on quality provision.

On the other hand, just as in Prescott's model,

**Proposition 2** *The equilibrium is optimum given the information structure.*

**Proof.** The planner chooses the number of rooms  $H$  to build and the distribution of quality to maximize total expected surplus. Index the rooms by  $j \in [0, H]$  and assume that  $x_j$  is weakly decreasing in  $j$ . Then, room  $j$  will be put into use if  $n \geq j$  whose probability is  $1 - F(j)$ . The expected surplus yielded by the construction of this room is

$$E[s_j] = [1 - F(j)]x_j - c(x_j). \quad (7)$$

The first order condition with respect to  $x$  yields

$$1 - F(j) = c'(x_j). \quad (8)$$

Set  $j = H(v)$  in (8). The equation then becomes identical to (5). This shows quality provision in equilibrium is optimum. Entry should cease at zero surplus. From (7) and (8), the quality of the last room built solves

$$c'(x) = \frac{c(x)}{x},$$

which by (A1) is equal to  $\bar{x}$ . This shows that entry is optimum. QED.

Optimality stipulates that the qualities of rooms should be disperse. The first one built should be of higher quality than the second one and the second one higher than the third one, etc. This is intuitive since the earlier units would be put into use with greater probabilities, it pays to have them to be of higher quality.

### 3. CONCLUSION

In this note, I extend Presocct's (1975) model of sequential trade by allowing sellers to compete in both prices and qualities. The most revealing lesson is that in equilibrium, prices and qualities need not be positively correlated. And even so, the equilibrium is optimum given the information structure.

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