Robustness, information–processing constraints, and the current account in small open economies

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A B S T R A C T

In this paper we examine the effects of two types of “induced uncertainty”, model uncertainty due to robustness (RB) and state uncertainty due to finite information–processing capacity (called rational inattention or RI), on consumption and the current account. We show that the combination of RB and RI improves the model’s predictions for (i) the contemporaneous correlation between the current account and income and (ii) the volatility and persistence of the current account in small open emerging and developed economies. In addition, we show that the two informational frictions improve the model’s ability to match the impulse response of consumption to income and the relative volatility of consumption to income growth.

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1. Introduction

Current account models following the intertemporal approach feature a prominent role for the behavior of aggregate consumption (see Sachs 1981). For given total income, consumption is the main determinant of national saving, and the balance of national saving in excess of investment is the major component of the current account. This important role for consumption has naturally led researchers to study current

account dynamics using consumption models. 1 For example, the standard intertemporal current account (ICA) model is based on the standard linear–quadratic permanent income hypothesis (LQ–PIH) model proposed by Hall (1978) under the assumption of rational expectations (RE). Within the PIH framework, agents can borrow in the international capital market and optimal consumption is determined by permanent income rather than current income; consequently, permanent income also matters for the current account. For example, consumption only partly adjusts to temporary adverse income shocks, which makes the current account tend to be in deficit. In contrast, consumption fully adjusts to permanent income shocks, with little impact on the current account.

However, many empirical studies show that the standard RE–ICA models are often rejected in the post-war data. 2 In addition, the standard models also cannot explain the different behavior of the current account and consumption in emerging and developed countries. 3 It is not surprising that the standard RE–ICA models are rejected because

1 See Obstfeld and Rogoff (1995) for a survey.
3 For example, see Neumeyer and Perri (2005), Aguiar and Gopinath (2007), Uribe (2009), among others.
the underlying standard PIH models have encountered their own well-known empirical difficulties, particularly the well-known ‘excess sensitivity’ and ‘excess smoothness’ puzzles. Specifically, the main problems with the standard RE–ICA models are as follows. First, the models cannot generate low contemporaneous correlations between the current account and net income (net income is defined as output minus investment and government spending). If net income is a persistent trend-stationary AR(1) process, the model predicts that the current account and net income are perfectly correlated, whereas in the data they are only weakly correlated. Note that in the data the current account is countercyclical with real GDP and more countercyclical in the emerging economy. (For example, see Neumeyer and Perri, 2005; Aguiar and Gopinath, 2007; Uribe, 2009). Second, they cannot generate low persistence of the current account. The standard RE models predict that the current account and net income have the same degree of persistence, whereas in the data the persistence of the current account is much lower than that of net income in emerging countries and insignificantly lower than that of net income in developed countries (see Table 1). Third, the models cannot generate observed volatility of the current account (Bergin and Sheffrin, 2000; Gruber, 2004). Fourth, they cannot generate more volatile consumption growth in emerging countries (Aguiar and Gopinath, 2007). Finally, the assumption of certainty equivalence in these models ignores some important channels through which income shocks affect the current account. As shown in Ghosh and Ostry (1997) in war-postwar quarterly data for the US, Japan, and the UK, the current account is positively correlated with the amount of precautionary savings generated by uncertainty about future net income. Fogli and Perri (2008) also show that in OECD economies changes in country-specific macroeconomic volatility are strongly correlated with changes in net external asset position.

It is, therefore, natural to turn to new alternatives to the standard RE–ICA model and ask what implications they have for the joint dynamics of consumption, the current account, and income. In this paper, we show that the data discussed above. Specifically, these two types of information imperfections interact with the fundamental shock (the income shock in our model) and give rise to closely related “induced uncertainty”: (i) model uncertainty and (ii) state uncertainty. These two types of induced uncertainty can affect the model’s dynamics even within the linear–quadratic (LQ) framework.

Note that here we follow Aguiar and Gopinath (2007) and Uribe (Chapter 1, 2009) and use the detrended data to compute the reported empirical second moments. Following Obstfeld and Rogoff (1995), Ghosh and Ostry (1997), Gruber (2004), Engel and Rogers (2006), among others, in this paper we net out investment and government spending because our model also suggests that consumption spending depends on income that is disposable for household consumption.

It is well known that given the length and structure of the data on real GDP, it is difficult to distinguish persistent trend-stationary AR(1), unit root, and difference-stationary (DS) processes for real GDP. (See Chapter 4 of Deaton, 1992 for a detailed discussion on this issue.) We focus on the AR(1) case in this paper; the results for the DS case are available from the authors upon request. In Section 3.2, we discuss the unit root case, in which the empirical second moments of the current account and net income are not finite. The RE model predicts that when net output follows a unit root process, the current account becomes constant.

See Table 1 for the average statistics for emerging and developed countries. Here we follow Aguiar and Gopinath (2007) by dividing the small economies into emerging and developed economies and use annual data from World Development Indicators.

Boz et al. (2010) also report the empirical autocorrelation of the current account and the correlation between the current account and real GDP in emerging countries, and examine how labor market frictions can improve the model’s predictions on these dimensions.

In this paper, we assume that there is only one shock to net income. If there are multiple structural shocks, the persistence of the detrended current account and that of detrended net income might be generated by the responses to the different shocks. See Kano (2008) for a detailed discussion.

Note that in the traditional linear–quadratic, linearized, or log-linearized models, uncertainty measured by the variance of the fundamental shock does not affect the model’s dynamics.

We adopt Hall’s LQ–PIH setting in this paper because the main purpose of this paper is to inspect the mechanisms through which the induced uncertainty affects the joint dynamics of consumption, the current account, and income, and it is much more difficult to study these informational frictions in non-LQ frameworks. After solving the models explicitly, we then examine how the induced uncertainty due to RB and RI can improve the model’s predictions on these important dimensions of the joint dynamics of the current account, consumption, and net income in emerging and developed countries we discussed above. In particular, we are interested in two key features of emerging market: consumption volatility exceeds income volatility and less procyclical current accounts with net income found in the data.

Hansen and Sargent (1995, 2007a) first introduced robustness (a concern for model misspecification) into economic models. In robust control problems, agents are concerned about the possibility that their model is misspecified in a manner that is difficult to detect statistically; consequently, they choose their decisions as if the subjective distribution over shocks was chosen by a malevolent nature in order to minimize their expected utility (that is, the solution to a robust decision-maker’s problem is the equilibrium of a max–min game between the decision-maker and nature). Robustness models produce precautionary savings but remain within the class of LQ–Gaussian models, which leads to analytical simplicity. A second class of models that produces precautionary savings but remains within the class of LQ–Gaussian models is the risk-sensitive model of Hansen et al. (henceforth HST, 1999).

Table 1

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Emerging vs. developed countries (averages).</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Emerging vs. developed countries (HP filter)</td>
<td></td>
</tr>
<tr>
<td>( \sigma(y)/\mu(y) )</td>
<td>3.19(0.20)</td>
</tr>
<tr>
<td>( \sigma(\Delta y)/\mu(\Delta y) )</td>
<td>3.82(0.19)</td>
</tr>
<tr>
<td>( \rho(y, y_{-1}) )</td>
<td>0.50(0.03)</td>
</tr>
<tr>
<td>( \rho(\Delta y, \Delta y_{-1}) )</td>
<td>1.35(0.08)</td>
</tr>
<tr>
<td>( \alpha(y) )</td>
<td>1.53(0.09)</td>
</tr>
<tr>
<td>( \beta(y) )</td>
<td>0.33(0.04)</td>
</tr>
<tr>
<td>( \rho(\alpha, \alpha_{-1}) )</td>
<td>0.30(0.05)</td>
</tr>
<tr>
<td>( \rho(\beta, \beta_{-1}) )</td>
<td>0.04(0.04)</td>
</tr>
<tr>
<td>B. Emerging vs. developed countries (linear filter)</td>
<td></td>
</tr>
<tr>
<td>( \sigma(y)/\mu(y) )</td>
<td>9.03(0.43)</td>
</tr>
<tr>
<td>( \sigma(\Delta y)/\mu(\Delta y) )</td>
<td>3.82(0.19)</td>
</tr>
<tr>
<td>( \rho(y, y_{-1}) )</td>
<td>0.80(0.02)</td>
</tr>
<tr>
<td>( \rho(\Delta y, \Delta y_{-1}) )</td>
<td>1.35(0.08)</td>
</tr>
<tr>
<td>( \alpha(\Delta y) )</td>
<td>0.80(0.06)</td>
</tr>
<tr>
<td>( \rho(\alpha, \alpha_{-1}) )</td>
<td>0.68(0.04)</td>
</tr>
<tr>
<td>( \rho(\beta, \beta_{-1}) )</td>
<td>0.53(0.04)</td>
</tr>
<tr>
<td>( \rho(\beta, \beta_{-1}) )</td>
<td>0.13(0.05)</td>
</tr>
</tbody>
</table>

Note that in the traditional linear–quadratic, linearized, or log-linearized models, uncertainty measured by the variance of the fundamental shock does not affect the model’s dynamics.

See Kano (2008) for a detailed discussion.

10 See Hansen and Sargent (2007a) and Sims (2003, 2006) for detailed discussions on the difficulties in solving the non-LQ models with information imperfections. The primary alternative model is based on Mendoza (1991), a small open economy version of an RBC model. That model would be significantly less tractable than the one we use, because it involves multiple state variables.


12 It is worth noting that although both robustness (RB) and information imperfections (RI) can significantly improve the model’s ability to process constraints (rational inattention or RI), can significantly improve the model’s ability to fit the data discussed above. Specifically, these two types of information imperfections interact with the fundamental shock (the income shock in our model) and give rise to closely related “induced uncertainty”: (i) model uncertainty and (ii) state uncertainty. These two types of induced uncertainty can affect the model’s dynamics even within the linear–quadratic (LQ) framework. We adopt Hall’s LQ–PIH setting in this paper because the main purpose of this paper is to inspect the mechanisms through which the induced uncertainty affects the joint dynamics of consumption, the current account, and income, and it is much more difficult to study these informational frictions in non-LQ frameworks. After solving the models explicitly, we then examine how the induced uncertainty due to RB and RI can improve the model’s predictions on these important dimensions of the joint dynamics of the current account, consumption, and net income in emerging and developed countries we discussed above. In particular, we are interested in two key features of emerging market: consumption volatility exceeds income volatility and less procyclical current accounts with net income found in the data.

13 See Hansen and Sargent (1995, 2007a) for detailed discussions on the difficulties in solving the non-LQ models with information imperfections. The primary alternative model is based on Mendoza (1991), a small open economy version of an RBC model. That model would be significantly less tractable than the one we use, because it involves multiple state variables.
We show that even if the parameter value of robustness is the same for all small open countries, the RB model has the potential to lead to the observed different joint behavior of consumption and current accounts across the developed and emerging economies. The reason is that the amount of model uncertainty that affects the model’s dynamics is determined by the interaction of the preference for robustness and income uncertainty; consequently, the model with the same parameter value of robustness can still lead to different behavior of consumption and the current account because income uncertainty is different across countries. Furthermore, we find that incorporating robustness can improve the model by along the following three dimensions in all small open countries: generating lower contemporaneous correlation between the current account and net income, lower persistence of the current account, and higher relative volatility of consumption growth to income growth. In addition, after calibrating the RB parameter using the detection error probability, we find that RB can help generate the different stochastic properties of the emerging and developed economies. Specifically, the current account in the emerging economy is (1) less correlated with net income, (2) less persistent, and (3) less volatile than that in the developed economy. However, quantitatively, we find that RB by itself cannot fully explain the joint behavior of consumption and the current account in the two small-open economies.

We therefore consider the model with imperfect state observation (state uncertainty) due to RI. Sims (2003) first introduced RI into economics and argued that it is a plausible method for introducing sluggishness, randomness, and delay into economic models. In his formulation agents have finite Shannon channel capacity, limiting their ability to process signals about the true state of the world. One key change relative to the RE case is that consumption has a hump-shaped impulse response to changes in income. Using the results in Luo (2008), it is straightforward to show that RI by itself still leads to counterfactual strongly-procyclical current accounts and cannot generate precautionary savings in the LQG setting. However, the combination of RB and RI produces a model that captures many of the facts that are seen as anomalous through the lens of an RE model, while producing consumption dynamics that are consistent with the data. The intuition is that RI introduces (i) slow adjustment to the income shock and (ii) an endogenous noise into the model, which amplifies the importance of model uncertainty in determining the model’s dynamics and further improves the model’s predictions on the joint behavior of consumption and the current account.

We briefly list the results of the RB–RI model. First, we can produce a low correlation between the current account and net income, and in fact can even produce negative correlations for some parameter settings; the key requirement to get low correlations is that the agent has a strong fear of model misspecification. Second, we can produce low persistence in the current account, a consequence of the slow movements in consumption that RI produces. Third, if information-processing is sufficiently restricted, current account volatility can match that observed in the data for emerging markets, although not for developed economies. Fourth, the model produces a hump-shaped consumption response to income, a consequence of RI, and can produce highly volatile consumption growth in emerging economies. Fifth, the precautionary savings effect generated by RB is consistent with the positive correlation between income volatility and average current accounts. We detail in the main body of the paper the intuition for all of these results.

The remainder of the paper is organized as follows. Section 2 presents key facts of small open economy business cycles. Section 3 reviews the standard RE–ICA model and discuss the puzzling implications of the model. Section 4 presents the RB–ICA model and discusses some results regarding the joint dynamics of consumption, the current account, and income. Section 5 solves the RB–RI ICA model and presents the implications for the same variables. Section 6 concludes.

2. Facts

In this section we document key aspects of small open economy business cycles. We follow Aguiar and Gopinath (2007) by dividing these small economies into two groups, labeled emerging economies and developed economies. Net income \(\gamma\) is constructed as real GDP–l–g, where \(l\) is Gross Fixed Capital Formation and \(g\) is General Government Final Consumption Expenditure. Consumption \(c\) in defined as Household Final Consumption Expenditure, \(c\) refers to the Current Account, and holdings of bonds \(b\) corresponds to Net Foreign Assets.

16 Habit formation also worsens the model’s predictions on the current account dynamics; consumption adjusts slowly with respect to income shocks under habit formation, as shown in Gruber (2004), generating procyclical current accounts. Luo (2008) compares the consumption predictions of habit formation and RI.
3.1. Model setup

The RE ICA model, the small-open economy version of Hall’s permanent income model, can be formulated as

$$\max_E E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

subject to the flow budget constraint

$$b_{t+1} = R b_t + y_t - c_t,$$

where $u(c_t) = \frac{1}{2} (c_t - \bar{c})^2$ is the utility function, $\bar{c}$ is the bliss point, $c_t$ is consumption, $R$ is the exogenous and constant gross world interest rate, $b_t$ is the amount of the risk-free foreign bond held at the beginning of period $t$, and $y_t$ is net income in period $t$ and is defined as output less than investment and government spending. Let $\beta R = 1$; then this specification implies that optimal consumption is determined by permanent income:

$$c_t = (R-1)s_t$$

where $s_t = b_t + \frac{1}{R} \sum_{j=0}^{\infty} R^{-j} E_t[y_{t+j}]$ is the expected present value of lifetime resources, consisting of financial wealth (the risk-free foreign bond) plus human wealth. As shown in Luo (2008) and Luo and Young (2010), in order to facilitate the introduction of RB and RI we reduce the above multivariate model with a general income process to a univariate model with iid innovations to permanent income $s_t$ that can be solved in closed-form. Specifically, if $s_t$ is defined as a new state variable, we can reformulate the above PIH model as

$$V(s_0) = \max \left\{ E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right] \right\}$$

subject to

$$s_{t+1} = R s_t - c_t + \zeta_{t+1},$$

where the time $(t+1)$ innovation to permanent income can be written as

$$\zeta_{t+1} = \frac{1}{R} \sum_{j=0}^{\infty} \left( R^{-j} \right) (E_{t+1} - E_t) [y_j];$$

$$V(s_0)$$ is the consumer’s value function under RE. Under the RE hypothesis, this model with quadratic utility leads to the well-known random walk result of Hall (1978).

$$\Delta c_t = R^{-1} (E_t - E_{t-1}) \left[ \sum_{j=0}^{\infty} \left( \frac{1}{R} \right) y_{t+j} \right] \approx (R-1) \zeta_t,$$

which relates the innovations in consumption to income shocks. In this case, the change in consumption depends neither on the past history of labor income nor on anticipated changes in labor income. In addition, the model specification also implies the certainty equivalence property holds, and thus uncertainty has no impact on optimal consumption.

Substituting Eqs. (2) and (3) into the current account identity, $c_a = b_{t+1} - b_t = (R-1) b_t + y_t - c_t$,

gives $c_a = -\sum_{i=t}^{\infty} \left( \frac{1}{R} \right)^{i-t} E_t [\Delta y_i]$, which means that the current account equals minus the present discounted value of future expected net income changes.\(^\text{22}\)

3.2. Model predictions for consumption and the current account

We close the model by specifying the stochastic process for net output. Specifically, we assume that the deviation of net output from its mean follows an AR(1) process

$$y_{t+1} - \bar{y} = \rho (y_t - \bar{y}) + \varepsilon_{t+1},$$

where $\rho \in (0,1]$ is the persistence coefficient of output and $\varepsilon_{t+1}$ is an iid normal shock with mean 0 and variance $\sigma^2$.\(^\text{23}\) In this case, Eq. (6) implies that $\zeta_{t+1} = \frac{1}{1-\rho} \varepsilon_{t+1}$ and $s_t = b_t + \frac{1}{1-\rho} y_t$. In the RE-ICA model,

$$c_a = \frac{1-\rho}{R-\rho} y_t,$$

which means that given $\rho$ and $R$, the current account inherits the properties of the stochastic process for net output (in particular, the persistence of net output), and the value of $\rho$ affects how output determines the behavior of the current account. Here we discuss two cases for the exogenous process of net output.

Case 1. $(0 < \rho < 1)$.

When $\rho < 1$, the shock is temporary and consumers adjust their optimal plans by only consuming the annuity value of the increase in income. In this case, the current account works as a shock absorber, and consumers borrow to finance negative income shocks and save in response to positive shocks. In other words, the current account in this case is procyclical: $\frac{\partial c_a}{\partial y_t} > 0$, which means that the current account improves during expansions and deteriorates during recessions. The solid line in Fig. 1 illustrates the impulse response of the current account to the income shock when $R = 1.04$ and $\rho = 0.7$. (We set $R$ to be 1 to be well above the threshold; we treat it as a compromise of different asset returns in the economy.) Eq. (10) also means that the contemporaneous correlation between the current account and income, $\text{corr}(c_a, y_t)$, is 1. This model prediction contradicts the empirical evidence: in small open economies the correlation between the current account and net output is positive but close to 0. As reported in Panel A (HP filter) of Table 1, $\text{corr}(c_a, y_t) = 0.04$ (s.e. 0.04) in emerging countries and 0.06 (s.e. 0.05) in developed economies. Similarly, in Panel B (linear filter) of Table 1, $\text{corr}(c_a, y_t) = 0.13$ (s.e. 0.05) in emerging countries and 0.17 (s.e. 0.05) in developed economies. In other words, the model predicts too high a correlation between the current account and net output.

Eq. (10) clearly shows that the volatility of the current account is less than that of income:

$$\mu = \frac{\partial^2(c_a)}{\partial y_t^2} = \frac{1-\rho}{(R-\rho)^2} < 1,$$

\(^{22}\) This expression also reflects the fact that consumers smooth income shocks by borrowing or lending in international financial markets. If income is expected to decline in the future, then the current account rises immediately as current consumption determined by permanent income is less than current income; the opposite occurs if income is expected to rise in the future.

\(^{23}\) The assumption of a single net income shock is very common in the literature of international finance and macro. For example, see Section 2.3 of Obstfeld and Rogoff (1996) and Chapter 2 of Uribe (2009). It is straightforward to model both permanent and transitory income shocks in the current setting. We can still solve the model explicitly and show that this multiple-shock specification does not affect our theoretical results on how RB affects the joint dynamics of the current account, consumption, and net income in our benchmark model. The detailed derivation is available from the authors by request.
where \( \text{sd} \) denotes standard deviation. Note that \( \frac{\partial \mu}{\partial \rho} < 0 \). Using the estimated \( \rho \) reported in Panel A (HP filter) of Table 1 and assume that \( \rho = 1.04 \), the RE model predicts that \( \mu = 0.926 \) in emerging countries and \( \mu = 0.933 \) in developed countries. However, in the data (using HP filter) reported in Table 1, \( \mu = 1.53 \) (s.e. 0.09) in emerging countries and \( \mu = 1.60 \) (s.e. 0.08) in developed countries.\(^{24}\) In other words, given the estimated income processes, the model cannot correctly predict the relative volatility of the current account to net output in emerging and developed economies.\(^{25}\)

Eq. (10) also implies that the persistence of the current account is the same as that of net output. However, in the data the current account is significantly less persistent than net output, and is less persistent in emerging economies than in developed economies. As shown in Panel B (linear filter) of Table 1, \( \rho(y_t, y_{t-1}) = 0.8 \) (s.e. 0.02) and 0.79 (s.e. 0.02) in emerging and developed countries, respectively, while the corresponding \( \rho(\text{ca}_t, \text{ca}_{t-1}) = 0.53 \) (s.e. 0.04) and 0.71 (s.e. 0.02).\(^{26}\)

Furthermore, given the AR(1) income specification, the change in aggregate consumption is \( \Delta c_t = \frac{R-1}{R-\rho} \varepsilon_t \), which means that consumption growth is white noise and the impulse response of consumption to the income shock is flat with an immediate upward jump in the initial period that persists indefinitely (see the solid line in Fig. 2). However, as well documented in the consumption literature (such as \( \text{Reis}, 2006 \)), the impulse response of aggregate consumption to aggregate income takes a hump-shaped form, which means that aggregate consumption growth reacts to income shocks gradually.

The relative volatility of consumption growth and income growth can be written as

\[
\mu_r = \frac{\text{sd}[\Delta c_t]}{\text{sd}[\Delta y_t]} \cdot \frac{R-1}{R-\rho} \sqrt{\frac{1+\rho}{2}},
\]

which is strictly increasing in \( \rho \), implying that consumption growth should be relatively more volatile in emerging economies (which is consistent with the data). However, given the values of \( \rho \) from Table 1, the volatility of consumption growth is much too low relative to net output. For example, if \( R = 1.04 \), the RE model predicts that the relative volatility of consumption growth to income growth in emerging and developed economies would be 0.28 and 0.24, respectively. In contrast, in the data, the corresponding \( \mu_r \) values are 1.35 and 0.98, respectively.\(^{27}\)

Case 2. (\( \rho = 1 \)).

When \( \rho = 1 \), net output follows a unit root process and the current account becomes constant because consumers allocate all of the increase in net income to current consumption. Intuitively, when the income shocks are permanent, the best response is to adjust consumption plan permanently. (Note that when \( \rho = 1 \) the empirical second moments of the current account and net income are not finite.) This principle is called “finance temporary shocks, adjust to permanent shocks” in the literature. As a result, \( \var[ca_t] = 0 \), which strongly contradicts the evidence that the current account is highly volatile in all small open economies.

In sum, comparing with the stylized facts reported in Table 1, it is clear that the stylized RE–ICA model with AR(1) income processes cannot account for the following key business cycle features in small open countries: (1) The contemporaneous correlation between the current account and net output is close to 0 in small open economies, and is slightly smaller in emerging markets. (2) The excess relative volatility of the current account to net output in emerging and developed economies. (3) The persistence of the current account is smaller than that of net output, and it is smaller in emerging economies than in developed economies. (4) The hump-shaped impulse responses of consumption to income shocks. (5) The relative volatility of consumption growth to income growth is larger in emerging economies than in developed economies.

Finally, in the standard ICA model the current account is independent of the uncertainty in output \( \omega^2 \); that is, the amount of precautionary savings does not affect the current account surplus. The reason is that the LQ setup satisfies the certainty equivalence property, ruling out any response of saving to uncertainty. However, as shown in Ghosh and Ostry (1997), in the post-war quarterly data for the US, Japan, and the UK, the greater the uncertainty in income, the greater will be the incentive for precautionary saving and, ceteris paribus, the larger the current account surplus.\(^{28}\)

\(^{24}\) Given the estimated \( \rho \) using the linear filter reported in Panel B of Table 1, the RE model predicts that \( \Lambda = 0.83 \) in emerging countries and \( \Lambda = 0.84 \) in developed countries. However, in the data reported in Table 1, \( \Lambda = 0.8 \) (s.e. 0.06) in emerging countries and \( \Lambda = 1.35 \) (s.e. 0.06) in developed countries.

\(^{25}\) Given the standard errors reported in parentheses in Panel B in Table 1, the result is significant.

\(^{26}\) As shown in Panel A of Table 1, using HP filter shows the same pattern.

\(^{27}\) Here we use the linear filter to obtain these results; using the HP filter leads to similar results.

\(^{28}\) Recent work examines the importance of precautionary savings for current account dynamics, including Mendoza et al. (2009) and Carroll and Jeanne (2009); such models are not analytically tractable (with the exception of Carroll and Jeanne, 2009) and the analysis is therefore somewhat less transparent.
4. Intertemporal models of current account with robustness

In this section, we introduce a concern for model uncertainty (robustness, RB) into the stylized intertemporal current account model (ICA) proposed in Section 3, and explore how this information imperfection affects the dynamics of consumption and the current account in the presence of income shocks.

4.1. Optimal consumption and the current account under robustness

A robust optimal control problem considers the question of how to make decisions when the agent does not know the probability model that generates the data. In the ICA model present in Section 3, an agent with a preference for robustness considers a range of models surrounding the given approximating model, Eq. (5), and makes decisions that maximize expected utility given the worst possible model. Following Hansen and Sargent (2007a), an RB version of the ICA model proposed in Section 3 can be written as

\[ v(s_t) = \max_{c_{t+1}} \left\{ -\frac{1}{2}(c_{t+1} - c_t)^2 + \beta \left[ \psi(c_{t+1}) + E_t[v(s_{t+1})] \right] \right\} \]

subject to the distorted transition equation:

\[ s_{t+1} = R_s - c_t + \xi_{t+1} + \omega_s v_t, \]

where \( v_t \) distorts the mean of the innovation and \( \delta > 0 \) controls how bad the error can be.\[^{29}\] As shown in HST (1999) and Hansen and Sargent (2007a), this class of models can produce precautionary behavior while maintaining tractability within the LOQ–Gaussian framework.

When net income follows an AR(1) process, Eq. (9), solving this robust control problem and using the current account identity yields the following proposition:

**Proposition 1.** Under RB, the consumption function is

\[ c_t = \frac{R - 1}{1 - \Sigma} s_t - \frac{\Sigma}{1 - \Sigma} \]

the mean of the worst-case shock is

\[ o_s v_t = \frac{(R - 1) \Sigma}{1 - \Sigma} s_t - \frac{\Sigma}{1 - \Sigma} c_t, \]

the current account is

\[ ca_t = \frac{1 - \rho}{R - \rho} y_t + \Gamma s_t + \frac{\Sigma}{1 - \Sigma} c_t, \]

and \( s_t = (b_t + \frac{1}{\rho - \rho^2} y_t \) is governed by

\[ s_{t+1} = \rho s_t + \xi_{t+1}, \]

where \( \xi_{t+1} = \varepsilon_{t+1}/(R - \rho) \), \( \Sigma = \text{Var} \varepsilon_t/(2\theta) \in (0, 1) \) measures the effect of the preference for robustness, \( \Gamma = \frac{\Sigma(R - 1)}{1 - \Sigma} < 0 \), and \( \rho_k = \frac{1 - R \Sigma}{1 - \Sigma} \in (0, 1) \).

**Proof.** See online Appendix A posted by the journal.

\[^{29}\] Formally, this setup is a game between the decision-maker and a malevolent nature that chooses the distortion process \( v_t \), \( \delta > 0 \) is a penalty parameter that restricts attention to a limited class of distortion processes; it can be mapped into an entropy condition that implies agents choose rules that are robust against processes which are close to the trusted one. In a later section we will apply an error detection approach to calibrate it.

In our univariate model the evil agent distorts the transition equation of permanent income \( s_t \), whereas in the multivariate HST model the evil agent distorts the income process \( y_t \). In other words, the key difference between the two models is that in the latter RB may affect the relative importance of the two state variables on the consumption function, whereas in the former the relative importance of the two effects are fixed by reducing the state space. However, after solving the two-state model numerically using the standard procedure proposed in Hansen and Sargent (2007a), we can see that the two models lead to the same decision rule. The reason is that in our univariate model the evil agent is not permitted to distort the law of motion for \( b_t \) as it is an accounting equation and has been used to obtain the \( s_t \) equation, whereas in the HST model we also only need to consider the distortion to \( y_t \), as there is no innovation to \( b_t \) in the resource constraint.

The effect of the preference for robustness, \( \Sigma \), is jointly determined by the RB parameter, \( \delta \), and the volatility of the permanent income, \( \omega_s \). This interaction provides a novel channel that the income shock can affect the consumption and the current account for different countries. That is, when there is a preference for robustness (i.e., \( \delta \to \infty \)), the different volatilities for the income processes in two countries can lead to different consumption and current account dynamics. This effect will disappear (i.e., \( \Sigma = 0 \)) if there is no preference for robustness (i.e., \( \delta \to \infty \)). Note that \( \Sigma < 1 \) comes from the requirement of the second-order condition of the optimization problem.\[^{31}\]

The consumption function under RB, Eq. (13), shows that the RB parameter, \( \delta \), affects the precautionary savings increment, \( -\frac{\Sigma}{1 - \Sigma} s_t \).

The smaller the value of \( \delta \) the larger the precautionary saving increment. The consumption function also implies that the stronger the preference for robustness; the more consumption responds initially to changes in permanent income; that is, under RB consumption is more sensitive to unanticipated income shocks. This response is referred to as “making hay while the sun shines” in the literature.

Note that Eq. (15) can be rewritten as \( ca_t = \frac{1 - \rho}{R - \rho} y_t = \frac{\Sigma}{1 - \Sigma} \left[ -(R - 1) \left( b_t + \frac{1}{R - \rho} y_t \right) + \frac{\Sigma}{1 - \Sigma} \right] \). It clearly shows that RB has greater impact on the level of the current account in the emerging economy as \( \delta \left( \frac{\Sigma}{1 - \Sigma} \right) / \beta \Sigma > 0 \) and the average value of \( \Sigma \) in emerging countries is larger. In addition, our model can generate stationary consumption and current account dynamics even if \( R = 1 \) as \( s \) is a stationary process under RB when \( \beta \rho = 1 \). Our RB model can thus generate persistent current account imbalances even if all countries have the same rate of time preference, as RB can lead to persistent imbalances via interacting with the fundamental uncertainty.

4.2. Implications for stochastic properties of consumption and current accounts

4.2.1. Impulse responses of the current account

When \( \rho \in (0, 1) \), the effect of a change in net output on the current account is determined by the first two terms in Eq. (15), and the current account includes a unit root. Specifically,

\[ \frac{\partial c_a}{\partial c_t} = \frac{\Gamma + 1 - \rho}{R - \rho}. \]

\[^{30}\] Note that the equivalence between the two models can be extended to the case with more state variables. We are grateful to an anonymous referee for suggesting us to check the possibility that the univariate and multivariate models are identical in the sense that they lead to the same solution.

\[^{31}\] The second-order condition for a minimization by nature can be rearranged into \( 0 > \frac{1}{2} |R\sigma|^2 \). Using the definition of \( \Sigma = |R\sigma|^2/(2\theta) \), we obtain \( 1 > R \Sigma \). Since \( R > 1 \), we must have \( \Sigma < 1 \).
which means that the current account will be procyclical if the effect of the robust preference is not sufficiently strong:

$$\Sigma \Sigma_1 = \frac{1 - \rho}{1 - \rho} \left( 1 - \frac{R - 1}{R - \rho} \right)$$  \hspace{1cm} (18)

For the special case that \( \rho = 1 \), introducing robustness generates countercyclical behavior of the current account as \( \Sigma > 0 \).^32

Fig. 1 shows the impulse response functions (IRF) of the current account to income shock under different values of \( \Sigma \). As they show, the current account can respond very differently to income shocks as the effect of the preference for robustness varies. For example, when \( \Sigma \) is zero (the RE model) or small, the current account responds positively to an income shock and slowly declines to zero. However, when \( \Sigma \) becomes large enough (such as when \( \Sigma = 0.95 \) as shown in Fig. 1), the current account initially responds negatively to a (positive) income shock. As we will discuss more in Section 5.2, these different shapes are supported by the VAR evidence from the studied emerging and developed countries. (See Figs. 10 and 11.)

It is worth noting that the trade balance \( (y_t - c_t) \) is also countercyclical if the same condition for the preference for robustness as specified in Eq. (18) holds, namely that \( \Sigma > (1 - \rho)/(R - \rho) \). The intuition for this result is very simple: the only difference between the trade balance and the current account is the net return on holding foreign bonds \((R - 1)\delta_t \), and this term is not affected by the income innovation at time \( t + 1 \).

4.2.2. Volatility of the current account

We now examine how RB affects the relative volatility of the current account to net income. Using Eq. (15), the relative volatility of the current account to net income can be written as

$$\mu = \frac{\sigma(c_{a_t})}{\sigma(y_t)} - \frac{\left(1 - \rho^2\right) \left[ \frac{1 - \rho}{1 + \rho} + \frac{\rho^2}{1 - \rho^2} + \frac{2(1 - \rho)^3}{1 - \rho^2} \right]}{\left(1 - \rho^2\right)^2} < 1.$$  \hspace{1cm} (19)

where we use the facts that

$$\text{var}(c_{a_t}) = \left[ \frac{1 - \rho}{1 + \rho} + \frac{\rho^2}{1 - \rho^2} + \frac{2(1 - \rho)^3}{1 - \rho^2} \right] \mu.$$  \hspace{1cm} (20)

Given \( R \) and \( \rho \), Eq. (19) shows that \( \mu \) is affected by the amount of robustness (\( \Sigma \)). Note that \( \mu \) is not a monotonic function of \( \Sigma \), as \( \frac{\partial^2}{\partial \Sigma^2} \) in Eq. (19) is increasing with \( \Sigma \) and \( 2(1 - \rho)^3 \) in Eq. (19) is decreasing with \( \Sigma \). Given the complexity of this expression, we cannot obtain an explicit result about how RB affects \( \mu \). Fig. 3 illustrates that how RB affects the relative volatility for different values of \( \rho \). It is clear that \( \mu \) is decreasing with \( \Sigma \) when \( \Sigma \) is large. The reason is that when \( \Sigma \) is large, the second term (the volatility term about \( s_t \)) in the bracket of Eq. (20) dominates the third term (the negative covariance term about \( s_t \) and \( y_t \)) there. (Note that \( \Gamma < 0 \).) RB thus has a potential to make the model fit the data better along this dimension. In addition, introducing RB can also explain that \( \mu \) is smaller in emerging counties than in developed countries if \( \Sigma \) is larger in emerging counties.\(^{33}\) The standard RE–ICA model predicts that the current account and income have the same degree of persistence, which contradicts the evidence that the current account is significantly less persistent than income in small open economies and the persistence of net income is larger in emerging countries than in developed countries.

4.2.3. Persistence of the current account

The persistence of the current account is measured by its first autocorrelation. Using Eq. (15), the first autocorrelation of the current account, \( \rho(c_{a_t}, c_{a_{t+1}}) \), can be written as

$$\rho(c_{a_t}, c_{a_{t+1}}) = \left[ \frac{1}{1 - \rho^2} \right] \left[ rac{(\rho - 1)^2}{1 - \rho^2} + \frac{\rho(1 - \rho)}{1 - \rho^2} \right] \left[ \frac{1}{1 - \rho^2} \right] \left[ \frac{2(1 - \rho)^3}{1 - \rho^2} \right] = \left[ \frac{1}{1 - \rho^2} \right] \left[ \frac{\rho^2}{1 - \rho^2} + \frac{(\rho - 1)^2}{1 - \rho^2} + \frac{2(1 - \rho)^3}{1 - \rho^2} \right]$$  \hspace{1cm} (21)

which converges to \( \rho \) (the persistence of net income) as \( \Sigma \) goes to 0. Given the complexity of this expression, we cannot obtain an explicit result about how RB affects \( \rho(c_{a_t}, c_{a_{t+1}}) \). Fig. 4 illustrates how RB affects the persistence of the current account for different values of \( \rho \). It is clear that \( \rho(c_{a_t}, c_{a_{t+1}}) \) is decreasing with \( \Sigma \) but RB thus has a potential to make the model fit the data better along this dimension. In addition, introducing RB can also explain that \( \rho(c_{a_t}, c_{a_{t+1}}) \) is smaller in emerging counties than in developed counties if \( \Sigma \) is larger in emerging counties.\(^{33}\) The standard RE–ICA model predicts that the current account and income have the same degree of persistence, which contradicts the evidence that the current account is significantly less persistent than income in small open economies and the persistence of net income is larger in emerging counties than in developed countries.

4.2.4. Correlation between the current account and income

An alternative description of the comovement of the current account and income is the contemporaneous correlation between the current account and income, \( \text{corr}(c_{a_t}, y_t) \). Under RB, the correlation can be written as:

$$\text{corr}(c_{a_t}, y_t) = \left( \frac{\Gamma}{1 - \rho^2} \right) \left[ \frac{1}{1 + \rho} + \frac{\rho^2}{1 - \rho^2} + \frac{2(1 - \rho)^3}{1 - \rho^2} \right] \left[ \frac{1}{1 + \rho} \right] \left[ \frac{2(1 - \rho)^3}{1 - \rho^2} \right]$$  \hspace{1cm} (23)

\(^{33}\) If net income is a pure random walk, the current account under RB can be written as

$$c_{a_t} = \Gamma s_t + \frac{\Sigma}{1 - \Sigma}$$

which clearly shows that the current account is countercyclical because \( \Gamma < 0 \). Given (16), the current account can be written as

$$c_{a_{t+1}} = \rho(c_a, c_{a+1}) + (1 - \rho) \frac{\Sigma}{1 - \Sigma}$$

which means that RB reduces the persistence of the current account because \( \partial \Sigma / \partial \Sigma^2 < 0 \).
which converges to 1 as $\Sigma$ converges to 0. Fig. 5 illustrates that how RB affects the correlation between the current account and net income for different values of $\rho$. It is clear that corr$(c_t, y_t)$ is decreasing with $\Sigma$ (note that in the figure we restrict the values of $\Sigma$ to be less than 0.83 such that corr$(c_t, y_t)$ is positive as generated in the data). RB thus aligns the model and the data more closely along this dimension. In addition, introducing RB can also account for the fact that corr$(c_t, y_t)$ is smaller in emerging countries than in developed countries, provided $\Sigma$ is larger in emerging countries.

4.2.5. Implication for consumption volatility

Although introducing robustness has a potential to improve the model's predictions on the dynamics of the current account and precautionary savings, it worsens the model's prediction for the joint dynamics of consumption and income. Given Eqs. (13) and (16), the change in aggregate consumption can be written as

$$c_{t+1} = \rho_t c_t - \frac{(1-R)\Sigma c_t}{1-\Sigma} + \frac{R - 1}{(1-\Sigma)(1-R)}\rho_E, \quad (24)$$

Therefore, aggregate consumption under RB follows an AR(1) process, which contradicts the evidence that in the data consumption reacts to income gradually and with delay. In other words, RB does not produce any propagation in consumption after an income shock. As emphasized in Sims (2003), VAR studies show that most cross-variable relationships among macroeconomic time series are smooth and delayed. Fig. 2 illustrates the response of aggregate consumption growth to an aggregate income shock $c_{t+1}$; comparing the solid line (RE) with the dash-dotted line, it is clear that RB raises the sensitivity of consumption growth to unanticipated changes in aggregate income.

Furthermore, the relative volatility of consumption growth to income growth, $\mu$, can be written as

$$\mu = \frac{\sigma(c_{t+1})}{\sigma(\Delta y_t)} = \frac{R - 1}{1 - \Sigma} \sqrt{\frac{1 + \rho}{1 + \rho_E}}. \quad (25)$$

It is clear from Eq. (25) that RB increases the relative volatility via two channels: first, it strengthens the marginal propensity to consume out of permanent income $\left(\frac{R - 1}{1 - \Sigma}\right)$; and second, it increases consumption volatility by reducing the persistence of permanent income measured by $\rho$; $\frac{\partial \mu}{\partial \Sigma} < 0$. Furthermore, if $\Sigma$ is larger in emerging economies, the RB–ICA model will predict that the relative volatility of consumption to income is greater in emerging economies than in developed economies.

4.2.6. Implications of macroeconomic uncertainty for the current account under RB

Finally, the last term in Eq. (15) determines the effect of precautionary savings on the current account. It is clear that with the preference for robustness, the greater the uncertainty in net income, the greater the amount of precautionary saving, and the larger the current account surplus, as

$$\frac{\partial c_{t+1}}{\partial \Sigma} > 0. \quad (26)$$

This result is consistent with the empirical evidence that the current account and macroeconomic volatility are positively correlated (Chosk and Ostry 1997, Fogli and Perri, 2008). This result is also related to Mendoza et al. (2009) and Carroll and Jeanne (2009) in which they solve the models with CRRA utility numerically and examine the importance of precautionary savings for current account dynamics. Our model therefore also contributes to this literature by providing a new mechanism through which precautionary saving due to induced uncertainty affects the current account. Note that the precautionary savings induced by a concern about robustness differs from the usual precautionary savings motive that emerges when labor income uncertainty interacts with the convexity of the marginal utility of consumption. This type of precautionary savings emerges because consumers facing more model uncertainty want to save more as protection against model misspecification and thus occurs even in models with quadratic utility.

4.3. Investment and the current account under RB

In the last subsection, we focus on examining how model uncertainty due to RB affects the joint behavior of consumption and the current account, and net income, and abstract from production and investment decisions. Since investment is an important force in determining the current account, in this subsection we briefly examine how the presence of investment decision affects the behavior of consumption and the current account. To maintain our analysis within...
the LQ setting, we follow Glick and Rogoff (1995), assume that output is determined by the following production function:

\[ y_t = a_t k_t^\alpha \left(1 - \frac{g_t}{2} \right) \]

where \( a_t \) is aggregate productivity, \( k_t \) is capital stock, \( i_t \) is investment, and the second term in the bracket captures the adjustment costs in capital. Taking a linear approximation to the first-order conditions of the firm’s optimizing problem yields the following investment policy

\[ i_t = \eta_i i_t - 1 + \eta_a \Delta a_t \]

(27)

where we use the fact that \( y_t = \eta_i i_t + \alpha \Delta k_t + \alpha_a \Delta a_t, \alpha_i < 0, \alpha_k < 0, \alpha_a > 0 \), and \( \alpha_i, \alpha_k, \alpha_a \) are the linearization coefficients, \( \eta_i, \eta_a \) are given, respectively. Furthermore, given the current account identity,

\[ \Delta c_t = (R-1)\alpha c_{t-1} + \Delta y_t - \Delta i_t - \Delta c_t, \]

(28)

we have

\[ \Delta c_t = (R-1)c_{t-1} + \eta_i i_{t-1} + \eta_a \alpha_a \Delta a_t, \]

(29)

where \( \eta_i, \alpha_i \) and \( \eta_a, \alpha_a \) are coefficients determined by the optimizing behavior of the household and firm sectors. We can see from Eq. (29) that endogenizing investment affects the current account dynamics by introducing a lagged investment term and the term of the change in aggregate productivity. It is also clear from Eq. (29) that RB affects the current account via two coefficients, \( \eta_i, \alpha_i \) and \( \eta_a, \alpha_a \). Given the structure of the current account specified in Eq. (29), it is impossible to obtain the explicit expression for the stochastic properties of the current account. However, we can still examine how RB affects the current account by inspecting Eq. (29). Specifically, as shown in Hansen and Sargent (2007a), introducing RB into the decision problem will strengthen the responses of the control variables to both endogenous and exogenous state variables. In other words, in the consumer problem, consumption is more sensitive to the income shock that is a linear function of productivity shocks, and in the firm problem capital stock and investment are more sensitive to the productivity shock (i.e., the values of \( \eta_i \) and \( \eta_a \) are larger under RB). Given Eqs. (29), (28), and \( \Delta y_t = \alpha_i \Delta k_t + \alpha \Delta a_t, \alpha_i < 0 \), it is straightforward to show that introducing RB will make the current account be more negatively correlated with the aggregate productivity by making \( \eta_a, \alpha_a \) more negative. In other words, the stronger the preference for RB, the more countercyclical the current account is.

4.4. Calibrating the RB parameter

Having examined the implications of RB for the relative volatility and persistence of the current account, and the correlation between the current account and income, it is clear that RB has a potential to improve the model’s predictions on the joint dynamics of the current account and net income. A requirement for matching these facts is that the fear of misspecification is stronger in emerging economies. This requirement is obviously subject to empirical testing, the task we turn to now.

Specifically, we use the procedure outlined in Hansen and Sargent (2007a) to calibrate the RB parameter (\( \theta \) or \( \Sigma \)). We calibrate \( \theta \) by using the notion of a model detection error probability (henceforth DEP) that is based on a statistical theory of model selection (the approach will be precisely defined below). We can then infer what values of the RB parameter \( \theta \) imply reasonable fears of model misspecification for empirically-plausible approximating models. The model detection error probability is a measure of how far the distorted model can deviate from the approximating model without being discarded; low values for this probability mean that agents are unwilling to discard very many models (as they want errors to be rare), implying that the cloud of models surrounding the approximating model is large.

Let model A denote the approximating model and model B be the distorted model. Define \( p_A \) as \( p_A = \text{Prob} \left( \frac{\alpha A}{L} < 0 | A \right) \), where \( \frac{\alpha A}{L} \) is the log-likelihood ratio. When model A generates the data, \( p_B \) measures the probability that a likelihood ratio test selects model B. In this case, we call \( p_B \) the probability of the model detection error. Similarly, when model B generates the data, we can define \( p_B \) as \( p_B = \text{Prob} \left( \frac{\alpha B}{L} > 0 | B \right) \).

Following Hansen et al. (2002) and Hansen and Sargent (2007a), the DEP, \( p \), is defined as the average of \( p_A \) and \( p_B \): \( p(\theta) = \frac{1}{2} (p_A + p_B) \), where \( \theta \) is the robustness parameter used to generate model B. Given this definition, we can see that \( 1-p \) measures the probability that econometricians can distinguish the approximating model from the distorted model. Now we show how to compute the model DEP in the RB model. Under RB, assuming that the approximating model generates the data, the state, \( s_t \), evolves according to the transition law

\[ s_{t+1} = R s_t - \gamma + \tilde{s}_{t+1} \]

(30)

where \( R = 1 - \frac{\gamma}{\Sigma} \) and \( \Sigma = 1 - \frac{\gamma}{\Sigma} \). In contrast, assuming that the distorted model generates the data, \( s_t \) evolves according to

\[ s_{t+1} = R s_t - \gamma + \tilde{s}_{t+1} + \alpha \Delta k_t \]

(31)

In order to compute \( p_A \) and \( p_B \), we use the following procedure.

Step 1: Simulate \( \{s_t\}_{t=0}^T \) using Eqs. (30) and (31) a finite number of times. The number of periods used in the simulation, \( T \), is set to be the actual length of the data for each individual country. Step 2: Count the number of times that \( \log \left( \frac{\alpha A}{L} \right) < 0 | A \) and \( \log \left( \frac{\alpha B}{L} \right) > 0 | B \) are each satisfied. Step 3: Determine \( p_A \) and \( p_B \) as the fractions of realizations for which \( \log \left( \frac{\alpha A}{L} \right) < 0 | A \) and \( \log \left( \frac{\alpha B}{L} \right) > 0 | B \), respectively.

In practice, given \( \Sigma \), to simulate the \( \{s_t\}_{t=0}^T \), we need to know \( a \) the volatility of \( s_t \) in Eqs. (30) and (31), and \( b \) the value of \( \Sigma \). For \( a \), we can compute it from \( sd(\tilde{s}) = \sqrt{1-R^2} sd(y) \) where \( sd(y) \) is the standard deviation of net income. For \( b \), we use the local coefficient of relative risk aversion \( \gamma = \frac{u''(c)}{u'(c)} - \frac{c}{\bar{c}} \) to recover the value of \( \bar{c} : c = \left(1 + \frac{1}{\gamma} \right) E[c] \) where \( E[c] \) is mean consumption. We choose \( \gamma = 2 \). Finally, we assume that consumers in our model economy are impatient enough such that they cannot resolve their model misspecification fears during the actual length of the data for each individual country.

4.5. Calibration results and main findings

After simulating the models and obtaining the DEP that circumscribes a neighborhood of models against which consumers want to assure robustness, we can find the values of \( \theta \) and \( \Sigma \) associated with that probability. Having shown how the RB parameter is related to the model DEP, in this section we report the calibrated values of the RB parameters by setting the model DEP to different targeted values.
As a benchmark, we choose the RB parameter to match the model DEP of $p = 0.1$. That is, the probability that the agent can distinguish the approximating model from the distorted model is 0.9.

Table 2 reports the average calibrated values of RB parameter, $\Sigma = \text{RE}^2/(2\theta)$, as well as the associated DEP $p$, the autocorrelation coefficient of GDP, $\rho$, and the ratios of the standard deviation of net income and permanent income to the mean of net income (undetrended), $\sigma(y)/\mu(y)$ and $\sigma(\zeta)/\mu(y)$, respectively, in both the emerging and developed countries.37 For simplicity here we only report the results using the linear filter; using the HP filter generates similar patterns from the model. We use $\sigma(y)/\mu(y)$ to measure the relative volatility of fundamental uncertainty. The table shows that on average:

1. Emerging countries face more volatile income processes than do developed countries. That is, macroeconomic uncertainty is higher in emerging countries.

2. After setting the detection error probability $p(\theta, \Sigma)$ to be the same in the two economies, the recovered $\Sigma$ is larger in emerging countries.

Therefore, the effect of the preference for RB (measured by $\Sigma$) in emerging countries is stronger than in developed countries. The intuition is simple: agents in the emerging economy are more concerned about model misspecification because they face larger macroeconomic uncertainty and instability than those in developed countries. It is worth noting that a larger $\Sigma$ does not necessarily imply a smaller value of $\theta$ since $\alpha_\theta$ (i.e., $\sigma(\zeta)$) can be different. As we have shown in Section 4.1, RB influences the countercyclical behavior of the current account and the relative volatility of consumption to income in the model through the interaction of $\theta$ and $\alpha_\theta$ in $\Sigma$ instead of $\theta$.

We first consider a comparison between the standard RE model and the RB model. In Tables 3–4, $p$ is set to 0.1 such that $\Sigma = 0.524$ in emerging countries and 0.205 in developed countries. In this case the first three columns of the tables clearly show that RB can improve the model’s predictions along the following three dimensions: the contemporaneous correlation between the current account and net income, the persistence of the current account, and the relative volatility of consumption growth to income growth, but worsens the model prediction on the relative volatility of the current account to net income. Specifically, for emerging countries, the calibrated $\Sigma$ RB reduces the correlation between the current account and net income from 1 to 0.62; reduces the first-order autocorrelation from 0.8 to 0.74; increases the relative volatility of consumption growth to income growth from 0.28 to 0.9; and reduces the relative volatility of the current account to income from 0.71 to 0.49. The intuition that RB reduces the volatility of the current account is that RB increases the response of consumption to income shock, and thus reduces the response of the current account.

In Tables 5–6, we reduce the DEP to 0.01 and find that in this case RB can improve the model’s predictions along all the four dimensions including the relative volatility of the current account to net income. When the RB parameter is large enough, the second term in the bracket of Eq. (20) dominates the third term, and thus the volatility of the current account increases. However, $p = 0.01$ is an extremely low value and means that agents rarely make mistakes and thus can distinguish the models quite well.38 As shown in Tables 5–6, even for this extremely low DEP, the RB model still cannot generate the observed volatility of the current account. In the next section, we will show that introducing another informational friction, rational inattention, helps resolve this anomaly.

### Table 2
Emerging vs. developed countries (averages, $p = 0.1$).

<table>
<thead>
<tr>
<th></th>
<th>Emerging countries</th>
<th>Developed countries</th>
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</thead>
<tbody>
<tr>
<td>$\Sigma$</td>
<td>0.524</td>
<td>0.205</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>$\sigma(y)/\mu(y)$</td>
<td>0.802</td>
<td>0.793</td>
</tr>
<tr>
<td>$\sigma(\zeta)/\mu(y)$</td>
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<td>0.044</td>
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<tr>
<td>$\sigma(y)/\mu(y)$</td>
<td>0.284</td>
<td>0.132</td>
</tr>
</tbody>
</table>

### Table 3
Implications of different models (emerging countries, $p = 0.1$).

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RE</th>
<th>RB</th>
<th>DEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0.9$</td>
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<td>0.13</td>
<td>0.28</td>
<td>0.90</td>
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<tr>
<td>$\theta = 0.8$</td>
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<td>0.13</td>
<td>0.28</td>
<td>0.90</td>
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<tr>
<td>$\theta = 0.7$</td>
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<td>0.13</td>
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<td>0.89</td>
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<tr>
<td>$\theta = 0.5$</td>
<td>0.13</td>
<td>0.13</td>
<td>0.28</td>
<td>0.89</td>
</tr>
</tbody>
</table>

### Table 4
Implications of different models (developed countries, $p = 0.1$).

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RE</th>
<th>RB</th>
<th>DEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0.9$</td>
<td>0.17</td>
<td>0.17</td>
<td>0.15</td>
<td>0.33</td>
</tr>
<tr>
<td>$\theta = 0.6$</td>
<td>0.17</td>
<td>0.17</td>
<td>0.15</td>
<td>0.33</td>
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<tr>
<td>$\theta = 0.3$</td>
<td>0.17</td>
<td>0.17</td>
<td>0.15</td>
<td>0.33</td>
</tr>
<tr>
<td>$\theta = 0.1$</td>
<td>0.17</td>
<td>0.17</td>
<td>0.15</td>
<td>0.33</td>
</tr>
</tbody>
</table>

37 The calibrated values of RB parameters, $\Sigma$ and $\rho$, for all individual countries can be found in the working paper version of the paper: http://www.kcb.frb.org/publicat/reswpap/pdf/rwp10-17.pdf.

38 Alternatively, low $p$ means that we impose weak limits on the evil nature who distorts the model.

---

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where $\xi_t$ is the conditional mean of the true state, and $\sigma^2_t = \text{var}[s_{t+1}|z_{t+1}]$ and $\sigma^2_t = \text{var}[s_{t+1}|z_t]$ are the posterior variance and prior variance of the state variable, respectively.  

5.1.2. Considering RB in the RI model

We now incorporate RI into the RB model and examine how the combination of the two types of information imperfections affect the joint dynamics of consumption, the current account, and income. A key assumption in the RB–RI model is that we assume that the consumer not only has doubts about the fundamental shock ($\xi_t$) but also distrusts her regular Kalman filter hitting the endogenous noise ($\xi_t$) and updating the estimated state. As a result, our agents have an additional dimension along which they desire robustness.

Specifically, the regular RI-induced Kalman filter equation updating $s_t$

$$\hat{s}_{t+1} = (1-\theta)(Rs_t - c_t) + \theta(\hat{s}_{t+1} + \hat{\xi}_{t+1}),$$

where $\hat{s}_t = E(s_t|z_t)$ is the conditional mean of $s_t$, $\hat{\xi}_t$ is the iid endogenous noise with $\alpha^2 = \text{var}[\zeta_{t+1}] = \frac{(\alpha^2)(R-p)^2 + R^2\sigma^2}{\alpha^2(R-p)^2 + (R^2-1)\sigma^2}$, $\theta = \alpha^2/\alpha^2 = 1 - 1/\exp(2\kappa)$ is the constant optimal weight on any new observation, and $s_0 - N(\hat{s}_0, \sigma^2)$ is fixed. Combining Eq. (35) with the $s$ transition equation, yields the following equation governing the dynamics of the perceived state $\hat{s}$ that matters in agents’ decision problems:

$$\hat{s}_{t+1} = R\hat{s}_t - c_t + \eta_{t+1},$$

where $\eta_t = \theta R(\hat{s}_t - \zeta_t) + \theta(\hat{s}_{t+1} + \hat{\xi}_{t+1})$ is the innovation to $\hat{s}_t$.

In the steady state, we can solve for $\alpha^2 = (\alpha^2)^{-1} - (\psi^2)^{-1}^{-1}$ using Eq. (34). In addition, Eq. (33) implies that in the steady state

$$\alpha^2 = \left(\frac{1}{R-p}\right)^2 \exp(2\kappa)/R^2,$$

where $\kappa$ is the consumer’s channel capacity, $\mathcal{H}(s_{t+1}|z_t)$ denotes the entropy of the state prior to observing the new signal at $t+1$, and $\mathcal{H}(s_{t+1}|z_{t+1})$ is the entropy after observing the new signal. The concept of entropy comes from information theory, and it characterizes the uncertainty in a random variable. The right-hand side of Eq. (32), being the reduction in entropy, measures the amount of information in the new signal received at $t+1$. Hence, as a whole, Eq. (32) means that the reduction in the uncertainty of the state variable gained from observing a new signal is bounded from above by $\kappa$.

Since the ex post distribution of $s_t$ is a normal distribution, $N(\hat{s}_0, \alpha^2)$, Eq. (32) can be reduced to

$$\log(\psi_t^2) - \log(\sigma^2_{t+1}) \leq 2\kappa.$$  

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$$\log(\psi_t^2) - \log(\sigma^2_{t+1}) \leq 2\kappa.$$  

To obtain (33), we use the fact that the entropy of a Gaussian random variable is equal to half of its logarithm variance plus a constant term.

Convergence requires that $\kappa > \log(3) \approx 1.1$; see Luo and Young (2010) for a discussion.

The RB–RI model proposed in this paper encompasses the hidden state model discussed in Hansen et al. (2002) and Hansen and Sargent (2007b); the main difference is that none of the states in the RB–RI model are perfectly observable (or controllable).

$\theta$ measures how much new information is transmitted each period or, equivalently, how much uncertainty is removed upon the receipt of a new signal.
Eq. (36) with a set of alternative models to represent his preference for robustness:

$$s_{t+1} = R_{t} - c_t + \omega_y p_t + \eta_{t+1}. \quad (37)$$

Under RI the innovation $\eta_{t+1}$ that the agent distrusts is composed of two MA($m$) processes and includes the entire history of the exogenous income shock and the endogenous noise, $s_{t+1}, s_{t}, \ldots, s_0; \xi_{t+1}, \xi_t, \ldots, \xi_0$. The optimizing problem for this RB–RI model is formulated as

$$v(s_t) = \max_{\tilde{c}_t} \min_{\eta_t} \left\{ \left( -\frac{1}{2} (c_t - \bar{c})^2 + \beta E_t [v(\tilde{s}_{t+1})] \right) \right\}, \quad (38)$$

subject to Eq. (37). Eq. (38) is a standard dynamic programming problem. The following proposition summarizes the solution to the RB–RI model.

**Proposition 2.** Given $\theta$ and $\theta$, the consumption function under RB and RI is

$$c_t = \frac{R - 1}{1 - \Sigma} \tilde{s}_t - \frac{\Sigma \bar{c}}{1 - \Sigma}, \quad (39)$$

the mean of the worst-case shock is

$$\omega_y p_t = \frac{(R - 1) \Sigma \bar{c}}{1 - \Sigma}, \quad (40)$$

and $\tilde{s}_t$ is governed by

$$s_{t+1} = \theta \tilde{s}_t + \eta_{t+1}, \quad (41)$$

where $\rho_1 = \frac{1 - R - \Sigma}{1 - \Sigma} \in (0, 1), \Sigma = \frac{R \rho_1}{(2 \theta)} > 0, \quad (42)$$

$$\omega_y^2 = \text{var}(\eta_{t+1}) = \frac{1 - (1 - \theta) R}{(1 - \theta) R} \omega_y^2. \quad (43)$$

**Proof.** See online Appendix A posted by the journal. \hfill \blacksquare

It is clear from Eqs. (39)–(43) that RB and RI affect the consumption function via two channels in the model: (1) the marginal propensity to consume (MPC) out of the perceived state $\frac{R - 1}{1 - \Sigma}$ and (2) the dynamics of the perceived state $(\tilde{s}_t)$. Given $\tilde{s}_t$, stronger degrees of RI and RB increase the value of $\Sigma$, which increases the MPC. Furthermore, from Eqs. (42) and (43), we can see that imperfect state observation due to RI can amplify the importance of model uncertainty measured by $\Sigma$ in determining consumption and precautionary savings.

Before proceeding, we want to draw a distinction between the model proposed above and similar ones used in Luo and Young (2010) and Luo et al. (2011). In those other papers, agents were assumed to trust the Kalman filter they use to process information, meaning that decisions were only robust to misspecification of the income process. An implicit assumption in the two papers is that the evil agent (the minimizing agent) has the same information set as the consumer (the maximizing agent). In that model $\Sigma$ was independent of $\theta$, and for the questions at hand here the resulting values were too small.44 By adding the additional concern for robustness developed here, we are able to strengthen the effects of robustness on decisions. In addition, our setup here is arguably more consistent with the underlying primitive structure of ambiguity that gives rise to robust decision-making (Gilboa and Schmeidler, 1989).

5.1.3. The joint dynamics of consumption, the current account, and net income under RB–RI

Furthermore, in the RB–RI model individual dynamics are not identical to aggregate dynamics. Combining Eq. (39) with Eq. (37) yields the change in individual consumption in the RI–RB economy:

$$\Delta c_t = \frac{(R - 1) \Sigma}{(1 - \Sigma)} (c_{t-1} - c_t) + \frac{R - 1}{1 - \Sigma} \left( \theta c_t + \theta R c_{t-1} \right), \quad (44)$$

where $L$ is the lag operator and we assume that $(1 - \theta) R < 1$.45 This expression shows that consumption growth is a weighted average of all past permanent income and noise shocks. Since this expression permits exact aggregation, we can obtain the change in aggregate consumption as

$$\Delta c_t = \frac{(R - 1) \Sigma}{(1 - \Sigma)} (c_{t-1} - c_t) + \frac{R - 1}{1 - \Sigma} \left( \theta c_t + \theta R c_{t-1} \right). \quad (45)$$

where $i$ denotes a particular individual, $E[\cdot]$ is the population average, and $\xi_t = E[\xi_t]$ is the common noise.46 This expression shows that even if every consumer only faces the common shock $\xi$, the RI economy still has heterogeneity since each consumer faces the idiosyncratic noise induced by finite channel capacity. As argued in Sims (2003), although the randomness in an individual’s response to aggregate shocks will be idiosyncratic because it arises from the individual’s own information–processing constraint, there is likely a significant common component. Therefore, the common term of the idiosyncratic error, $\xi_t$, lies between 0 and the part of the idiosyncratic error, $\xi_t$, caused by the common shock to permanent income, $\xi_t$. Formally, assume that $\xi_t$ consists of two independent noises: $\xi_t = \xi_t + \xi_t$, where $\xi_t = E[\xi_t]$ and $\xi_t$ are the common and idiosyncratic components of the error generated by $\xi_t$, respectively. A single parameter, $\lambda = \frac{\text{var}(\xi_t)}{\text{var}(\xi_t)} \in [0, 1]$, can be used to measure the common source of coded information on the aggregate component (or the relative importance of $\xi_t$ vs. $\xi_t$).47 Fig. 2 also shows how RI can help generate the smooth and hump-shaped impulse response of consumption to the income shock, which, as argued in Sims (2003), fits the VAR evidence better.48

Substituting Eq. (39) into the current account identity, the current account in the RB–RI model economy can be written as

$$ca_t = \frac{1 - \rho}{R - \rho} p_t - \frac{\Sigma (R - 1)}{(1 - \Sigma)} (s_t - \tilde{s}_t) + \frac{\Sigma}{1 - \Sigma} \bar{c}. \quad (46)$$

44 Due to limited space, we do not report the results of this RB–RI model; they are available from the authors by request.

45 This assumption requires $\kappa > 2 \log (R) \approx \frac{1}{1 - \theta}$, which is weaker than the condition needed for convergence of the filter.

46 For simplicity, here we use the same notation $c$ for aggregate consumption.

47 It is worth noting that the special case that $\lambda = 1$ can be viewed as a representative-agent model in which we do not need to discuss the aggregation issue.

48 In a recent paper, Angelotto and Lo’O (2009) show how dispersed information about the aggregate productivity shock contributes to significant noise in the business cycle and helps explain cyclical variations in Solow residuals and labor wedges. In contrast, Lorenzoni (2005) examines how demand shocks, defined as noisy news about future aggregate productivity, contribute to business cycles fluctuations in a new Keynesian model. Here we will show that the common noise due to RI simultaneously increases the relative volatility of consumption growth and income growth and reduces the contemporaneous correlation between the current account and income, which makes the RB–RI model fit the data better.
where \( s_t - \hat{s}_t = \frac{(1-\theta)\xi_t}{1-(1-\theta)\rho \frac{R}{L}} - \frac{\theta E_2(\xi_t)}{1-(1-\theta)\rho \frac{R}{L}} \), is the error in estimating \( s_t' \). It is clear that when \( \theta = 1 \), Eq. (45) reduces to Eq. (15) in Section 4.1. Eq. (45) clearly shows that the current account under RB and RI is determined by four factors: (1) The income term, \( -\frac{\rho}{R-p} \Delta y_t \). Holding other factors constant, the current account deteriorates in response to a positive income shock. (2) The overreaction in consumption due to the preference for RB, \( -\Sigma(R-1) \frac{1}{1-\Sigma} s_t \). This expression means that the stronger the preference for RB, the more countercyclical the current account is. Under RB, consumption is more sensitive to the unanticipated income shock, and thus the increase in consumption is larger than that of income itself; consequently, the current account deteriorates. (3) The forecast error term due to RI, \( \frac{1}{1-\Sigma} (s_t - \hat{s}_t) \). Consumers with finite capacity cannot observe the state perfectly, and thus adjust optimal consumption gradually and with delay. For a positive income shock, a gradual adjustment in consumption improves the current account. (4) The precautionary savings term, \( \frac{\Sigma c}{\Sigma} \). The precautionary saving premium due to the fear of model misspecification induces a bias toward current account surplus.

5.1.3.1. Impulse responses of the current account. Fig. 1 also plots the impulse response of the current account to the income shock when \( \Sigma = 0.95 \) and \( \theta = 80\% \). It clearly shows that the current account also responds to the income shock smoothly and gradually, which can better fit the VAR evidence. This shows that most cross-variable relationships among macroeconomic time series are smooth and delayed. Using Eq. (46) it is straightforward to show that the current account is procyclical if the following inequality is satisfied: \( \Sigma = 1 - \frac{R-1}{R-p} \). As will be shown in Section 5.2 that the RB–RI model also has the potential to generate the different shapes of the IRFs in different countries.

5.1.3.2. Volatility of the current account. Under RB–RI, using Eq. (46), the relative volatility of the current account and net income can be written as

\[
\mu = \frac{\sigma(c_{\text{cat}})}{\sigma(y_{\text{cat}})} = \sqrt{1-\rho^2} \frac{1 + \rho}{1 + \rho + \frac{\rho^2}{1-\Sigma} + \frac{R-1}{R-1-\Sigma}} + \frac{\rho^2}{1-\rho} \frac{1}{1-\rho} \frac{1}{\rho \rho \rho \rho (1/(1-R))}.
\]

Given the complexity of this expression, we cannot obtain an explicit result about how the interactions of RI and RB affect the relative volatility. As in the RB case, we thus use a figure to illustrate how RB and RI affect the relative volatility. Fig. 6 illustrates the effects of RI on the relative volatility when \( \text{Ro}_1^2/(2 \theta) = 0.5 \) and \( \rho = 0.8 \). Note that in the RB–RI case, \( \Sigma = \text{Ro}_1^2/(2 \theta) = \frac{\theta}{1-(1-\theta)R^2} \text{Ro}_1^2/(2 \theta) \) as \( \alpha^2_\eta = \frac{\theta}{1-(1-\theta)R^2} \). It is clear from the figure that giving the aggregation factor (\( \lambda \)), the relative volatility is decreasing with the degree of attention (\( \theta \)); given \( \theta \), the relative volatility is increasing with \( \lambda \). The intuition for the first result is that holding the aggregation factor fixed (i.e., given the impact of the common noise), reducing \( \theta \) increases the smoothness of aggregate consumption, and thus increases the volatility of the current account. The intuition for the second result is that holding \( \theta \) fixed, increasing \( \lambda \) strengthens the importance of the common noise, which leads to more volatile consumption and current accounts. Therefore, RI measured by \( \theta \) and \( \lambda \) has the potential to make the model fit the data better along this dimension. In the next section, we will examine how RI and RB improve the model’s quantitative predictions.

5.1.3.3. Persistence of the current account. Under RB–RI, using Eq. (46), the first-order autocorrelation of the current account can be written as:

\[
p(c_{\text{cat}}, c_{\text{cat}+1}) = \frac{\rho(1-\rho) + \rho^2 \frac{R-1}{R-1}}{1 + \rho + \frac{\rho^2}{1-\Sigma} + \frac{R-1}{R-1-\Sigma}} + \frac{(\rho_\theta(1-\theta^2))}{1-\rho \rho \rho \rho (1/(1-R))} + \frac{(\rho + \rho_\theta)(1-\rho)^2}{1-\rho \rho \rho \rho (1/(1-R))} + \frac{\rho_\theta(1-\theta^2)}{1-\rho \rho \rho \rho (1/(1-R))}.
\]

Using this explicit expression, Fig. 7 illustrates the effects of RI on \( \rho(c_{\text{cat}}, c_{\text{cat}+1}) \) when \( \text{Ro}_1^2/(2 \theta) = 0.5 \) and \( \rho = 0.8 \). It clearly shows that given \( \theta \) the persistence of the current account is decreasing with \( \lambda \). In contrast, the effects of \( \theta \) on the persistence depend on the values of the aggregation factor (\( \lambda \)). When \( \lambda \) is large, (e.g., \( \lambda = 1 \)), the persistence is decreasing with the degree of RI; when \( \lambda \) is small, (e.g., \( \lambda = 0.1 \)), the persistence is increasing with the degree of RI. The intuition behind these results is as follows. Given the degree of attention (\( \theta \)), \( \lambda \) has no impact on the covariance between \( c_{\text{cat}} \) and \( c_{\text{cat}+1} \), but increases the variance of the current account, which in turn reduces

Fig. 7. Persistence of \( c_{\text{cat}} \).
\( \rho(\text{ca}, \text{ca}_{t+1}) \). It is obvious that RI and RB have the most significant impact on \( \rho(\text{ca}, \text{ca}_{t+1}) \) in the representative agent case (\( \lambda = 1 \)) because the impact of the noise due to RI on the variances of ca, that appear in the denominator of Eq. (48) is largest in this case. In the next section using the calibrated model we show that the aggregate noise can quantitatively improve the model's predictions for the first-order autocorrelation of the current account.

5.1.3.4. Correlation between the current account and income. Similarly, under RB–RI, the correlation between the current account and net income can be written as:

\[
\text{corr}(\text{ca}, y_t) = \frac{(1 - \rho)_{R} (1 - \rho^c)_{R} \rho_{RI} + \left( \frac{\rho_{RI}}{\rho_{c}} \right) (1 - \theta)_{R} (1 - \rho)_{R}}{\sqrt{\rho_{c} \rho_{RI} (1 - \rho^c)_{R} (1 - \rho)_{R} (1 - \theta)_{R}}}
\]

Using this expression, Fig. 8 illustrates the effects of RI on the correlation when \( \rho_{R}/(2\theta) = 0.5 \) and \( \rho = 0.8 \). The figure also shows that given \( \theta \), the correlation is increasing with \( \lambda \). In contrast, the effects of \( \theta \) on the correlation are complicated and depend on the value of \( \lambda \). Specifically, when \( \lambda \) is large (\( \lambda = 1 \)), the persistence is decreasing with the degree of RI; when \( \lambda \) is small (\( \lambda = 0.1 \)) the correlation could be increasing with the degree of RI. The intuition behind these results is similar as that for \( \rho(\text{ca}, \text{ca}_{t+1}) \): given \( \theta \), a large persistence has an impact on the covariance between ca, and \( y_t \), but increases the volatility of the current account, which in turn reduces \( \text{corr}(\text{ca}, y_t) \).

5.1.3.5. Implication for consumption volatility. Using Eq. (45), the relative volatility of aggregate consumption growth relative to income growth can be written as

\[
\nu_1 = \frac{\sigma(\Delta c_t)}{\sigma(\Delta y_t)} = \frac{\theta(R - 1)}{2} \left( \sum_{j=1}^{j=R} \rho_{RI}^j \right) \left( \sum_{j=1}^{j=R} \rho_{c}^j \right) \left( \sum_{j=1}^{j=R} \rho_{RI}^j \rho_{c}^j \right) \left( \sum_{j=1}^{j=R} \rho_{RI}^j \rho_{c}^j \right)
\]

where we use the facts that \( \rho_1 = \rho_c = \frac{1 - \rho \Sigma}{1 - \Sigma} \), \( \rho_{RI} = (1 - \theta) \), \( R \in (0, 1) \), and \( \rho_{RI}^j = \sum_{k=0}^{j=R} (\rho_1 - k, \rho_2) - \sum_{k=0}^{j=R} (\rho_1 - k, \rho_2) \), for \( j \geq 1 \) and \( \rho_{RI}^j = 1 \). Fig. 9 illustrates how the combination of \( \theta \) and \( \lambda \) affects the relative volatility of consumption growth to income growth when \( \rho_0^2/(2\theta) = 0.5 \), \( \rho = 0.8 \), and \( R = 1.04 \). It is clear that given \( \theta \), the relative volatility \( \nu_1 \) is increasing with \( \lambda \). The effect of \( \theta \) on \( \nu_1 \) is not monotonic, and depends on the values of \( \lambda \). Specifically, when \( \lambda \) is large (\( \lambda = 1 \)), the relative volatility is decreasing with the degree of attention (\( \theta \)); when \( \lambda \) is small (\( \lambda = 0.1 \)), the relative volatility is decreasing with \( \theta \) first and then increasing with \( \theta \). The intuition behind this result is as follows. Given \( \lambda \) is small, when \( \theta \) is low, the presence of the common noise, \( \varepsilon \), dominates the smoothness of consumption caused by the gradual responses to fundamental shocks; in contrast, when \( \theta \) is large, the gradual response effect dominates the common noise effect, which reduces the relative volatility.

5.2. Comparing the implications of different models

To illustrate the quantitative implications of the RB–RI model on the stochastic properties of the joint dynamics of consumption, the current account, and net income, we fix the RB parameter at the same levels we obtain in Section 4.5 and vary the two RI parameters, \( \lambda \) and \( \theta \).\textsuperscript{49} As in Section 4.5, we first set the detection error probability, \( p \), to be a plausible value, 10%. Tables 3–4 compare the model performance under different assumptions (RB, RE, and RB–RI) on matching four important dimensions of the data we documented in Section 2: (1) the contemporaneous correlation between the current account and net income, (2) the volatility of the current account, (3) persistence of the current account, and (4) the relative volatility of consumption growth to income growth. The tables clearly show that RI could help further improve the RB model's predictions along all these four dimensions. Specifically, for emerging countries, in the representative agent case (\( \lambda = 1 \)), when \( \theta = 0.5 \), the interaction of RB and RI reduces the correlation between the current account and net income from 1 to 0.58; reduces the first-order autocorrelation from 0.8 to 0.36; increases the relative volatility of the current account to income from 0.71 to 0.79, and increases the relative volatility of consumption growth to income growth from 0.28 to 1.36, bringing all of them closer to the data.

We make three comments about this result. First, we have seen that in this case (\( \lambda = 1 \) and \( \theta = 0.5 \)) the interaction of RB and RI make the model fit the data quite well along dimensions (3) and (4),

\textsuperscript{49} The reason why we use the calibrated RB parameter values and vary the two RI parameters is that we want to distinguish the different effects of RB and RI on the model's dynamics. If we use the DEP to calibrate RB in the RB–RI model, it is difficult to separate the different effects of RB and RI within the model. We recalibrated the value of \( \theta \) using the DEP in the RB–RI model and found that it does not change our main conclusions. The calibration procedure and results are available from the authors by request.
while also quantitatively improving the model’s predictions along dimensions (1) and (2). Second, this improvement does not preclude the model from matching the first two dimensions as well (i.e., the contemporaneous correlation between the current account and net income and the volatility of the current account). For example, holding $\lambda$ equal to 1 and further reducing $\theta$ generates a smaller correlation between the current account and net income which is closer to the data. And holding $\theta = 0.5$ and reducing $\lambda$ to 0.1 makes the relative volatility of the current account to net income very close to the data. Third, and mostly importantly, all these quantitative results are consistent with the theoretical results we obtained in Section 5.1.

As being mentioned earlier, Fig. 1 shows the impulse response functions from the RB and RB–RI models under different parameter values. To have a comparison, Figs. 10 and 11 report the empirical IRFs of the current account to the income shocks for all small open economies studied in this paper.50 As these figures show, the shape of the IRFs are very different among different countries. For example, in some countries (such as those shown in Panel A of Fig. 10), the

50 To get the empirical IRFs, we run the following bivariate VAR:

$$\begin{bmatrix} y_{t+1} \\ c_{t+1} \end{bmatrix} = A \begin{bmatrix} y_t \\ c_{t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \end{bmatrix}$$

where $A$ is a $2 \times 2$ coefficient matrix, $c_{t}$ and $y_t$ are the detrended current account and net income, and $\varepsilon_{1t+1}$ and $\varepsilon_{2t+1}$ are the VAR innovations to net income and the current account, respectively. We use a triangular rotation matrix with net income ordered first.
current account responds positively to the income shock, while in some other countries (such as those shown in Panel D of Fig. 11), the current account responds negatively to income shocks. And there are also some countries (such as those shown in Panel C of Fig. 11) whose current accounts initially respond negatively and then increase to positively before the effects diminish to zero. These (empirical) shapes of IRFs are consistent with those generated by the RB and RB–RI models in Fig. 1. Actually, as shown in Fig. 1, without RB and RI, the RE model can only generate a positive response of the current account to income shock. But the RB–RI model can help generate more flexible shapes of the IRFs consistent with the data. These results further show that introducing RB and RI into the standard model can help better explain the data.

To check how robust these results are, we set the DEP to be 0.01 and report the results in Tables 5–6. From these tables, it is clear that in this case RI can improve the model’s predictions on the correlation of the current account and the first-order autocorrelation of the current account. For example, for emerging countries, in the representative agent case ($\lambda = 1$), when $\theta = 95\%$ (the agent can process almost all available information about the state), the combination of RB and RI reduces the correlation between the current account and income to 0.09 and reduces the first-order autocorrelation of the current account to 0.52. It is worth noting that given the high calibrated $\Sigma$ when $p = 0.01$, the model generates very volatile processes of consumption growth (the relative volatility of consumption growth to income growth increases to 2.09 in this case).

6. Conclusion

We have examined how introducing two types of information imperfections, robustness and rational inattention, into an otherwise standard intertemporal current account model changes the dynamic effects of income shocks on the joint dynamics of consumption and the current account. We have shown that a model with agents who have both a preference for robustness and limited information processing capacity has the potential to better account for the data along a number of dimensions.

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51 This response is consistent with the empirical evidence reported in Kano (2008) in which he found that the current account in Canada and UK responds positively to a positive transitory country-specific shock to net output initially and monotonically converges to zero in subsequent periods.
The model proposed in this paper can also be used to address the international diversification and consumption correlations puzzles (Backus et al., 1992). In Luo et al. (2011) we show that the model incorporating model uncertainty and state uncertainty reduces the correlation of consumption across countries, and can in fact produce consumption correlations lower than income correlations. RB will lower the international consumption correlations by generating heterogeneous responses of consumption to income shocks across countries, provided countries differ in terms of their preference for robustness. In addition, in contrast to the intertemporal consumption approach we consider here, the ‘new rule’ approach to the current account assigns the preeminent role to portfolio choice (for conflicting views on the relevance of the new rule, see Kraay and Ventura, 2003). An interesting extension to our study would be to permit portfolio choice and study the dynamics of the current account in the RB–RI model. Finally, to explore the mechanisms through which the two informational frictions interact and work, in this paper we have set up the model in a parsimonious way so that we can obtain a closed-form solution. We think that the mechanisms and insights we have explored in this simple framework can be carried over to more general cases. In particular, extending the model to incorporate the global interest rate shock emphasized by Nason and Rogers (2006) will be critical for demonstrating conclusively the utility of the RB–RI framework.

Appendix A. Supplementary data

Supplementary data to this article can be found online at doi:10.1016/j.jinteco.2012.02.004.

References


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