The Spirit of Capitalism and Excess Smoothness

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In a recent paper [Luo, Smith, and Zou (2009)] we showed that the spirit of capitalism could in theory resolve the two fundamental anomalies of modern consumption theory, excess sensitivity and excess smoothness. However, that basic model could not plausibly explain the empirical magnitude of excess smoothness. In this paper we develop two extensions of the model — one with transitory and permanent shocks to income, the other with a stochastic interest rate — that where the spirit of capitalism can explain excess smoothness.

Key Words: The spirit of capitalism; Consumption smoothing; Interest rate risk.

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1. INTRODUCTION

Weber (1948) famously asserted that a defining characteristic of capitalist societies is a desire to accumulate wealth as an end in itself, not merely as an instrument with which to acquire consumption goods. Over the last decade economists have developed the insight of the great sociologist by exploring the implications of the “spirit of capitalism” for a number of economic issues, ranging from savings [Zou (1995)], growth [Gong and Zou...

In a recent paper [Luo, Smith, and Zou (2009)] we studied how the spirit of capitalism affects precautionary savings and the dynamics of consumption. We incorporated a preference for wealth into an otherwise canonical model of precautionary savings, with an exogenous, autoregressive income process and a riskless interest rate. We established that, even in this stripped-down setting, the spirit of capitalism could explain the two great empirical anomalies of modern consumption theory.

First, if people obey the permanent income hypothesis and expectations are formed rationally, then consumption should be a martingale [Hall (1978), 1988]. Empirically, however, consumption growth can be predicted with anticipated changes in income, as has been shown in many empirical studies. This is the excess sensitivity puzzle [Flavin (1981), Campbell and Mankiw (1989), Deaton (1992)]: Why is consumption growth so sensitive to anticipated changes in income? In Luo, Smith, and Zou (2009) we demonstrate that a dependence of expected consumption growth on expected income growth is exactly what theory predicts in the presence of the spirit of capitalism.

Second, the permanent income hypothesis implies that, if income is stationary, people should try to smooth consumption over time: innovations to income should induce smaller innovations to consumption, which is what we observe. However, Campbell and Deaton (1989) and Deaton (1992) argue that income is non-stationary. The permanent income hypothesis should then require innovations to consumption to be larger than innovations to income. This is the excess smoothness puzzle: If income is non-stationary, then why is consumption so smooth? In Luo, Smith, and Zou (2009) we show that the spirit of capitalism mitigates the effect of an income innovation on consumption growth as long as income is non-stationary.

In Luo, Smith, and Zou (2009) we therefore established that in theory the spirit of capitalism can explain both excess sensitivity and excess smoothness. However, a plausible calibration of our basic model suggested that as a practical matter the spirit of capitalism by itself was unlikely to explain excess smoothness.

In this paper we enrich our basic model in two ways, by incorporating a more realistic income process, and by allowing for uncertainty about the interest rate. We show that these extensions heighten the ability of the spirit of capitalism to account for excess smoothness.
2. THE BENCHMARK MODEL

Let us begin by reviewing the benchmark model from Luo, Smith, and Zou (2009). This will allow us to show how the spirit of capitalism may explain the consumption anomalies, as well as suggest why it may not be sufficient to explain excess smoothness. We proceed in two steps. First we develop very general model of savings and the spirit of capitalism, one with minimal restrictions on either preferences or the income process. Then, in order to arrive at more concrete predictions, we look at a special case with a specific utility function and income process.

2.1. The General Model

Consider a discrete-time model where time is divided into discrete intervals of length $\Delta t$. To facilitate the passage to the continuous-time limit, as, $\Delta t \to 0$ we adopt the framework suggested by Obstfeld and Rogoff (1996, p. 745).

Imagine a consumer who can invest in a riskless bond with rate of return $r$ and who receives wage income (or more generally, non-asset income) of $y_t$ in each period. His flow budget constraint is

$$w_{t+\Delta t} = w_t(1 + r\Delta t) + y_t \Delta t - c_t \Delta t. \quad (1)$$

The consumer faces a random stream of wage income. We adopt a very general income process, assuming only that it is a discrete-time diffusion

$$\Delta y_t = y_{t+\Delta t} - y_t = \mu_{y,t} \Delta t + \sigma_{y,t} \Delta z_t, \quad (2)$$

Here $\Delta z_t$ is the increment to the Wiener process $z_t$. The conditional expectation and standard deviation of the growth of income, $\mu_{y,t}$ and $\sigma_{y,t}$, need not be constant, but may vary over time. Notice that although the current value of income is known at time $t$, its future evolution is uncertain. In other words, there is what Merton (1975) called “future,” rather than “current” uncertainty. This is the standard assumption in the literature on precautionary savings [Carroll (2001)].

The consumer lives forever and has a constant rate of time preference $\theta > 0$. He maximizes the lifetime expected utility of time-separable preferences defined over consumption $c_t$ and wealth $w_t$:

$$E_0 \sum_{t=0}^{\infty} \frac{1}{1 + \theta \Delta t} U(c_t, w_t) \Delta t. \quad (3)$$

The felicity function $U(c_t, w_t)$ is twice continuously differentiable in $c_t$ and $w_t$, with $U_c > 0, U_w > 0, U_{cc} < 0$, and $U_{ww} < 0$. Following Zou (1994), the

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1 Gourinchas and Parker (1999) and Turnovsky and Smith (2006) also allow for current uncertainty.
presence of wealth in the utility function captures the notion of the spirit of capitalism: the consumer derives pleasure from wealth itself, as well as from consumption.

The consumer maximizes the expected lifetime utility in Equation (3), subject to the budget constraint in Equation (1), given initial wealth \(w_0\) and the income process in Equation (2). In Appendix A we show that the Euler equation for this problem is

\[
1 = \frac{1}{1 + \theta \Delta t} E_t \left[ \frac{U_c(c_{t+\Delta t}, w_{t+\Delta t})}{U_c(c_t, w_t)} (1 + r \Delta t) + \frac{U_w(c_{t+\Delta t}, w_{t+\Delta t})}{U_c(c_t, w_t)} \Delta t \right].
\]  

(4)

Using the limiting arguments of Grossman and Shiller (1982), we can use this first-order condition to infer the stochastic process for the optimal consumption path. Define \(\sigma_{c,t}^2\) as the (possibly time-varying) instantaneous variance of the growth of consumption. In Appendix A we prove Proposition 1. In the continuous-time limit, as \(\Delta t \to 0\), the expected growth of consumption is

\[
E_t dc_t = \left[ \frac{U_c(c_t, w_t)}{U_{cc}(c_t, w_t)} \right] (\theta - r) + \frac{U_{cw}(c_t, w_t)}{U_{cc}(c_t, w_t)} \, dw_t + \frac{U_{ccc}(c_t, w_t)}{U_{cc}(c_t, w_t)} \, \sigma_{c,t}^2 \Delta t.
\]  

(5)

Although expressions like this are common in the literature [Carroll (1992), Baxter and Jermann (1999), Campbell (1994), Lettau and Ludvigson (2001)] as approximations, Equation (5) holds exactly in the continuous-time limit.

Two of the terms in Equation (5) are standard. The first term is the slope of the consumption profile encountered in the continuous-time models of consumption without uncertainty. The third term is a risk premium [Carroll (1992), Carroll and Kimball (1997)] that alters the slope of the consumption profile.

The twist here is in the second term. When there is a spirit of capitalism (and preferences are non-separable between consumption and wealth, so \(U_{cw} \neq 0\)) then the expected growth of consumption \(E_t dc_t\) depends upon the expected growth in wealth, \(E_t dw_t\). Intuitively, wealth is a second state variable, the evolution of which alters the marginal utility of consumption. Therefore, anticipated changes wealth can be used to predict growth in consumption. The random walk hypothesis does not hold in the presence of the spirit of capitalism.

2.2. An Example

Proposition suggests a resolution to the excess sensitivity puzzle, but says nothing about excess smoothness. To say more, we consider a special case
that permits a closed-form solution for the optimal consumption policy. The only known closed-form solution to the precautionary savings problem is with constant-absolute-risk aversion (CARA) utility function and a normally distributed innovation to income.\textsuperscript{2} To arrive at a closed-form solution with the spirit of capitalism, we therefore consider a CARA-like utility function defined over consumption and wealth. Time is now continuous and that the utility function is of the following form:

\begin{equation}
\frac{1}{a} E_0 \int_0^\infty e^{-\theta t - \alpha c_t - b w_t} dt.
\end{equation}

\textsuperscript{6}

\begin{equation}
a > 0 \text{ is the coefficient of absolute risk aversion with respect to consumption, while } b \geq 0 \text{ governs the strength of the spirit of capitalism. If } b = 0 \text{ there is no spirit of capitalism, and the model reduces to the standard CARA utility defined over consumption alone.}
\end{equation}

The consumer’s budget constraint is

\begin{equation}
dw_t = (rw_t + y_t - c_t) dt.
\end{equation}

\begin{equation}
\text{Income is Ornstein-Uhlenbeck, a continuous-time, first-order autoregressive process}
\end{equation}

\begin{equation}
dy_t = \rho \left( \frac{\mu}{\rho} - y_t \right) dt + \sigma dz_t.
\end{equation}

\begin{equation}
The mean of income in the steady-state is } \bar{y} = \mu/\rho, \text{ while the parameter } \rho \text{ governs the speed of convergence or divergence. If } \rho > 0 \text{ the process is mean-reverting, so that deviations from the steady state are temporary, while if } \rho < 0 \text{ the process is non-stationary and innovations to income are “super-permanent.” We permit the latter in order to catch the flavor of Campbell and Deaton’s (1992) argument that income is non-stationary.}\textsuperscript{3}
\end{equation}

The consumer maximizes expected lifetime utility [Equation (6)], subject to the budget constraint [Equation (7)], and given the income process [Equation (8)]. This nests two classic versions of the precautionary savings problem. If } b = \rho = 0 \text{ the model is the same as that in Blanchard and Mankiw (1988), and Hall (1988), with CARA utility and random-walk}

\textsuperscript{2} Examples include Blanchard and Mankiw (1988), Hall (1988), Blanchard and Fischer (1989), Caballero (1990), Alessie and Lusardi (1997), and Smith (1998, 2002). Weil (1990) solves the precautionary savings problem with CARA risk preferences and a constant elasticity of intertemporal substitution. Turnovsky and Smith (2007) provide a solution that obtains in general equilibrium for the constant-relative-risk aversion case when there are aggregate income shocks.

\textsuperscript{3} Empirically, income seems to be well described by a non-stationary second-order process [Campbell and Deaton (1992)]. I order to get a closed-form solution we are stuck with the first-order process in Equation (8). Permitting } \rho < 0 \text{ allows a form of non-stationarity.
income. If \( b = 0 \) but \( \rho > 0 \) it reduces to the model in Caballero (1990), where income is autoregressive.

Using methods of Merton (1971) and Wang (2006) we demonstrate in Appendix B that the solution to this problem is the consumption function

\[
c(w_t, y_t) = \Omega(w_t, y_t) - P_{b>0}
\]

where

\[
\Omega(w_t, y_t) = \frac{\theta - r - b/a}{ar} + \frac{1}{r + b/a + \rho} \mu + \frac{r + b/a}{r + b/a + \rho} y_t + rw_t
\]

and

\[
P_{b>0} = a \frac{r + b/a}{(r + b/a + \rho) \sigma^2} \frac{\sigma^2}{2}
\]

Consumption has two parts, certainty-equivalent consumption \( \Omega(w_t, y_t) \) and a risk adjustment \( P_{b>0} \).\(^4\) Certainty-equivalent consumption is a linear function of financial wealth \( w_t \) and wage income \( y_t \).

To see how the spirit of capitalism, observe that this consumption function is identical to what the consumption function would be in another model without the spirit of capitalism, but with an interest rate of \( r + b/a \) rather than \( r \). What does this mean? Suppose that you save a dollar of your wage income by investing in a riskless bond of one-period maturity. The market rate of return is \( r \), so that a year from now the return from investment will be \( 1 + r \). Absent the spirit of capitalism, the appropriate rate of discount your wage income is clearly the market rate, \( r \). If you share the capitalist spirit, however, you also derive pleasure from accumulating wealth. Investing your dollar also yields a psychic rate of return of \( b/a > 0 \) (a measure of the marginal utility of wealth relative to consumption). Thus, the spirit of capitalism raises the “effective” interest rate: \( r \) is the market rate, while \( r + b/a \) is the effective “psychological” rate at which the consumer discounts wage income.

This insight allows us to rewrite the consumption function in terms of human wealth. Define human wealth \( h_t \) as the expected present value of future labor income discounted at the appropriate rate. But we have just seen that in the presence of the spirit of capitalism the appropriate rate of interest for discounting wage income is \( r + b/a \). Therefore human wealth is

\[
h_t = E_t \int_t^\infty e^{-(r+b/a)(s-t)} y_s ds.
\]

\(^4\)It is “certainty-equivalent” in the sense that it is the consumption predicted by a non-stochastic model with CARA utility. “Certainty-equivalent” is often used to describe linear-quadratic preferences, which do not generate a precautionary premium.
As shown in Cox, Ingersoll, and Ross (1985), straightforward calculations imply that
\[
h_t = \frac{1}{r + b/a + \rho} \left( y_t + \frac{\mu}{r + b/a} \right)
\]  
(13)

We assume that \( r + b/a + \rho > 0 \) so that the \( e \) integral in Equation (12) converges. In other words, in order for human wealth to be well-defined, income cannot be “too” non-stationary.

The consumption function in equations (9) - (11) can now be rewritten as
\[
c(w_t, y_t) = \frac{1}{a} \left( \frac{\theta - r - b/a}{r + b/a} + (r + b/a)h_t + rw_t - P_{b>0} \right)
\]  
(14)

Consumption is a linear function of both forms of wealth, financial and human.

We refer the reader to Luo, Smith, and Zou (2009) for a detailed discussion of how the spirit of capitalism (captured by the parameter “\( b \)”) affects the precautionary premium. Here our concern is with its implications for the time-series properties of consumption.

2.3. Excess Sensitivity and Smoothness

As a benchmark, consider what consumption dynamics would be in this model if there were no spirit of capitalism. Using Equations (7), (8) and (9) and setting \( b = 0 \), the growth of consumption
\[
dc_t = r \left[ \frac{r - \theta}{ar} + a \frac{r}{(r + \rho)^2} \frac{\sigma^2}{2} \right] dt + \frac{r}{r + \rho} \sigma dz_t
\]  
(15)

Note two properties of consumption growth in this benchmark case. First, it reflects Hall’s (1978, 1988) classic result that consumption should be a random walk under rational expectations. As shown by Caballero (1990), Hall’s conclusion is not affected by the persistence of income: Current and lagged consumption and income cannot help predict the growth of consumption. In fact, it has been documented again and again [Flavin (1981), Campbell and Mankiw (1989), and Deaton (1992), to mention just some classic references] that changes in income predict changes in consumption. This is the excess sensitivity puzzle.

Second, the innovation to consumption is equal the annuity value of the innovation to income [also a result due to Caballero (1990)]. This implies

\[E_t[y_{t,s} = y_{t,s} e^{-\rho_i(s-t)} + \mu_i(1 - e^{-\rho_i(s-t)}) \text{ for } i = 1, 2.\]
that if income is stationary ($\rho > 0$) the variance of consumption growth is less than the variance of income growth, as one would expect from the consumption smoothing suggested by the permanent income hypothesis. If income is non-stationary ($\rho < 0$) however, the variance of consumption growth exceeds the variance of income growth. This leads to the excess smoothness puzzle, or Deaton (1992) paradox: if income is non-stationary, observed consumption growth is actually too smooth relative to what the permanent income hypothesis predicts.

How does the spirit of capitalism ($b > 0$) alter these predictions? The expected growth of consumption is now

$$E_t dc_t = r \left[ \frac{r + b/a - \theta}{ar} + a \frac{r + b/a}{(r + b/a + \rho)^2} \frac{\sigma^2}{2} \right] - \frac{b}{a} dt. \quad (16)$$

This is the analogue of Equation (5) in the general model in Section 2.1. Compare it to Equation (15). Clearly, if $b > 0$ then the expected change in consumption can be predicted by the growth in wealth: The spirit of capitalism causes the random walk hypothesis to fail.

Now use the budget constraint in Equation (7) to rewrite consumption growth in terms of income growth:

$$dc_t = r \left[ \frac{1}{a} \frac{r - \theta + b/a}{r + b/a} + \frac{\Sigma \sigma^2}{2} \right] dt + \frac{b/a}{r + b/a + \rho} E_t dy_t + \frac{r + b/a}{r + b/a + \rho} \sigma dz_t \quad (17)$$

This has important implications for both excess sensitivity and excess smoothness.

Excess Sensitivity

Equation (17) shows that if $b > 0$ expected growth of wage income can be used to predict changes in consumption. This establishes

**Proposition 2.** The spirit of capitalism is a sufficient condition to explain the excess sensitivity of consumption to income.

Needless to say, the spirit of capitalism is not the only explanation of excess sensitivity. Campbell and Mankiw (1989), for example, attribute excess sensitivity to the presence of “rule of thumb” consumers. They regress consumption growth on income growth, and find that the estimated coefficient attached to income growth is large and statistically significant. They interpret this coefficient as the proportion of “rule-of-thumb” consumers in the economy. We argue that this coefficient can also be interpreted as a measure of the strength of the spirit of capitalism, $b/a$.

Excess Smoothness

Consider the innovation to consumption growth in Equation (17). The standard deviation of consumption growth in the presence of the spirit of
capitalism is

$$\text{std}(dc_t) = \frac{r + b/a}{r + b/a + \rho} \sigma. \tag{18}$$

Excess smoothness hinges upon the volatility of consumption growth relative to that of income growth. We therefore define the excess smoothness ratio as

$$\lambda = \frac{\text{std}(dc_t)}{\text{std}(dy_t)} = \frac{r + b/a}{r + b/a + \rho} \tag{19}$$

Since $a > 0$, it is clear that $\lambda < 1$ as $\rho < 0$. Indeed, a simple calculation reveals that $\partial \lambda / \partial b > 0$ as $\rho < 0$. Therefore, if income is non-stationary the spirit of capitalism mitigates the volatility of consumption growth. This leads to

**Proposition 3.** The spirit of capitalism can explain the excess smoothness puzzle, by reducing the volatility of consumption growth when income is non-stationary.

The following figure plots the relationship between the excess smoothness ratio and the spirit of capitalism ($b$) when labor income is non-stationary. It is obvious from this figure that the spirit of capitalism can reduce the ratio of the standard deviation of consumption growth to the standard deviation of labor income growth, that is, the excess smoothness ratio. As $b$ increases, the ratio converges to 1. In the US aggregate data, the ratio is close to .58. Hence, the spirit of capitalism itself cannot resolve the excess smoothness puzzle in this simple model. In the next Section, we will show how the spirit of capitalism can help resolve this puzzle in two, more realistic, setups.

### 3. EXTENSIONS

In this section we enrich the basic model by incorporating two new features. First we introduce a more realistic income process. Second, we allow for interest rate risk. Both extensions enhance the ability of the spirit of capitalism to explain the excess smoothness puzzle.

#### 3.1. Extension 1: A More Realistic Labor Income Process

In this section, we consider a more realistic labor income process. The empirical literature often specifies labor income as a sum of two distinct components: one is a permanent (or very persistent) process, for example,
a unit-root process, and the other is a transitory process, for example, a white noise process.\footnote{See Pischke (1995).} Here we specify labor income as

\[ y_t = y_{1,t} + y_{2,t}. \]  

(20)

Thus

\[ dy_t = dy_{1,t} + dy_{2,t} \]

(21)

where we assume

\[
\begin{align*}
    dy_{1,t} &= \rho_1 \left( \frac{\mu_1}{\rho_1} - y_{1,t} \right) dt + \sigma_1 dz_{1,t} \\
    dy_{2,t} &= \rho_2 \left( \frac{\mu_2}{\rho_2} - y_{2,t} \right) dt + \rho_{12} \sigma_2 dz_{1,t} + \sqrt{1 - \rho_{12}^2} \sigma_2 dz_{2,t}
\end{align*}
\]

(22)

and \( z = (z_1, z_2)' \) is a standard Brownian motion in \( R^2 \). \( \rho_{12} \) is the instantaneous correlation coefficient between the two labor income components, and the parameters \( \rho_1 \) and \( \rho_2 \) measure the persistence of the two individual components of labor income, respectively. Without loss of generality, we assume that \( y_{1,t} \) is more persistent than \( y_{2,t} \), that is, \( \rho_1 < \rho_2 \).

Following the same procedure used in the benchmark model, we can derive the consumption function as follows

\[ c_t = \Omega(w_t, y_{1,t}, y_{2,t}) - P_{b>0} \]

(23)

where

\[
\begin{align*}
    \Omega(w_t, y_{1,t}, y_{2,t}) &= rw_t + \frac{r + b/a}{r + b/a + \rho_1} y_{1,t} + \frac{r + b/a}{r + b/a + \rho_2} y_{2,t} \\
    &\quad + \frac{1}{r + b/a + \rho_1} \frac{\mu_1}{\rho_1} + \frac{1}{r + b/a + \rho_2} \frac{\mu_2}{\rho_2} + \frac{1}{a} \frac{\theta - r - b/a}{r + b/a}
\end{align*}
\]

(24)

and

\[
\begin{align*}
    P_{b>0} &=\frac{r + b/a}{(r + b/a + \rho_1)^2} \frac{\sigma_1^2}{2} + \frac{r + b/a}{(r + b/a + \rho_2)^2} \frac{\sigma_2^2}{2} \\
    &\quad + 2a \rho_{12} \frac{r + b/a}{(r + b/a + \rho_1)(r + b/a + \rho_2)} \frac{\sigma_1 \sigma_2}{2}
\end{align*}
\]

(25)
The appropriate measure of human wealth in this case turns out to be

\[ h_t = \frac{1}{r + b/a + \rho_1} \left( y_{1,t} + \frac{\mu_1}{r + b/a} \right) + \frac{1}{r + b/a + \rho_2} \left( y_{2,t} + \frac{\mu_2}{r + b/a} \right) \]  

(26)

This allows the consumption function to be expressed as

\[ c_t = \frac{1}{a} \left( \theta - r - b/a \right) + (r + b/a)h_t + rw_t - P_{b>0} \]  

(27)

This is the same as Equation (14) in the simple model, except for two things: first, the more complicated expression in Equation (26) has been substituted Equation (13) for human wealth; second, the precautionary premium in Equation (25) now involves the variances and covariances of the two shocks, as well as their autoregressive parameters.

**Implications for Excess Sensitivity and Excess Smoothness**

We can now derive the expression for consumption growth as follows

\[ dc_t = rdw_t + a \left( \frac{r + b/a}{r + b/a + \rho_1} dy_{1,t} + \frac{r + b/a}{r + b/a + \rho_2} dy_{2,t} \right) \]

\[ = r \left[ \frac{1}{a} \left( \theta - r - b/a \right) + P_{b>0} \right] dt \]

\[ + \left[ \frac{b/a}{r + b/a + \rho_1} y_{1,t} + \frac{b/a}{r + b/a + \rho_2} y_{2,t} \right] dt \]

\[ + \left[ \frac{r + b/a}{r + b/a + \rho_1} \sigma_1 dz_{1,t} + \frac{r + b/a}{r + b/a + \rho_2} (\rho_{12} \sigma_2 dz_{1,t} + \sqrt{1 - \rho_{12}^2} \sigma_2 dz_{2,t}) \right] \]

(28)

As before, the spirit of capitalism can explain the excess sensitivity of consumption to income, that is, consumption growth can be predicted by expected income growth. Furthermore, the spirit of capitalism can also mitigate the excess smoothness puzzle. To see this, note that in this case

\[ std(dy_1) = std(dy_{1,t} + dy_{2,t}) = \sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2} \]  

(29)

and

\[ std(dc_1) = \sqrt{\left( \frac{r + b/a}{r + b/a + \rho_1} \sigma_1 + \frac{r + b/a}{r + b/a + \rho_2} \rho_{12} \sigma_2 \right)^2 + (1 - \rho_{12}^2)\sigma_2^2} \]  

(30)

\[ ^8 \text{In this case we assume } r + b/a + \rho_i > 0, \ i = 1, 2 \text{ to ensure that human wealth is well-defined.} \]
We can then write the excess smoothness ratio as follows

\[
\lambda = \sqrt{\frac{(r + b/a) \sigma_1 + r + b/a + \rho_1 \sigma_2}{\sigma_1^2 + \sigma_2^2 + 2 \rho_{12} \sigma_1 \sigma_2}} \quad (31)
\]

The properties of the income process affect excess smoothness of consumption in complicated ways. For example, if the two components have the same persistence \((\rho_1 = \rho_2)\), \(\lambda = a \frac{r + b/a}{r + b/a + \rho_1}\). This means that, as in the benchmark case, the spirit of capitalism does reduce the smoothness of consumption growth, but it cannot eliminate the excess smoothness puzzle because \(\lambda \geq 1\) when \(\rho_1 < 0\). However, if the two components have different degrees of persistence (without loss of generality, we suppose that \(\rho_1 < \rho_2\)), \(\lambda\) could be less than 1 if the spirit of capitalism is strong.

We can see this from a simple numerical example. For simplicity, assume that the two components are perfectly correlated \((\rho_{12} = 1)\). The excess smoothness ratio becomes

\[
\lambda = \frac{r + b/a}{r + b/a + \rho_1} \frac{\sigma_1 + \frac{r + b/a + \rho_1}{r + b/a + \rho_2} \sigma_2}{\sigma_1 + \sigma_2} \quad (32)
\]

Note that since \(\rho_1 < \rho_2\), \(\frac{\sigma_1 + \frac{r + b/a + \rho_1}{r + b/a + \rho_2} \sigma_2}{\sigma_1 + \sigma_2} < 1\). In the absence of a spirit of capitalism

\[
\lambda = \frac{r + b/a}{r + b/a + \rho_1} \frac{\sigma_1 + \frac{r + \rho_1}{r + \rho_2} \sigma_2}{\sigma_1 + \sigma_2} \quad (33)
\]

Setting \(a = 1, r = 0.04, \rho_1 = -0.02, \) and \(\rho_2 = 0\), we get \(\lambda > 1\) for any positive values of \(\sigma_1\) and \(\sigma_2\). Introducing the spirit of capitalism, \(\lambda\) becomes less than 1 for some plausible values of \(\sigma_1\) and \(\sigma_2\). For example, when \(\sigma_1 = 0.03\) and \(\sigma_2 = 0.02\), without the spirit of capitalism, \(\lambda = 1.6 > 1\), while with the spirit of capitalism \(\lambda = 0.82 < 1\). In sum, in this model where there are two distinct components in the income process, \(\lambda\) converges to \(\frac{\sigma_1 + \frac{r + \rho_1}{r + \rho_2} \sigma_2}{\sigma_1 + \sigma_2} < 1\) when the spirit of capitalism is strong enough. Hence, in some cases with our more realistic income process, the spirit of capitalism could help resolve the excess smoothness puzzle.

### 3.2. Extension 2: Interest Rate Risk

So far we have assumed consumers only face labor income risk and they smooth their consumption over time by borrowing or lending at a constant risk-free interest rate. However, in reality, consumers also face substantial risk for holding financial wealth that would largely affect their optimal consumption and saving decisions. In this section, we will explore the
implications of soc for precautionary saving and consumption dynamics in the model with both labor income risk and interest rate risk. In this case, the consumer’s budget constraint becomes

$$dw_t = (rw_t + y_t - c_t)dt + dω_t$$

where $ω_t$ is a Brownian motion with $E[dω] = 0$ and $var[dω] = ϖ^2$ and summarizes interest rate risk. Further, interest rate risk is instantaneously correlated with labor income risk, that is, $ρ_{wy} ≠ 0$.

In Appendix C we show that the solution to this problem is:

$$c_t = Ω(w_t, y_t) - P_{b>0},$$

where

$$Ω(w_t, y_t) = \frac{1}{a} \left[ \frac{θ - r - b/a}{r + b/a} + \frac{r + b/a}{r + b/a + ρ} y_t + rw_t + \frac{1}{r + b/a + ρ} μ \right]$$

and the precautionary saving premium is

$$P_{b>0} = a \frac{r + b/a}{(r + b/a + ρ)^2} \frac{σ^2}{2} + a^2 (r + b/a) \frac{ϖ^2}{2} + a^2 \frac{r + b/a}{r + b/a + ρ} ρ_{wy} σ ϖ.$$ (37)

Implications for Excess Sensitivity and Excess Smoothness
Setting $b = 0$, the growth of consumption is

$$dc_t = r \left[ \frac{θ - r}{a} + P_{b=0} \right] dt + \frac{r}{r + ρ} σ dz_t + rdω_t,$$ (38)

while with the spirit of capitalism, it becomes

$$dc_t = r \left[ \frac{1}{a} \left( \frac{θ - r + b/a}{r + b/a} + P_{b>0} \right) \right] dt + \frac{b/a}{r + b/a + ρ} E_t dy_t$$

+ \frac{r + b/a}{r + b/a + ρ} σ dz_t + rdω_t. \quad (39)

Hence, in the first case, the excess smoothness ratio is

$$\lambda = \frac{std(dc_t)}{std(dy_t)} = \sqrt{\left( \frac{r}{r + ρ} \right)^2 + \frac{r}{r + ρ} \left( \frac{σ}{σ} \right)^2 + 2ρ_{wy} \frac{r^2 ϖ}{r + ρ σ}}; \quad (40)$$

and in the second case, the ratio is

$$\lambda = \sqrt{\left( \frac{r + b/a}{r + b/a + ρ} \right)^2 + \frac{r}{r + ρ} \left( \frac{σ}{σ} \right)^2 + 2ρ_{wy} \frac{(r + b/a)r ϖ}{r + b/a + ρ σ}}. \quad (41)$$
Note that since $\rho_{wy} \in [-1, 1], \lambda \in \left[ \frac{r+b/a}{r+b/a+p} - r \frac{\sigma}{\sigma}, \frac{r+b/a}{r+b/a+p} + r \frac{\sigma}{\sigma} \right]$. Hence, when $\rho < 0$, both negative correlation between labor income risk and interest rate risk and the spirit of capitalism reduce the excess smoothness ratio. As documented in Campbell and Viceira (2002), in the US data (CRSP data on the NYSE value-weighted stock return relative to the Treasury bill rate), the correlations between labor income and stock returns are positive for all education groups (0.328 for the group with no high school education, 0.371 for the group with high school education, and 0.516 for the group with college education). However, the stock returns used in their study are not equivalent to the stochastic process for the interest rate used in this paper. We haven’t modeled the stochastic process for the interest rate explicitly, so it is difficult to find the empirical counterpart of this process. Theoretically, the correlation could be any value between $-1$ and $1$. Therefore, for given $r$ and $a$, incorporating SOC could lower the ratio $\lambda$ to a value less than 1 in the presence of interest rate risk and thus resolve the excess smoothness puzzle.

4. CONCLUSION

Recent research in economics has shown that Weber (1948) notion of the spirit of capitalism has found important implications for a range of economic phenomena. In Luo, Smith and Zou (2009) we showed how it could theoretically explain the two core empirical puzzles of consumption theory, excess sensitivity and excess smoothness. However, the example used in that paper — with CARA-like utility and an AR(1) income process — provided a simple explanation for excess sensitivity. However, it had difficulty explaining the quantitative magnitude of excess smoothness: the model predicted that the excess smoothness ratio should not fall below one, while in fact it is much smaller than one. In other words, the spirit of capitalism by itself does not provide a plausible explanation for excess smoothness. In this paper we enquire whether the spirit of capitalism can explain excess smoothness in a richer economic environment. The answer is in the affirmative. With the introduction of two realistic features — one an income process with temporary and permanent components, the other a random interest rate — the model can predict the magnitude of excess smoothness fairly well.
The derivation of the Euler equation in discrete-time follows Obstfeld and Rogoff (1996, p. 745).

The Lagrangian for the discrete-time problem is

\[ L(c_t, \lambda_t) = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \theta \Delta t} \right)^t E_t \{ U(c_t, w_t) \Delta t + \lambda_t [w_t(1 + r \Delta t) + y_t \Delta t - c_t \Delta t - w_{t+\Delta t}] \}. \]

The first-order conditions are

\[ U_c(c_t, w_t) = \lambda_t \] \hspace{1cm} (A.2)

\[ \lambda_t = \frac{1}{1 + \theta \Delta t} E_t[(1 + r \Delta t)\lambda_{t+\Delta t} + U_w(c_{t+\Delta t}, w_{t+\Delta t}) \Delta t] \] \hspace{1cm} (A.3)

Combining Equations (A.2) and (A.3) yields the Euler equation

\[ U_c(c_t, w_t) = \frac{1}{1 + \theta \Delta t} E_t[(1 + r \Delta t)\lambda_{t+\Delta t} + U_w(c_{t+\Delta t}, w_{t+\Delta t}) \Delta t] \] \hspace{1cm} (A.4)

To derive Proposition 1 we employ the limiting argument pioneered by Grossman and Shiller (1982), and more recently employed by Bakshi and Chen (1996a, 1996b) and Smith (2001).

First, assume that the optimal consumption policy is a discrete-time diffusion:

\[ \Delta c_t = c_{t+\Delta t} - c_t = \mu_{c,t} \Delta t + \sigma_{c,t} \Delta z_{c,t} \] \hspace{1cm} (A.5)

Similarly, express the equilibrium growth in wealth as

\[ \Delta w_t = \mu_{w,t} \Delta t. \] \hspace{1cm} (A.6)

Now take a second-order Taylor series of the Euler equation (A.4) around \( \Delta t = 0 \), \( c_{t+\Delta t} = c_t \), and \( w_{t+\Delta t} = w_t \). This leads to

\[ 0 \approx E_t \left[ U_{cc}(c_t, w_t)(c_{t+\Delta t} - c_t) + U_{cw}(c_t, w_t)(w_{t+\Delta t} - w_t) + \frac{1}{2} U_{ccc}(c_t, w_t)(c_{t+\Delta t} - c_t)^2 \right. \\
\left. + (r - \theta)U_c(c_t, w_t) \Delta t + U_w(c_t, w_t) \Delta t. \right] \] \hspace{1cm} (A.7)

Using Equations (A.2) and (A.3), applying the Ito multiplication rules, and rearranging leads to Equation (5) in the text.
Derivation of the Consumption Function in the CARA Example

The derivation is a straightforward application of the methods in Merton (1971) and Wang (2006).

Define the value function as $J(w_t, y_t)$. The Bellman equation for this problem is then

$$0 = \max_c -e^{-ac_t-bw_t} - \theta J + J_w [rw_t + y_t - c_t] + J_y \rho \left( \frac{\mu}{\rho} - y_t \right) + J_{yy} \frac{\sigma^2}{2}. \tag{B.1}$$

Performing the indicated optimization yields the first-order condition

$$e^{-ac_t-bw_t} = J_w. \tag{B.2}$$

Substitute Equation (B.2) back into Equation (B.1) to arrive at the partial differential equation

$$0 = -J_w a + \theta J + J_w \left( rw_t + y_t + \ln J_w + bw_t \right) + J_y \rho \left( \frac{\mu}{\rho} - y_t \right) + J_{yy} \frac{\sigma^2}{2}. \tag{B.3}$$

Conjecture that the value function is of the form

$$J(w_t, y_t) = -e^{-\alpha_0 - \alpha_1 w_t - \alpha_2 y}, \tag{B.4}$$

where $\alpha_0$, $\alpha_1$, and $\alpha_2$ are constants to be determined. Using this conjecture, Equation (B.3) reduces to

$$0 = -\frac{1}{a} \theta + \frac{\alpha_0 + (\alpha_1 - b)w_t + \alpha_2 y}{a} + \frac{\alpha_2}{\alpha_1} \rho \left( \frac{\mu}{\rho} - y_t \right) + \frac{\alpha_2^2 \sigma^2}{\alpha_1}. \tag{B.5}$$

Collecting terms, the constants turn out to be

$$\alpha_1 = ra + b \tag{B.6}$$

$$\alpha_2 = \frac{\alpha a_1}{\alpha_1 + a \rho} \tag{B.7}$$

$$\alpha_0 = \frac{\theta a}{ra + b} - 1 + \frac{a^2}{a(r + \rho) + b \mu} - \frac{ra + b}{[a(r + \rho) + b]^2} \frac{\sigma^2}{2}. \tag{B.8}$$

Substituting these back into the first-order condition (B.2) yields the consumption function in Equations (9), (10), and (11) of the text.

The value function must also satisfy the transversality condition

$$\lim_{t \to \infty} E e^{-\theta t} J(w_t, y_t) = 0. \tag{B.9}$$
Some tedious algebra reveals that a sufficient condition for this to be satisfied is that the effective rate of interest be positive, \( r + b/a > 0 \).

**APPENDIX C**

**Derivation of the Consumption Function with Interest Rate Risk**

The derivation is similar to that in Appendix B. The Bellman equation for this problem is

\[
0 = \max_{c_t} \left[ -\frac{\exp(-ac_t - bw_t)}{a} - \theta J + J_w(rw_t + yt - c_t) \right] + J_{ww} \frac{\sigma^2}{2} + J_y \rho \left( \frac{\mu}{\rho} - yt \right) + J_{yy} \frac{\sigma^2}{2}, \tag{C.1}
\]

The first order condition implies that

\[
\exp(-ac_t - bw_t) = J_w \tag{C.2}
\]

Substituting it back into Equation (C.1) yields

\[
0 = -J_w \frac{a}{a} - \theta J + J_w \left( rw_t + yt + \frac{\ln J_w + bw_t}{a} \right) + J_{ww} \frac{\sigma^2}{2} + J_y \rho \left( \frac{\mu}{\rho} - yt \right) + J_{yy} \frac{\sigma^2}{2} + J_{wy} \rho \sigma \omega. \tag{C.3}
\]

Guess that the value function takes the form

\[
J(w_t, y_t) = -\frac{\exp(-\alpha_0 - \alpha_1 w_t - \alpha_2 y_t)}{\alpha_1}, \tag{C.4}
\]

where \( \alpha_0, \alpha_1, \) and \( \alpha_2 \) are undetermined coefficients. Using this conjecture, we have

\[
\begin{align*}
J_w &= \exp(-\alpha_0 - \alpha_1 w_t - \alpha_2 y_t), \\
J_{ww} &= -\alpha_1 J_w, \\
J_y &= \frac{\alpha_2}{\alpha_1} J_w, \\
J_{yy} &= -\frac{\alpha_2^2}{\alpha_1} J_w, \\
J_{wy} &= -\frac{\alpha_2}{\alpha_1} J_w, \\
J &= -\frac{1}{\alpha_1} J_w.
\end{align*} \tag{C.5}
\]
and

\[ 0 = -1 + \frac{\theta}{\alpha_1} + \left[ r w_t + y_t + \left( -\alpha_0 - \alpha_1 w_t - \alpha_2 y_t \right) + bw_t \right] - \alpha_1 \frac{\sigma^2}{2} + \alpha_2 \rho \left( \frac{\mu - y_t}{\rho} \right) - \alpha_2 \rho_w \sigma \omega. \]  

(C.6)

Collecting terms yields

\[ 0 = \left[ -1 + \frac{\theta}{\alpha_1} - \frac{\alpha_0}{\alpha_1} - \frac{\alpha_2}{\alpha_1} \frac{\mu}{\sigma^2} + \frac{\alpha_2}{\alpha_1} \frac{\sigma^2}{2} - \alpha_2 \rho_w \sigma \omega \right] + \left[ r + \frac{b}{\alpha_1} w_t + \left[ 1 - \frac{\alpha_2}{\alpha_1} - \frac{\alpha_2}{\sigma} \right] y_t. \right. \]  

(C.7)

Matching the terms yields

\[ \alpha_1 = ra + b \]

\[ \alpha_2 = \frac{a \alpha_1}{\alpha_1 + \rho a} \]

\[ \alpha_0 = -1 + a \frac{\theta}{\alpha_1} - a \frac{\alpha_1 \sigma^2}{2} + a \frac{\alpha_2}{\alpha_1} \frac{\mu}{\sigma^2} - a \frac{\alpha_2}{\alpha_1} \frac{\sigma^2}{2} + a \alpha_2 \rho_w \sigma \omega \]

\[ = -1 + a \frac{b \theta}{ra + b} + a^2 \frac{\mu}{(r + \rho) a + b} - a^3 \frac{ra + b}{(r + \rho) a + b} \frac{\sigma^2}{2} - a (ra + b) \frac{\omega^2}{2} + a^2 \frac{ra + b}{(r + \rho) a + b} \rho_w \sigma \omega. \]  

(C.8)

Hence, we have

\[ c_t = -\frac{\ln J_y + bw_t}{a} \]

\[ = \frac{\alpha_2 + (\alpha_1 - b) w_t + \alpha_2 y_t}{a} \]

\[ = \Omega(w_t, y_t) - P_{b>0}. \]

where \( \Omega(w_t, y_t) \) and \( P_{b>0} \) are defined in Equations (36) and (37) in the text.

REFERENCES


